# AUTOMATIC LAYOUT OF METRO MAPS USING MULTICRITERIA OPTIMISATION

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### Abstract

This thesis is concerned with the automatic layout of metro maps using a hill climbing multicriteria optimiser. Metro maps are a widely used tool for the visualisation of the interconnecting services on a public transport network, allowing the user of the network to quickly and easily plan their route between stations. The difficulty of manually drawing these maps means that their use has been restricted and it is difficult to apply the metro map metaphor to other application areas beyond public transport networks.

This thesis introduces a method for drawing metro maps based on a graph model using a hill climbing multicriteria optimisation method. This method combines a number of criteria for positioning nodes (stations) taking into account aesthetic properties such as the angular resolution of edges, the octilinearity of edges, the length of edges and the straightness of lines. Another set of criteria handle the labelling of station names by considering the number of occlusions that the labels make and the positioning of the labels relative to other labels. A number of clustering and partitioning methods are also used in response to particular local minima in the search space.

The metro map layout method is demonstrated for a number of real-world examples, which are then evaluated empirically. This evaluation compares metro maps drawn with our method against metro maps currently in use as well as geographic maps of the metro network. This tests the hypothoses that maps drawn using this method are preferred by users and that they can be better for route-finding tasks in terms of the accuracy and time taken to complete those tasks.

This thesis concludes with a discussion describing how the metro map layout method can be applied to other application areas such as the visualisation of project plans and website navigation maps. Also described are details of ongoing and future research.

## Dedication

Dedicated to the memory of my grandad, Les, and my grandpa, Geoff, whose inspiration and encouragement when I was young will always be remembered.

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### Chapter 1

### Introduction

Metro maps can be seen worldwide and have possibly become one of the most memorised cartographic items in the world [Dow05, Gar94, Ove03]. Ever since construction of the first railways began in the early 19th century, there has been a need to map the networks. This was particularly the case with metro networks, where people needed to plan short journeys across a city or metropolitan area. Before long, the metro networks were rapidly expanding and the traditional topographic maps were quickly becoming cluttered and increasingly difficult to read  $\bigstar$  Not justified/referenced  $\bigstar$ .

#### 1.1 Metro Maps: A Brief History of Design

Early diagrammatic maps started to appear at the start of the 20th century. In 1900, a poster to advertise the ferry service between Harwich in England and the Hook of Holland was produced by a Dutchman by the name of Henrik Willem Mesdag [Wil47]. These maps were usually seen in advertising and promotional posters where the aim of using a simplified map of the network might have been to emphasise its ease of use. Most diagrammatic maps around this time showed only small networks or sections of larger networks.

The London Underground diagram [TfL05], designed by Harry Beck and first published in 1933 [Gar94] (Figure 1.2), marked a significant departure from the more traditional geographic maps and built on the ideas introduced by earlier simple diagrammatic

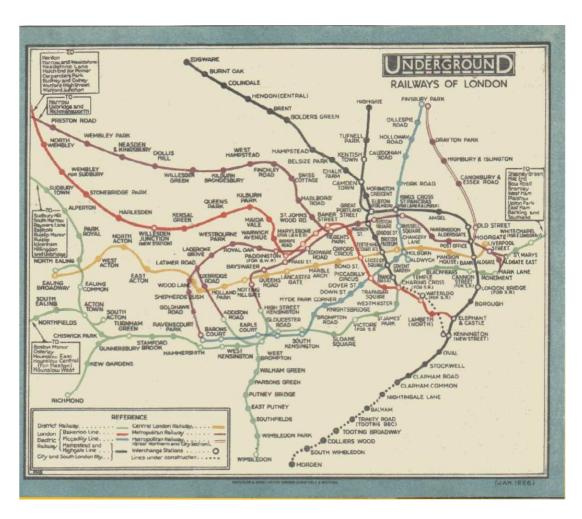


Figure 1.1: London Underground map published in 1926, designed by Fred Stingemore.

maps. Prior to this, geographic maps of the London Underground were used such as the example designed by Fred Stingemore and published in 1926 (Figure 1.1). Beck introduced more formalised design rules which have been followed right through to this day [Rob05], as can be seen in the 2005 map designed by Clockwork (Figure 1.3).

The diagram works by straightening meandering lines with line segments drawn either horizontally, vertically or diagonally at 45° and by using a non-linear scale so that the central area of the diagram is shown at a larger scale than the extremities. The effect is to produce a diagram that proved to be extremely clear and concise and has even been embraced as an iconic image of London. Following on from the success of Beck's version of the London Underground map, most metro networks now have a schematic map using a similar concept. Schematic maps have also been adapted for use



Figure 1.2: London Underground map published in 1933, designed by Harry Beck.

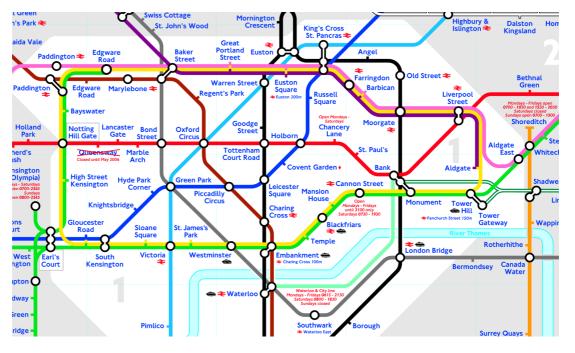


Figure 1.3: Extract from the London Underground map published in 2005 by Clockwork. Reproduced with permission from Transport for London.

with other public transport networks such as overground railways and bus routes. As a result of the prevalence of such maps many people, particularly in the developed world, are familiar with them, and are confident in their use.

At first glance, it might appear that metro maps are simple to design. However, this is seldom the case, and it often requires an experienced cartographer with considerable skill and design knowledge to be able to produce effective maps \( \Delta \) Not justified/referenced \( \Delta \). There are many aspects which together contribute to produce an effective design: the position of stations and the lines between stations, the width of lines; the symbols used to represent stations (circles, dots or ticks are most often used, but not exclusively so); the font and size of text used for labelling; the amount of geographic context and metadata to include (such as roads, rivers or coastlines); and the size, shape and resolution of the medium being used to display the maps. Even what may appear to be trivial changes—for example, slightly increasing the size of labels—can drastically alter the overall appearance and effectiveness of a map. The challenge faced by metro map designers is to balance these issues so that the map is as easy to use as possible by people travelling on the metro system.

#### 1.2 Motivation

The motivation for our research comes from the apparent difficulty in drawing metro maps. We are keen to explore effective methods of automatically drawing metro maps or sections of metro maps which can help map designers create alternative (and possibly improved) designs of existing maps and to provide targetted maps for travellers giving enhanced route information.

We believe that the metro map as a conceptual metaphor has value as a design form beyond its current use for navigation of public transport networks  $\star$  justification?  $\star$  and can be applied to other application areas such as for the visualisation of project plans or website navigation. However, if these application areas are to be realised, some way for metro maps to be more easily produced is necessary. This thesis ultimately describes how we have implemented a novel method for the layout of metro maps,

empirically evaluated the layout and then described how it can be applied to other application areas.

#### 1.3 Contributions of this Research

The main contributions of this research are:

- A novel method for the layout of metro maps which uses a hill climbing multicriteria optimiser that combines the following elements:
  - the movement of individual nodes to more optimal locations based on a set of six node movement criteria which take into account such features as the length of edges, the angular resolution of edges around a node, edge crossings, the straightness of lines and the regularity of angles of edges;
  - the identification of clusters by finding overlength edges and partitioning the graph and the movement of these clusters using the same set of criteria as for individual nodes;
  - the repositioning of labels in more optimal locations based on a set of seven labelling criteria which incorporate label occlusions, label position, the consistency of label position, the proximity of labels to other nodes and the distance of a label from the node it belongs to.
- An empirical evaluation that provides statistical evidence to support our hypotheses that automatically-drawn metro maps are better for route planning tasks than geographical maps.
- The adaptation with examples of our metro map layout method to novel application areas where metro maps have not previously been used, particularly that of the visualisation of project plans and website navigation.

#### 1.4 Thesis Outline

The structure of this thesis is as follows:

- Chapter 1 (this chapter) introduces the metro map as a tool for visualisation and outlines our motivation and contributions of this research.
- Chapter 2 provides detail on the areas that this research draws upon, such as graph theory, graph drawing, cartography, map labelling and schematic diagrams. Also given is a comprehensive analysis of the characteristic features of metro maps which draws upon the historical evolution of the metro map as a design form.
- Chapter 3 describes the process by which we draw metro maps using hill climbing multicriteria optimisation. It details the process involved in selecting optimum positions for nodes including the various criteria and rules that we have implemented. The chapter also describes the criteria that we developed in order to position station labels.
- Chapter 4 addresses the problem of moving many nodes in a graph at the same time by presenting methods in which clusters of nodes can be identified; we also show how the graph can be partitioned.
- Chapter 5 describes how the method described in Chapters 3 and 4 was implemented.

  We explain how we determined effective criteria weightings as well as provide an analysis of particular local minima.
- Chapter 6 provides a set of results generated using our metro map layout method.

  The chapter also contains a discussion of the method used with reference to a worked example.
- Chapter 7 describes how we evaluated our results using an empirical evaluation. The design, conduct and analysis of results are also detailed.
- Chapter 8 describes the modifications to our method that would be required for two application areas which apply the metro map metaphor to diagrams other than of public transport networks.
- Chapter 9 summarises our conclusions and presents directions for further research.

#### 1.5 Publications

Four peer-reviewed publications have resulted from this research:

- Jonathan M. Stott and Peter Rodgers. Metro map layout using multicriteria optimization. In *Proceedings of the 8th International Conference on Information Visualisation (IV04)*, pp. 355-362. IEEE, July 2004 [SR04].
- Remo A. Burkhard, Michael Meier, Peter Rodgers, Matthias T. J. Smis, and Jonathan Stott. Knowledge visualization: a comparative study between project tube maps and gantt charts. In K. Tochtermann and H. Maurer, editors, Proceedings of the 5th International Conference on Knowledge Management (I-KNOW 2005), pp. 388-395. Know-Center, Austria, June 2005 [BMR+05].
- Jonathan M. Stott, Peter Rodgers, Remo A. Burkhard, Michael Meier, and Matthias
   T. J. Smis. Automatic layout of project plans using a metro map metaphor.
   In Proceedings of the 9th International Conference on Information Visualisation (IV05), pp. 203-206. IEEE Computer Society, July 2005 [SRB+05].
- Jonathan M. Stott and Peter Rodgers. Automatic metro map design techniques. In *Proceedings of the 22nd International Cartographic Conference*, p. 10. International Cartographic Association, July 2005 [SR05].

### Chapter 2

## Background

This chapter introduces the various background material which is relevant to this area of research. It begins with a discussion of the characteristics of metro maps, followed by a description of a number of different methods for graph drawing and laying out schematic diagrams. Aesthetics of graphs is very important, so a number of papers that consider graph aesthetics from an empirical point of view are also described. Map labelling and the applications of the metro map metaphor to the visualisation of abstract concepts are also covered. The relevance of this background material to this thesis is highlighted throughout.

#### 2.1 Definitions

The following definitions of terminology are used throughout this thesis.

Cartography The drawing of maps.

Diagram An illustration of the appearance of an object, retaining shape and relations of various parts, without being an exact representation of the object. A diagram can also be used to provide a concrete visualisation of an abstract concept. Examples of diagrams include Gantt charts, Euler diagrams, flowcharts and electronic circuit diagrams. ★ Figures to illustrate these examples? ★ these examples? ★

- **Embedding** An embedding is a particular instance of some structure, X, in another structure, Y, such that there exists a mapping  $f: X \to Y$  that preserves the structure of X. In the context of two-dimensional graph drawing, an embedding is a mapping of nodes and edges to particular points and lines in a plane.
- **Generalisation** A cartographic process involving the selection, simplification and combination of information when drawing a map.
- **Graph** A graph is a structure that expresses the connectivity of a set of objects. More formally, a graph, G, is a pair G = (V, E) where V is a set of elements representing objects (nodes) and E is a set of unordered pairs representing the connectivity of nodes (edges). The endpoints of an edge, e, are the nodes belonging to that edge. The degree,  $\rho(v)$ , of a node, v is the number of edges that have v as an endpoint. A  $planar\ graph$  is one which can be embedded in a plane such that no two edges intersect.

Graph drawing The process of finding acceptable embeddings of graphs.

- Map A representation of a geographic feature such that each part of the feature is drawn corresponding to some fixed scale or projection.
- Metro map A schematic that represents a transport network. As a minimum, a metro map always consists of the following elements: *stations* which show places where passengers can board or alight from services, *metro lines* which show the routes taken by services, and *labels* which show the name of stations.
- Metro map metaphor A conceptual metaphor that allows the understanding of one idea in terms of a metro map. More specifically, the application of the design characteristics of metro maps to concepts other than the illustration of transport networks. The metro map metaphor is defined in more detail in Section 2.3.
- **Schematic** A diagram that represents the appearance of an object using abstract, graphic symbols.

**Topography** The surface features of a place or region.

**Topology** The study of the nature of space. In terms of the embedding of graphs or drawing of diagrams or schematics, topology is the consideration of the structure of the embedding or drawing and the way that constituent features (e.g. nodes and edges) are interrelated or arranged.

An important distinction must be made between the use of "map", "diagram" and "schematic". This is often confusing: for example, a "metro map" is a schematic and not a map in the geographic sense. "Diagram" and "schematic" are also often used interchangeably or even as the noun "schematic diagram". To avoid confusion, where possible, we have used the appropriate phrase when taking into account the above definitions and have highlighted places where the phrasiology in common use deviates from these defintions.

#### 2.2 Characteristics of Metro Maps

Metro map diagrams have been around for almost as long as the networks themselves. Section 1.1 introduced some of the earliest examples of the metro map as opposed to previous geographic maps.

The requirements of metro maps evolved during the early years of public transport networks in urban areas such as London. Some of the requirements would be altruistic and be intended to help the passenger solve route-planning tasks easily. A passenger might need a map for a number of different route planning tasks including "how do I get from station A to station B"; "how many stops is it before I have to get off"; "where do I have to change trains"; "what route do I need to take after I have changed trains"; "what are the alternative routes to get to my destination" [Dow05]. Commercial pressure may also have driven the change of diagrams. For example, diagrams that appear clean and simple give the impression that a transport network is simple to use; straight lines give an impression of faster and more direct services than might actually be the case; distortion of scale and equal spacing of stations on a line can give the impression of shorter journeys (particularly from outlying regions of urban networks).

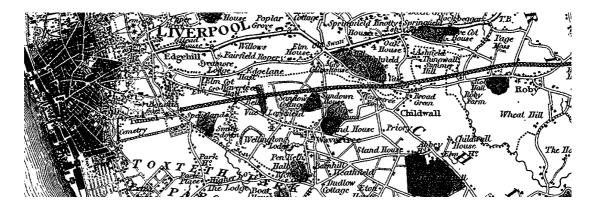


Figure 2.1: Extract of part of an 1829 map of the Liverpool to Manchester Railway, drawn by G. Hennet [Hen29].

One of the important features of metro map design is that the topology of the map should remain invariant, but the designer is afforded freedom to modify the location of stations and path of lines [Mor96b].

The next section describes how the schematic style for metro maps evolved.

#### 2.2.1 Evolution of the Schematic Design for Metro Maps

It is important to understand the evolution of the schematic style for metro maps in order to appreciate the features which characterise the schematics over equivalent maps [AH06, Dow05, Gar94, Ove03]. The railway revolution in Britain during the mid 19th century saw the construction of nearly 10,000km of new railways and in many cases the British Government's Board of Ordnance (the precursor of the Ordnance Survey) could not keep up with surveying the new tracks. The very earliest railways were very often just etched onto existing geographic maps such as the example of the Liverpool to Manchester Railway shown in a map from 1829 (Figure 2.1) [Hen29]. Unfortunately, the burgeoning networks soon resulted in a kind of map spaghetti and it became impossible to provide useful maps using the etching method. The very nature of densely populated areas implies a very dense railway network with stations much closer together than in more sparsely populated rural areas. Specialised maps soon appeared which excluded almost all surface topography. Examples of these minimalist maps include one of London's Metropolitan Railway in 1874 (Figure 2.2) and another of Berlin's railways (Figure 2.3).

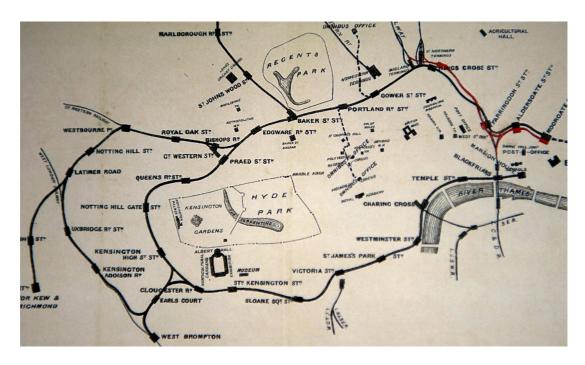


Figure 2.2: Map of London's Metropolitan Railway, published in 1874. This map illustrates an early example of the removal of much of the surface topography to produce a clearer map.

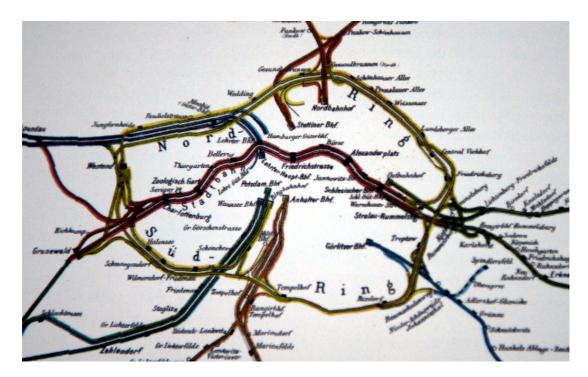


Figure 2.3: Late nineteenth-century map of Berlin's railways. This map has all surface topography removed to enhance the clarity of the railway.

However, these maps did not last very long in cities such as London, Berlin and New York as the networks continued to expand and even these "simpler" maps soon became cluttered. The need arose to make more space in the maps, which could only be achieved by distorting the scale. This spatial distortion was also essential with maps covering both rural areas (where stations could be miles apart) and urban areas (where stations could be just a few hundred yards apart). If the map were to remain to scale and still be large enough for stations in the urban area to still be legible, the map would have to be excessively large. An early example of a schematic using distorted scale in this manner is that of the Metropolitan Railway, published in 1896 (Figure 2.4) where stations in the top-left corner of the map are actually much further apart than stations in the bottom-right corner. In a talk at the London Transport Museum on 6 May 2003, K. Garland, the author of Mr. Beck's Underground Map, described this selective distortion of scale as the "convex effect". Harry Beck used this device in a large diagram of a complete network in his 1933 London Underground map (Figure 1.2): George Dow predates this in his 1929 LNER diagrams (Figure 2.5).

The next step in the evolution of the schematic form of the metro map came with the introduction of the practice of simplifying lines by removing most of the deviations a line makes and replacing it with a straight line on the diagram. This process is also known as shape generalisation. Early examples of schematics which began to use this feature date back to the mid-1920s. This feature was adopted by George Dow in his London and North Eastern Railway schematics of 1929 (Figure 2.5) and positively seized upon by Beck for his seminal 1933 London Underground schematic (Figure 1.2). The great step made by both Dow and Beck was to make use of parallel lines and lines of common angle; Beck extended this concept to use lines entirely composed of horizontal, vertical and 45° diagonal components. Between 1929 and 1933, Dow was to produce a number of other "Dowagrams" of rail networks in south-east England, all conforming to the same design guidelines.

After Beck's 1933 London Underground schematic, many others appeared following similar forms. However, there was very little innovation and new ideas tended to be restricted to using different symbols for stations or other minor cosmetic design tweaks.

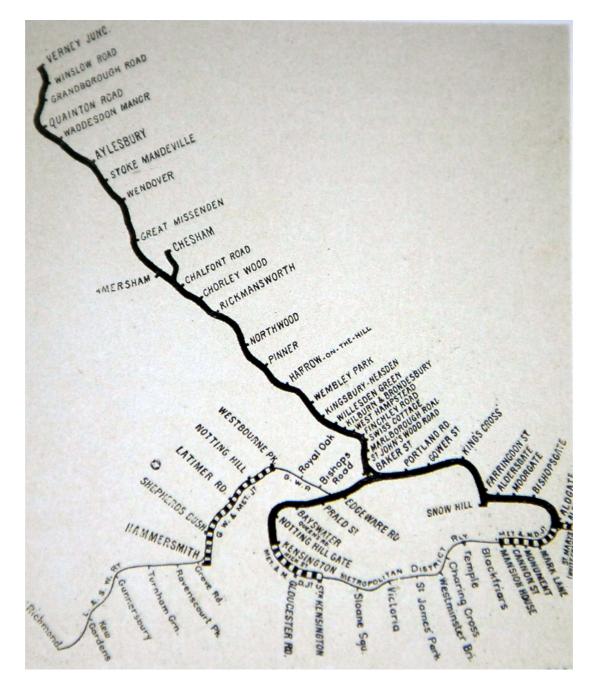


Figure 2.4: Schematic of the London Metropolitan Railway network, published in 1896. This map has large distortions in the scale—stations in the top-left corner of the map are up to six miles apart while some stations in the bottom-right corner are just a few hundred yards apart.

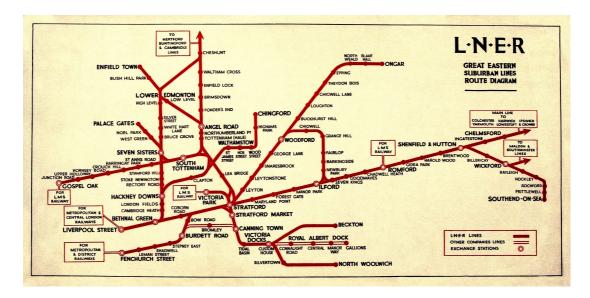


Figure 2.5: Schematic of the London and North Eastern Railway Great Eastern suburban network, published in 1929 and drawn by George Dow. Courtesy of Andrew Dow.

It is surely testament to the forms introduced by both Dow and Beck that we still use schematics with the same features more than 70 years later.

More recently, Transport for London have produced a series of diagrams centred around individual bus stops [TfL07]. These *spider* or *octopus* diagrams feature a schematic showing all the bus routes that pass by a particular bus stop [Mor96a]. An example of a spider diagram showing bus routes from Monument in Central London is shown in Figure 2.6. The Transport for London spider diagrams include an area in the centre showing a street map which allows the location of individual bus stops to be shown in relation to the road network. As there will be many hundreds or thousands of bus stops, manual generation of these diagrams would be time consuming, so some automation of the process is necessary.

#### 2.2.2 Metro Lines

Depiction of metro lines on metro maps involves a number of characteristic features. Probably the most prominent feature of many metro maps is the frequent use of lines of common angle (angle generalisation). For example, the use of solely horizontal, vertical and 45° diagonals causes lines to be parallel with other lines. This tends to be very

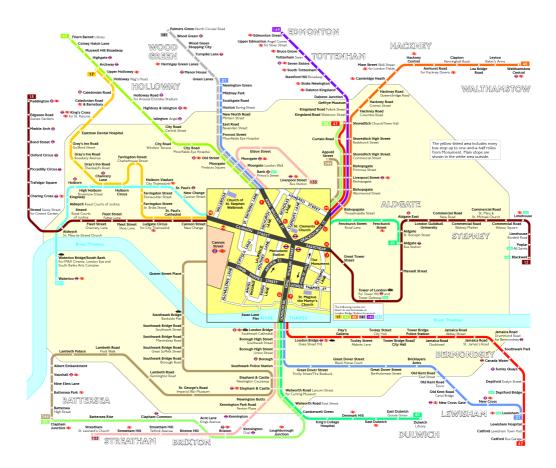


Figure 2.6: Spider diagram showing bus routes from Monument in Central London [TfL07].

effective owing to the eight-fold symmetry of the horizontal, vertical and diagonal lines. The introduction of lines of common angle also forces the use of shape generalisation where the meandering path of a line has to be straightened to fit the common angles in use. Another prominent feature is the convex effect of increased scale at the centre of the map with decreasing scale towards the extremities of the map (scale generalisation) coupled with regular spacing of stations along a line. Metro lines are also usually distinguished on the map by using different colours. The colours of metro lines should normally be chosen such that lines that run together or intersect have great enough contrast in order to be able to tell them apart and to emphasise the continuation of a line through a junction or station.

Examples of the features of metro lines can be seen in six map excerpts in Figure 2.7. Examples (a), (b), (c) and (d) all show shape, scale and angle generalisation with regular spacing between stations and horizontal, vertical and 45° diagonal lines. Example (e) is similar except that it uses 35° diagonals instead of 45° diagonals. Example (f) is unusual in that it does not exhibit much use of angle or shape generalisation and the spacing between stations is much less regular than in the other examples. The last example is much closer to the geographic layout of the metro map than any of the other five examples.

### 2.2.3 Labelling

Labelling of station names is clearly an essential part of metro maps. It also introduces many challenges to make sure that labels are clear and unambiguous. Many factors contribute to the way in which labels are applied to the maps and the labels also have a significant contribution to the way in which the maps are drawn. The font and size of lettering has a direct impact on the space required for each label: the size is usually dictated by the use that is intended for the map (for example, a large-print version of a map for visually impaired users or a version for a poster in a station would likely require

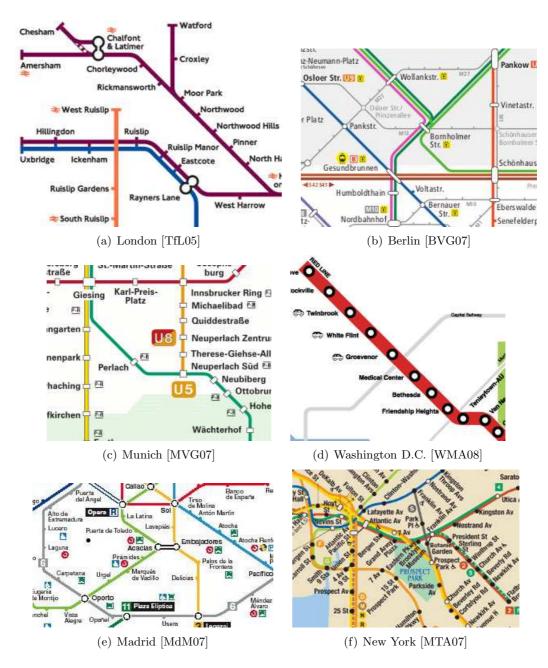


Figure 2.7: Examples of metro lines on published maps. Examples (a), (b), (c) and (d) illustrate horizontal, vertical and 45° diagonal lines with even spacing between stations. Example (e) shows a map that uses 35° diagonals and example (f) uses a geographic layout.

larger font sizes). Most maps use sans serif fonts with proper case lettering<sup>1</sup>.  $\bigstar$  What about Figure 2.3 (Berlin map)?  $\bigstar$ 

A common feature of station labels is that they tend to be placed along a single side of a line when that line is straight for any length. A possible justification for this is that the labels then form a straight list along one side of the line, making it easier for the user to scan the labels to find the station that they require. Examples (a), (c) and (d) in Figure 2.8 are good examples of this feature.

Station labels tend to be predominantly horizontal<sup>2</sup>, even to the point where a designer would prefer to use vertical or diagonal lines to avoid the need to use diagonal labels [Ros04]. Horizontal lines cause a problem when using horizontal labels as there usually is not enough space for all the labels to be on the same side of the line (at least without making the line excessively long). Diagonal labels are quite often used along horizontal lines where they are needed, but occasionally, the labels will alternate from one side of the line to the other, as in the labels for Ickenham, Hillingdon and Uxbridge in Figure 2.7(a). An example of diagonal labelling can be seen in Figure 2.8(f).

The biggest challenge when placing labels is to ensure that there is enough space for them without any cases where a label is drawn on top of (occludes) a line or a station. It is also desirable that the label is positioned so that it is unambiguously associated to just a single station. The example in Figure 2.9(f) shows a map where labels have been allowed to be drawn on top of lines and some of the labels could even be claimed to be ambiguous.

#### 2.2.4 Topographic Metadata

Topographic metadata are surface features such as roads, rivers, landmarks, parks and shorelines that do not serve any purpose for showing the interconnections of the public transport network. Having said this, metadata can perform the task of contexualising the transport network, for example to show which stations are on either side of a river.

<sup>&</sup>lt;sup>1</sup>Interestingly, proper case lettering on maps appeared quite late on, perhaps as late as 1936 in an LNER map.

 $<sup>^{2}</sup>$ The first map to use only horizontal station labels was probably a 1908 map of the District Line in London.

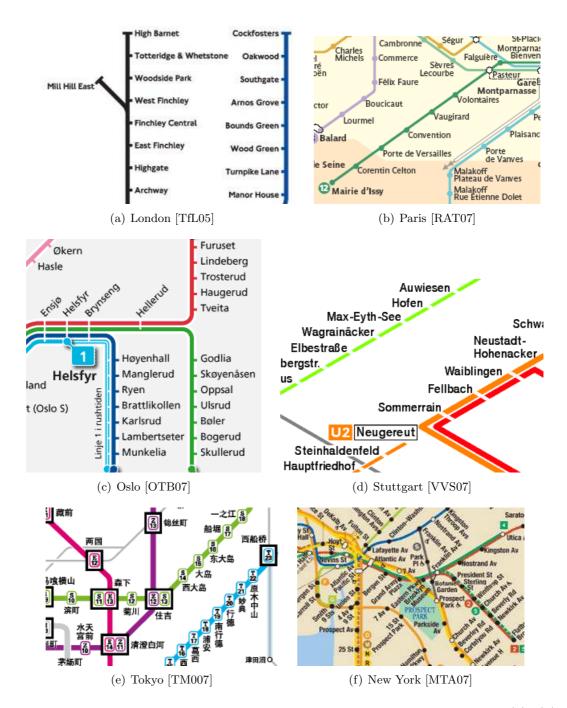


Figure 2.8: Examples of metro map labelling on published maps. Examples (a), (b), (c) and (d) all show horizontal station labels consistently on the same side of a line. Example (e) places information about the station inside the device used to represent the station and example (f) uses diagonal labels of various orientations.

Topographic metadata is very commonly removed entirely or at least nearly completely from most published metro maps today and those maps that do feature metadata tend to be of fairly small networks. The general trend is for less metadata to be present as the size and complexity of the transport network increases. Examples of maps with no metadata at all are illustrated in Figure 2.9, and include Madrid (a), Mexico City (b) and Stockholm (c).

Nearly all metro maps exhibit some form of topographic metadata, but in most cases it is usually not very prominent on the map. Coastal maps or maps of places with a major river nearly always show the river on the map (such as examples (d), (e) and (f) in Figure 2.9). In many cases the metadata is also styled to take on the characteristics of the map itself, including using the same lines of common angle and shape generalisation of coastlines. However, the inclusion of geographic metadata like coastlines and rivers can make the task of applying scale generalisation to the map difficult without distorting the metadata features beyond recognition.

★ What topographic metadata might there have been on the maps that neglect to use it? (Madrid, Mexico, Stockholm) ★

### 2.2.5 Stations, Termini and Interchanges

The representation on a map of a station is also an important characteristic of metro maps. It does, however, have less of an impact on the overall layout of the map. There are a number of different ways of representing stations, often using devices such as dots, rings, ticks, squares and diamonds. In many cases, terminus stations and interchange stations are indicated using different symbols. In the examples from London (Figure 2.10(a)) and Barcelona (Figure 2.10(b)), ticks are used against the lines to indicate stations. Interchanges can very often be quite complex, such as those at Euston or King's Cross St. Pancras in London (Figure 2.11(a)) where the interchange is indicated with a ring. The rings are quite often bridged together which implicitly indicates a connection between lines. The example from Hamburg (Figure 2.11(b)) uses rectangles of differing sizes to join together lines where interchange is possible, most notably for the *Hauptbahnhof*. Examples of different devices used to represent stations include dots

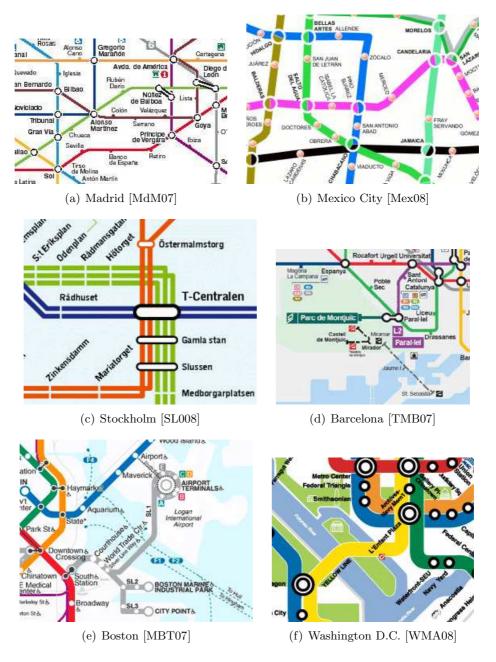


Figure 2.9: Examples of metro map topographic metadata on published maps. Examples (a), (b) and (c) have no topographic metadata at all. Examples (d), (e) and (f) show examples of topographic metadata including coastline, parks, rivers and landmarks.

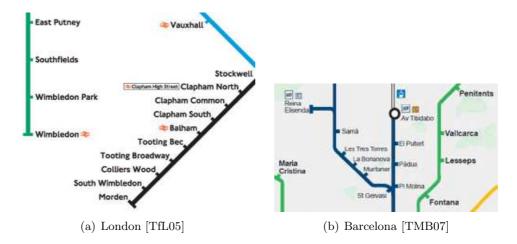


Figure 2.10: Examples of metro maps using ticks to represent stations on published maps.

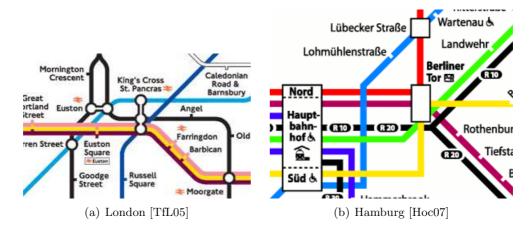


Figure 2.11: Examples of devices used for distinguishing interchange stations on published metro maps.

within the line (Lisbon, Figure 2.12(a)) and slices taken through the line (Stockholm, Figure 2.12(b)).

## 2.3 The Metro Map Metaphor

The previous section has highlighted a significant variety of characteristics for drawing metro maps. However, it is necessary to identify those characteristics that form the stereotypical view of how a metro map should appear. This will allow us to define more clearly what we mean when we talk of the *metro map metaphor*. While there will

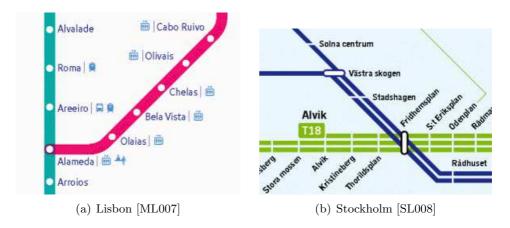


Figure 2.12: Examples of metro maps using different devices for representing stations on published maps.

obviously be variation in how particular metro map designs appear, the following list of minimum requirements brings together the most commonly observed characteristics:

- Metro lines should be drawn using line segments that are either horizontal, vertical or at 45° diagonal. A metro line should be drawn using a single colour that allows the distinction of that line from others.
- Stations should be indicated on metro lines using a common symbol. Stations should be spaced evenly along metro lines—this implicitly requires that the scale of a metro map be undefined.
- Labelling of station names should be carried out using horizontal text. The possible locations for labels should be limited: labels on horizontal metro lines should be positioned immediately above or below the station symbol; labels on vertical lines should be immediately to the left or right of the station symbol; labels on diagonal lines should be above and to the left or right or below and to the left or right of the station label depending on the orientation of the line.

There is no requirement that a metro map include any **topographic metadata**. This is because the decision as to whether to include topographic metadata on a particular map depends on the context of that map meaning that there is no single rule that could fit all metro maps.

The selection of these characteristics comes about from our observations of realworld metro maps spanning the last eighty years. They are therefore based on historical convention and stereotypical perception rather than any formally validated rules determined empirically or otherwise.

This definition of the metro map metaphor drives the selection of the criteria for our metro map layout method described in Chapters 3 and 4 as well as the application of the metaphor to other application areas as described in Chapter 8.

### 2.4 Graphs and Graph Drawing

★ This section is far too brief. Needs an extensive introduction, with a clear distinction made between graphs and graph drawings ★

 $\star$  If complexity is to be given for algorithms, then all the relevant algorithms need it.  $\star$ 

Graph drawing is the process of taking a graph, G, and applying a graph drawing algorithm to produce a particular embedding of G [dBETT94, dBETT99, KW01]. This visualisation process is deeply rooted in the human interpretation of the graph as a structure and as such, the quality of the aesthetics of a particular drawing of a graph are very important. The readability of a graph is the key to allowing users to interpret the diagram accurately and clearly.

However, graph drawing is a generally difficult problem. Many aspects are NP-complete [GJ83, MO85]  $\star$  WHICH ASPECTS?  $\star$ , and by its very nature, the assessment of the quality of a particular drawing of a graph is entirely subjective. Therefore, acceptable heuristics are generally required to find good drawings of graphs in reasonable time, and measures of the aesthetic quality (either explicit or implicit) are required in order to be more objective about a particular drawing.

 $\bigstar$  For many of the algorithms described, we are not shown any examples of their output  $\bigstar$ 

★ The section on empirical studies seems inappropriately placed in a section whose purpose is to introduce (and hence define) graph drawing aesthetics—move it to the front of this section! ★

### 2.4.1 Force-Directed Graph Drawing

★ Frustratingly brief! Where are repulsive forces applied? What does "non-adjacent" mean? What is the meaning of "equilibrium" in this context? ★

Natural physical models appear to offer an excellent basis for a number of heuristics for the layout of graphs. Examples include a mechanical model of springs and electrical forces and a physical model based on simulated annealing. Section 2.8.1 introduces the previous work on the automatic layout of metro maps which uses a force directed approach.

 $\star$  explain relation to metro maps (would even out spacing between nodes, space out lines, etc)  $\star$ 

The spring embedder [Ead84] uses a model of springs and electrical forces. In this case, nodes are represented as rings and edges as springs attached to the rings. The force of the spring causes connected nodes to attract each other (the force is calculated in terms of the logarithm of the distance between the nodes). A repulsive force is also applied using an inverse square law. This ensures that non-adjacent nodes are kept apart. The forces should allow for a reasonable separation of the nodes while edges are kept to be of roughly similar length.  $\bigstar$  Need to describe spring embedder in more detail if we are to reference it later  $\bigstar$ 

The spring embedder typically starts with a random embedding of the graph and a number of iterations are applied until some equilibrium is reached. For larger graphs, a greater number of iterations is typically required. Figure 2.13 and Figure 2.14 show how the spring embedder lays out the complete graph with six nodes,  $K_6$ .

Fruchterman and Rheingold [FR94] extend Eades' algorithm by basing the force calculations on an optimal distance between connected nodes, k, which depends on the number of nodes and the area of the drawing area.  $\bigstar$  More careful description here?  $\bigstar$ 

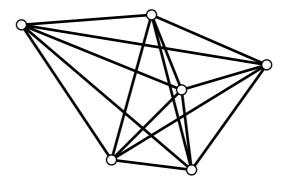


Figure 2.13: Randomised embedding of the complete graph with six nodes,  $K_6$ .

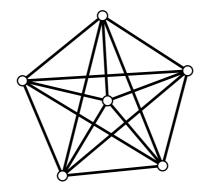


Figure 2.14: Embedding of  $K_6$  using the basic spring embedder.

Kamada and Kawai show another approach to force directed graph drawing [KK89]. Their approach uses the relationship between the graph theoretic distance and the geometric distance between nodes to produce good embeddings. The algorithm works particularly well for symmetric graphs and is relatively good at minimising edge crossings. They use Floyd's shortest paths algorithm [Flo62] to find the minimum graph theoretical distances which runs in  $O(n^3)$  time (faster shortest path algorithms are available); the other parts of their algorithm run in either linear or constant time.

There are many other force directed algorithms, including an algorithm based on magnetic springs [SM94, SM95], an adaptive approach (which is generally faster  $\bigstar$  overall or just for some specific graphs?  $\bigstar$ ) [FLM95] and a method that ensures that edge crossings properties  $\bigstar$  Wht does properties mean? Remove 'properties'?  $\bigstar$  are preserved [Ber99].

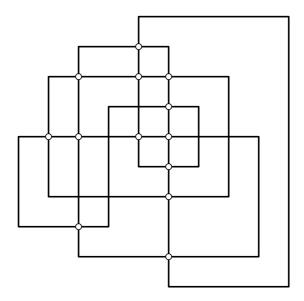


Figure 2.15: An orthogonal embedding of a graph.

### 2.4.2 Orthogonal Graph Drawing

### $\bigstar$ Why is this section here - explain relation to metro maps $\bigstar$

Orthogonal graph drawing is mainly concerned with drawing graphs where edges are restricted to horizontal and vertical lines [EFK00, EFK01]. In many approaches, polylines (lines with more than one straight segment) are allowed where a line with a single segment cannot be drawn orthogonally (see Figure 2.15). As such, it is particularly interesting with regards to metro map layout where a predominantly orthogonal embedding is required. Orthogonal diagrams also have applications outside of graph drawing, particularly in the field of VLSI (very large scale integration) design [Ger99] and in diagrams used for information systems design such as entity relationship diagrams. Many orthogonal graph drawing algorithms are called dynamic—that is they construct an embedding from the bottom up rather than considering the whole graph in a top-down approach  $\bigstar$  what does this mean? Doesn't make sense out of context  $\bigstar$ . Orthogonal diagrams that allow a relaxation permitting some non-orthogonal edges are called quasi-orthogonal diagrams [KM98, GM98].

A number of approaches are based on the Kandinsky model which dictates that a graph should be drawn with nodes represented as finite-sized rectangles and edges should be drawn using only horizontal and vertical components. Tamassia [Tam87] presents

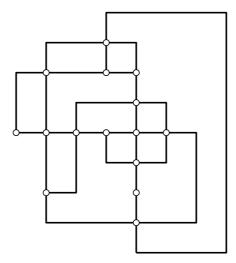


Figure 2.16: A planar orthogonal embedding of a graph.

an algorithm for embedding a planar graph on a grid in such a way as to minimise the number of bends. (A planar graph is a graph which can be drawn without edge crossings as shown in Figure 2.16). Highly connected graphs are not catered for—the maximum degree of a node for an orthogonal drawing using this algorithm is four. (The degree of a node is the number of edges which are incident to the node. For example, the degree of every node in Figure 2.15 is four.) The algorithm runs in  $O(n^2 \log n)$  time where n is the number of nodes in the graph. The algorithm is based on network flow techniques where the flow is related to the number of bends  $\star$  what are bends? ref to fig 2.15 for an example  $\star$  on an edge and the capacity of nodes (how many more edges could be added incident to the node without exceeding the maximum of four incident edges)  $\bigstar$  description of algorithm too vague  $\bigstar$ . The aim of the algorithm is to find the minimum cost flow for the graph which should be that with the fewest edge bends. Later in the paper, an extension to k-gonal graphs is given. A k-gonal graph is one where edges are formed of polylines with segments at multiples of 180/k degrees. An orthogonal graph is therefore one where k=2. This is a relatively simple extension of their algorithm and one which would prove useful for metro map layout as metro maps tend to be 4-gonal. (The maximum degree graph that Tamassia's algorithm handles is therefore equal to 2k.)

Fößmeier and Kaufmann [FK95] extend Tamassia's algorithm to handle graphs with degree greater than 2k. They do this be extending Tamassia's representation. They show an algorithm which first creates a nearly-orthogonal representation in  $O(n^2 \log n)$  time. The nearly-orthogonal  $\bigstar$  meaning what?  $\bigstar$  representation is then used to create an embedding on the grid using a similar compaction algorithm  $\bigstar$  what algorithm?  $\bigstar$  to Tamassia. To be able to draw high degree nodes  $\bigstar$  first mention of node drawing!  $\bigstar$ , large nodes are represented by 8s smaller nodes (where s is the length of the side of the node). Larger nodes are preferred over increasing the number of bends (smaller nodes would be possible but only by increasing the number of bends in the graph).  $\bigstar$  Example needed - not clear  $\bigstar$ 

Another algorithm is given by Papakostas and Tollis in [PT97]. This algorithm also considers graphs with nodes of degree greater than four, but is more general as it also takes into account simple non-planar graphs  $\star$  definition  $\star$ . They use three algorithms to size the nodes (taking into account the degree of the node), number and group the nodes into a particular order for processing and the placement of the nodes such that the increased size of the nodes is taken into account. Nodes are paired together so that edges between pairs of nodes can share a row or column  $\star$  of what?  $\star$ . The algorithm is able to produce an embedding of a graph in O(e) time where e is the number of edges in the graph. Maximum bounds on the size of the graph can be given in terms of the number of edges and there will not be more than e bends as each edge has at most one bend. However, the resulting embeddings tend to have a relatively high number of edge crossings and a planar embedding for a plane graph may not necessarily be found.  $\star$  too vague to be useful  $\star$ 

Brandes et al. [BEKW02] use a non-dynamic algorithm for drawing graphs where a previous embedding (possibly a rough sketch) is known. The Kandinsky model is again used. The graph is planarised by inserting dummy nodes at edge crossings. They use the concept of "readability"—the total number of bends in the graph—and "stability"—by how much the angles of the final embedding differ from the sketch. Their algorithm is claimed  $\bigstar$  any reason not to believe the claim?  $\bigstar$  to run in  $O(n^2 \log n)$  time. They claim  $\bigstar$  claim again  $\bigstar$  to maintain stability further by attaching nodes at the

extremity of the graph to a bounding box surrounding the graph. After the algorithm has completed, the dummy nodes are removed and a compaction step is used to minimise the area of the graph. This paper is of relevance as it is particularly interested in embedding schematic diagrams  $\star$  papers aren't interested in stuff!  $\star$ .

An interesting paper by Six, Kakoulis and Tollis [SKT00] deals with post-processing of orthogonal embeddings of graphs. This is done by considering a number of cases where improvements could be made, such as by straightening out U-turns, removing superfluous bends, moving degree-two nodes to improve their placement, removing self-crossings of edges, dealing with stranded nodes and reducing excess area. In general, significant reductions of the area of the graph, the number of edge bends and crossings and the length of edges are produced, resulting in more compact and aesthetically pleasing embeddings.  $\star$  This paragraph is without context in that the "problems" of these algorithms which this paper may or may not fix have not been discussed  $\star$ 

### 2.4.3 Graph Drawing Aesthetics

# ★ Check references of Purchase's work—is there anything else that could be highlighted here? ★

Many graph drawing algorithms make common assumptions that certain aesthetic criteria of the graph will have a detrimental effect on the readability of the graph. These criteria include edge crossings, edge length, angular resolution of incident edges, proximity of one node or edge to another node or edge and node/edge occlusion [DM90]. Little work has been done in order to quantify and justify these aesthetic criteria.  $\star$  Has it?  $\star$ 

The first attempt at an empirical study into graph aesthetics was by Purchase, Cohen and James [PCJ95]. In this work they hypothesised that an increase in either the number of edge bends or edge crossings would lead to a decrease in the understandability of a graph and that an increase in the local symmetry in a graph would increase the understandability. In order to quantify the understandability of a graph, the performance of a person when answering a number of questions was considered. These

questions involved such tasks as finding the shortest path between two given nodes, and finding the minimum number of nodes or edges that need to be removed in order to disconnect a graph. They conclude by saying that attempting to minimise edge crossings and edge bends has a significant positive effect on the aesthetic quality of a graph layout. They were unable to make any conclusive assessment of the effectiveness of the local symmetry hypothesis.

A later paper by Ware, Purchase, Colpoys and McGill [WPCM02] contributes to this work by measuring the aesthetic quality of graphs based on the time taken for the shortest path to be found in each graph. They consider five primary factors (continuity; number of edge crossings; average edge crossing angle; number of branches; shortest path length) as well as two additional factors (total geometric line length and total number of edge crossings). Their experiment tested 43 subjects. They conclude that the continuation of edges (where edges pass through nodes as straight as possible) contributes most to the quality of the graph when it is used for finding shortest paths. They also claim that their results show that the total number of edge crossings in the graph is less important than the number of edges that cross the shortest path. Finally, they observe that for very short paths with length less than three, the path is possibly perceived by the brain in approximately constant time.

### 2.4.4 Optimisation Methods for Drawing Graphs

- $\bigstar$  No introduction to multicriteria optimisation processes, which are referred to extensively in chapters 3 and 4.  $\bigstar$
- ★ Is there a more general definition of "optimisation"? Is there a bone fide defintion of "optimisation" when used in the context of drawing graphs? ★

A number of other graph drawing methods based on optimisation exist including a heuristic optimisation approach [Tun94], approaches using genetic algorithms [KMS91, BBS97, RSOR98, Tet98] and the use of simulated annealing [DH96]. Optimisation is a process whereby incremental improvements are made to a graph. Each successive iteration should produce a more optimal graph (in most cases—some algorithms occasionally allow less optimal graphs in order to avoid local minima). Optimisation methods which

never allow degradation in the graph are referred to as hill climbers and have previously been used for drawing graphs [RSORS99] and Euler diagrams [FRM03].

Tunkelang [Tun94] takes the approach of creating an aesthetic cost function which is then minimized using a local optimisation procedure. The algorithm is flexible due to the way in which the aesthetic cost function is modular  $\star$  What does this mean?  $\star$ . The cost function used by Tunkelang is based on three aesthetic criteria, namely: uniform edge lengths, even distribution of nodes and a minimal number of edge crossings. A naïve implementation of the cost function runs in  $O(e^2)$  time where e is the number of edges in the graph. This can be improved to a linear function by not recalculating the cost function from scratch each time. The local optimisation procedure involves examining the locality of a node to see whether a better position can be found. This algorithm is capable of producing acceptable embeddings for moderately sized graphs but struggles with large and dense graphs.  $\star$  Why does it struggle?  $\star$ 

# $\bigstar$ More detail of the mentioned algorithms. What has been done to improve their effectiveness? $\bigstar$

Simulated annealing is another approach to drawing graphs [KGJV83, Č85]. It is based on physical processes of the way that liquids cool into a crystalline form (annealing). This uses the analogy that the minimum energy state of the system (the crystal state) is equivalent to the minimum energy of a simulated annealing system. Davidson and Harel demonstrate an algorithm which is based on simulated annealing [DH96]. As with force directed drawing algorithms, they start with a random embedding of the graph and iteratively generate an improved embedding. The simulated annealing process produces an erratic improvement with some chance of subsequent iterations being apparently worse than the previous iteration. This has the advantage that it is possible to escape from local minima in the search space. Results are reasonably comparable to that of force directed methods, but the running time of the algorithm is generally poor, especially for large graphs (over sixty nodes).

# 2.5 Schematic Diagrams and Automatic Cartographic Generalisation

★ The notion that a map can only be a model, and that a model is a representation of reality would be useful. This section appears to relate scale and resolution to abstraction—this is not necessarily the case: there may be abstraction even if there is no change in scale. The abstraction in a model is related both to the purposes for which the model is required as well as the amount of space available for the representation of the model. ★

#### 2.5.1 Generalisation

Cartographic generalisation is the process of taking a map and modifying it by removing information whilst ensuring that enough detail remains to serve the purpose of the map. Generalisation in this context usually involves selecting the information to display, simplifying information or combining information from a number of features into a single feature. In cartography, this process is necessary for producing maps of differing scale where smaller scale maps will show much less information than larger scale maps to avoid the map becoming too cluttered and illegible. Figure 2.17 shows four map extracts centred on the same place (Canterbury, England), each with increasing scale. The top-left extract is at the smallest scale and shows street names on small roads in the city centre, but as the scale increases, first the street names, then the smaller roads themselves disappear. By the time the largest scale map is drawn, Canterbury is not even marked on the map.

The decision as to which features are removed when a map is generalised is by no means arbitrary. Take the example of the roads in Figure 2.17: on a large scale map it just is not possible to show all the minor roads because the resolution of the map does not allow this, and even if it was possible it would still be nearly impossible for a user of the map to distinguish these features visually. However, as the scale increases, we remove first the minor roads but leave the main roads and motorways, then many of the main roads are progressively removed. In other words, the order that features are

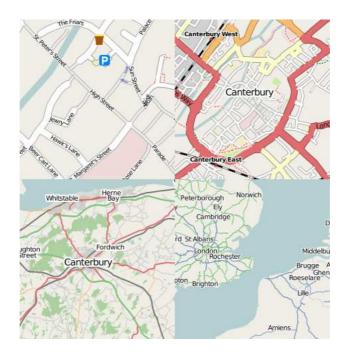


Figure 2.17: Examples of cartographic generalisation required for changing scale [OSM07]. All four map extracts are centred on the same place (Canterbury, England) but the increasing scale as you "zoom out" requires a generalisation of features in order to avoid the map becoming unreadable.

removed as scale increases is dependent on their relative importance. The same applies to the labels for place names: the towns of Whitstable, Herne Bay, Canterbury and Fordwich are less important than London and Rochester. In this case, importance can be determined by considering the population of a town or city or the grade (motorway, major, minor) of roads.

An excellent online example of generalisation is that of Google Maps [Goo07] where it is possible to view maps from a scale of roughly 1:357,000,000 right down to 1:2,700 at 18 different scales.

### 2.5.2 Automatic Cartographic Generalisation

The advent of computers provided a great resource for cartographers to automate many of the tedious processes involved in creating maps. This became essential when geograpical information systems began to appear [HCC98]. The process of selecting which features to show on a map of a particular scale has received a great deal of interest from researchers, including the publication of several books [BM91, MS92, MLW95].

★ What is in these books that means they should be cited?! ★ Generalisation can be broken down into a number of different areas depending on which features are being generalised. Line generalisation is one of the most significant areas (and the one most relevant to metro map layout), but other areas include settlement selection, where particular point features are selected for inclusion at a specific scale or combination, where irregular features (such as trees or houses) are combined into single contiguous features (such as forests or cities). It may also be necessary to generalise scale or to base the map on some other scale basis other than geographical area. A conceptual framework for automated map generalisation is given by Brassel and Weibel [BW88]. Cartograms are an example of this where the apparent size of a particular feature is proportional to a particular value associated with that area [Den99, Cam01]. An example of a cartogram would be a map of the world where the area of each country is proportional to some metric derived for that country, such as the gross domestic product or average life expectancy.

For metro map layout, it is important that all the information in the map is retained  $\bigstar$  As it is not clear what is meant by "the map" in this context it is unclear to the reader that this statement is really true  $\bigstar$ , so selection or combination are not relevant here. However, line generalisation forms a significant part of metro map layout and so we will concentrate on that in the next section.

### 2.5.3 Automatic Line Generalisation

- ★ No definition of what line generalisation is. It may be useful to clearly list the different types of generalisation, their definitions, and how they can be applied to features on a map in section 2,3,2. It is also not clear in this section whether the generalistaion methods are applicable to a map or a schematic diagram (or both)  $\bigstar$
- $\bigstar$  This section should start with the Douglas-Peucker algorithm and then introduce the other algorithms in terms of what they fo better and how they do it better.  $\bigstar$ 
  - ★ Are these methods algorithms or heuristics? ★

# $\bigstar$ Unintelligible unless you happen to know all the algorithms and what the different criteria are. $\bigstar$

Graph theory  $\star$  really graph theory here?  $\star$  is quite often used as a basis for generalisation [MB93, TR95] as roads or other line features can be represented as a set of edges connecting nodes. Approaches that use graph theory as a basis include those by Imai and Iri [II86a, II86b, II88] who consider the shortest path  $\star$  how?  $\star$  in a directed, acyclic graph representing the network to be simplified. The generalised lines are based on a subset of the points that make up the complex line. Their algorithm runs in  $O(n^3)$  time. Melkham and O'Rourke describe a similar algorithm which runs in  $O(n^2 \log n)$  time [MO88].  $\star$  Worth mentioning how they get the speed-up. Is there a trade-off based on the size of the graph?  $\star$  Other algorithms which can be applied to line generalisation in this way include those presented by Bose et al. [BCC+06], Chan and Chin [CC96], Chen and Daescu [CD03] and Toussain [Tou85]. Van Kreveld and Peschler describe how road network maps can be generalised using selection based on a number of geometric, typological and semantic requirements [vKP98, Wei96b].

In general, an approximation of a line must satisfy some kind of error criteria. A number of error criteria are used such as the parallel-strip (or infinite beam) criterion [CD03, ET94, Tou85], the tolerance zone criterion [BCD+02, CDH+05, II86a, II88, MO88] or the uniform metric [AV00, Goo95]. The criteria might also be attempting to preserve the distance between the two endpoints of the line [GNS07] or the area  $\star$  area of a line?!  $\star$  [BCC+06]. Line generalisation algorithms tend to either minimise the error criteria (the min- $\epsilon$  problem) or to ensure that the line contains the minimum number of segments, but that the error criteria do not exceed some tolerance (the min-# problem).

# ★ Don't you end up with the original line if you impose too strict a tolerance? ★

The Douglas-Peucker algorithm [DP73] is very often used as a basis for line generalisation heuristics. The algorithm works by taking a path between two points,  $P_0, P_1, \ldots, P_n$ , and recursively splitting the line at the point which is furthest from the direct line from  $P_0$  to  $P_n$ . An illustration of how the Douglas-Peucker algorithm

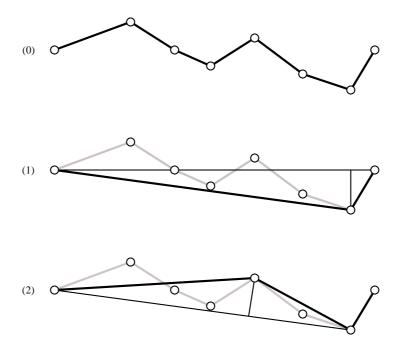


Figure 2.18: An example of line generalisation using the Douglas-Peucker algorithm [DP73]. The original line is recursively subdivided at the node which is furthest from a line between the two end-points until all nodes are within a certain error criterion.

works is given in Figure 2.18. A generalised path can be found in  $O(n \log n)$  time, but as this is a heuristic, it is not necessarily the most optimal solution [HS98]. Weibel provides details on a selection of spatial data generalisation algorithms [Wei96a]. The original Douglas-Peucker algorithm can occasionally introduce circumstances where the topology of the diagram is modified by inadvertent self-crossings. Saalfield modifies the Douglas-Peucker algorithm to ensure the finished simplifications are topologically consistent [Saa99].

### 2.5.4 Schematic Diagrams and Route Maps

Schematic route maps are used to depict a route between two points on a map. A common application of route maps is for planning driving directions between two locations. Route maps can depict these directions by massively exaggerating the scale of the start and end points to show the route through city roads and shortening long inter-city motorway sections. This normally involves a process of generalisation so that

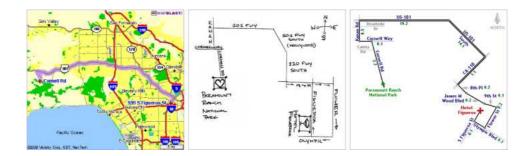


Figure 2.19: Three route maps rendered to show the same route [AS01]. The left-hand image uses a standard geographic map; the middle image is a schematic sketch while the right-hand image is a computer-generated schematic.

the essential topological information for following a route is preserved (such as the intersections and approximate direction), while at the same time exaggerating distances to make the schematic cleaner and easier to read [Ave02, Cab04]. An example of a schematic route map is shown in Figure 2.19. These route maps are particularly useful for route-planning applications and especially with today's modern in-car satellite navigation devices, where the aim is to communicate a route to the driver as quickly as possible.  $\bigstar$  Appears to imply that satnavs exaggerate distance—do they?  $\bigstar$ 

Agrawala and Stolte describe the LineDrive system to generalise and schematise 
★ What do generalise and schematise mean? ★ route maps using three types of generalisation [AS01]. The aspects of the map which are generalised are:

- Length of roads. Shorter roads are drawn longer and longer roads are drawn shorter such that routes involving roads with lengths that differ by several orders of magnitude can be drawn on a compact schematic map.
- Angle generalisation. Very acute angles at intersections are made more obtuse and roads are aligned to the horizontal or vertical axis.
- Shape generalisation. The exact meandering of a road is not important, so the shape of the road is straightened or smoothed out.

Special care is taken to ensure that the topology of the map is preserved in the schematic, so that intersections, turn direction and the overall route shape are all maintained. Avelar and Huber [AH01] show a similar method but model their route maps on the characteristics of public transport networks.

Another approach uses an algorithm for generalising the shapes of curves in a schematic diagram by eliminating unnecessary curves (in a similar way to the shape generalisation part used by Agrawala and Stolte) [BLR00]. Casakin *et al.* [CBKF01] provide a taxonomy of various aspects of the schematisation of route maps (particularly intersections), and use their taxonomy to provide an empirical assessment of schematised graphs. Yates and Humphreys [YH98] give a discussion of various aspects of schematic diagrams and show a prototype (which uses a heuristic provided as a sample applet in the Java 1.1.6 SDK). Cabello *et al.* [CdBvDvK01] presents a relatively efficient combinatorial algorithm for the generation of schematic maps which takes into account a number of requirements such as choosing the minimum separation of nodes and not moving nodes. Their algorithm runs in  $O(n \log^3 n)$  time. Later research by Cabello and van Kreveld [CvK02, CdBvK02] implements and evaluates this algorithm.

Elroi describes how the orientation of lines in simplified schematic diagrams can be restricted by placing paths on a grid [Elr88a, Elr88b]. Avelar and Müller present an algorithm for ensuring that a schematic map is topologically correct [AM00]. In schematic diagrams used for wiring diagrams or public transport networks it is important to be able to recognise easily the continuation of wires or lines as they pass through components or stations. Benkert et al. [BNUW07] consider the problem, where several lines or edges run through a node, of ensuring that the relative order remains the same as they do so. Their algorithm also attempts to minimise the number of edge crossings and runs in  $O(n^2)$  time.  $\bigstar$  Figure to illustrate this?  $\bigstar$ 

Lauther and Stübinger [LS01] present a demonstration of software which is capable of laying out schematic diagrams using a force-directed approach with the aim of visualising cable plans schematically.

★ How successful were these methods? Did you use any of their work in what you did? ★

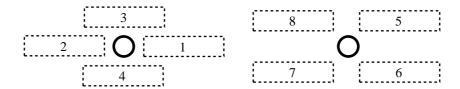


Figure 2.20: Search space for labelling.

### 2.6 Labelling

In many fields, such as cartography, labelling a map or diagram is an essential part of drawing the diagram. Typically, when labelling maps and diagrams, three types of labels are needed: node labels (which label point features, such as cities or stations on a railway), edge labels (which label such features as roads, railways or rivers) and area labels (for labelling such things as oceans or countries). In cartographic terms, these are point-, line- and area-feature labels. We are mainly concerned with the labelling of point features—in this case stations [WS07]. Map labelling can be a very tedious task, with cartographers only able to place labels at a rate of approximately 20 to 30 labels per hour [CJ90]. In particular, the problem of finding a labelling solution where no labels overlap and where no point features are overlapped by labels is NP-complete [KI88, MS91, FW91].

Point feature label placement is discussed in detail in a survey by Christensen, Marks and Shieber [CMS95].  $\star$  What did they say?!  $\star$  Typically, labels are placed in one of a finite number of positions (the labelling space) surrounding a point feature (or with a near infinite number of positions by sliding a label around the point feature as described in [vKSW98]). A typical labelling space is shown in Figure 2.20. An order of preference is usually specified as to which positions a label should take up (for example, one could prefer a label to appear to the right of a point feature as opposed to the left).  $\star$  In metro maps you actually might want to change the preference order of the labelling of a node (or indeed the positions available) depending on the direction of the metro line or where the stations appear  $\star$ 

Deciding how to place labels in the labelling space requires the use of an objective function as the position of each label will affect the quality of the labelling [Imh75].

Describing the objective function in terms of a number of rules is a common approach [AF84, FA87, Jon89, CJ90, WB91, DF92]. ★ More detail—examples at least! ★ Yoeli [Yoe72] uses three rules for determining the quality of a labelling: how much overlap there is between labels and other features of the map, a ranked order of preferred label positions, and the number of points left unlabelled.

There are many different algorithms that can be applied to labelling point features. The different categories of algorithm are discussed in the following sections.

### 2.6.1 Exhaustive Search Labelling Algorithms

These algorithms are based on ultimately performing a complete search of every possible label position [AF84, Nom87, FA87, Jon89, CJ90, DF92]. A backtracking methodology is used so that if a label cannot be placed, the search returns to the last successfully labelled point and tries a different location. However, exhaustive search is by its nature not ideally suited to situations with large numbers of point features.  $\bigstar$  What's large in this context? Is the average number of nodes in a metro map large??  $\bigstar$  Heuristics can be applied to reduce the size of the search space or to optimise the way in which the exhaustive search algorithm explores the search space [Kor88].

### 2.6.2 Greedy Search Labelling Algorithms

Exhaustive searches suffer from having too large a search space to explore using the backtracking method. A greedy search can improve on this from a time perspective by compromising on the quality of the labelling. Conflicts where labels would overlap can be resolved either by leaving out that label, leaving it in but making do with the overlap, or obtain the assistance of an expert user [Yoe72].

### 2.6.3 Gradient Descent Labelling Algorithms

Rather than attempting to label all the features in the map in a single go, it is possible to start with a (random) labelling and make incremental improvements (optimisations) by altering the positions of individual labels. This requires the computation of the objective function for all the labels in the map and an iterative process which calculates

the function for each potential label position in the label space for a particular label. The label is then moved to a new location which improves (minimises) the objective function.  $\star$  what objective function?  $\star$  The gradient descent method is quite effective  $\star$  reference to justify this statement  $\star$ , but problems with local minima can mean that a more optimal solution is not explored.

### 2.6.4 Force-Based Optimisation for Labelling

Rather than using a finite set of label positions, it is also possible to use an infinite set of positions and then use a modified gradient descent algorithm.  $\star$  How modified?  $\star$  The advantage in this case is that potential local minima can be avoided as there is a greater freedom of movement afforded for each label. Clearly the disadvantage is that the search space is now inifinitely large, so a heuristic is required to decide where labels can be positioned. Hirsch [Hir82] describes a method which uses the concept of an overlap vector which can be used to "force" the two labels apart. The vector in this case is analogous to a repulsive force of two objects in a physical system. It is also similar to the analogy used in force-directed graph drawing (Section 2.4.1 and [Ead84]). Two problems exist with methods using these force-based approaches: errors in calculating the overlap vectors can arise when the vectors have relatively large magnitudes  $\star$  What errors? explain!  $\star$ , and local minima in the search space can also become manifest.

### 2.6.5 Stochastic Search for Labelling

To handle the local minima cases when using a gradient descent algorithm, some element of stochasticity can be introduced. This means that each subsequent movement need not always result in an improvement. Simulated annealing is a common stochastic optimisation method [KGJV83, Č85] where the element of randomness of the system decreases in line with an annealing schedule. Simulated annealing is based on a model of a physical system undergoing cooling. The probability, P of a label being moved into a new position is given by  $P = e^{-\frac{\Delta E}{T}}$  where  $\Delta E$  is the change in the objective function and T is the temperature.  $\bigstar$  Simulated annealing is introduced earlier—reference back  $\bigstar$ 

### 2.6.6 Labelling with Linear Programming

Linear programming is a mathematical method which minimises or maximises a particular objective function by taking into account a set of constraints. Zoraster [Zor86, Zor90] implemented an algorithm for labelling point features of a map using 0-1 integer programming (ZOLP). Finding the optimal solution for ZOLP is NP-hard, so Zoraster incorporates a number of heuristics to improve results. Zoraster also describes a number of modifications required to mitigate the risk of ending up in local minima.

★ How good are the heuristics in practice? Is this method applicable to metro maps? ★

### 2.6.7 Labelling Graphs

### ★ Are these methods applicable to metro maps? If not why not? ★

While most of the research on labelling is concerned with cartographic maps, it can also be applied to labelling graphs. Kakoulis and Tollis [KT98] show a method for labelling graphical features specifically of graphs. Their approach firstly reduces the search space for labels by seeing where a label would overlap another node or edge. The subset of potential label positions is then reduced further by detecting overlapping labels. Their results show that their algorithm produces a more efficient labelling  $\bigstar$  by how  $much? \bigstar$  for orthogonal diagrams  $\bigstar$  no angled lines?  $\bigstar$  than previous attempts. They also say that the size of the labels is a significant factor in being able to produce a successful labelling of a graph  $\bigstar$  obvious? if not, why not?  $\bigstar$ .

## 2.7 Applications of the Metro Map Metaphor

While the majority of examples of diagrams using the metro map metaphor are of public transport networks, there are a number of existing applications that use the metaphor to illustrate other abstract concepts. Burkhard and Meier [BM04, BMR<sup>+</sup>05] show how the metro map metaphor can be applied to the visualisation of project plans (Figure 2.21). They created a large wall-mounted poster showing the plan for a project involving a number of groups of people with lines representing each group and stations

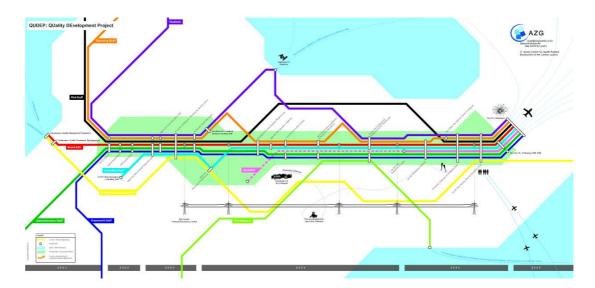


Figure 2.21: Representation of a project plan using a metro map metaphor  $[BM04, BMR^+05]$ .

representing milestones in the project. The poster was displayed in a prominent location in a communal area in the organisation. They found that people were more readily attracted to the poster and that interest in the project plan was increased. Most people surveyed found the metaphor to be easy to understand but said that they would prefer a more interactive version of the map to be available online. The map was hand-drawn  $\star$  and therefore useless!!!  $\star$  and it was noted that a static display would not be suitable for a project plan that frequently changed without some automation of the drawing process.

An "open source map" produced by the publisher O'Reilly in 2003 [ORe] illustrates the relationships between different computing technologies (Figure 2.22). The map shows each technology (such as "Perl", "UNIX" or "Java") as a separate line with stations representing publications by O'Reilly. Another map of cancer pathways was created by Hahn and Weinberg and designed by Claudia Bentley [HW02] (Figure 2.23). In this map, the progression of the biological steps involved in cancer are shown. As this is a progressive map, lines are augmented with the direction of the pathways and whether that step is activative or inhibitive.

Other examples of the use of the metro map metaphor include the visualisation of the trains of thought that run through a Ph.D. thesis [Nes04] (Figure 2.24), the

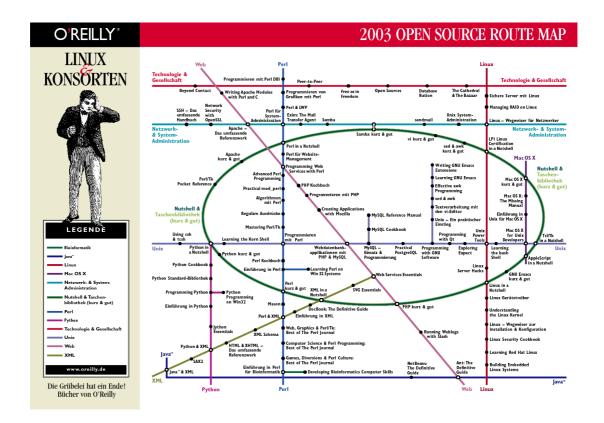


Figure 2.22: Open source route map [ORe].

organisation of web-based learning resources [BGH02] and the illustration of guided tours on the web [SGSK01]. Enterprise portals can also be visualised using the metro map metaphor [Zie04] where pages, functions and links in the portal are represented by stations and lines in a metro map.

The usefulness of the metro map as a metaphor is somewhat limited to simple examples by the time required to manually produce these maps. As such they are generally only useful for applications that do not change frequently. This limitation could be removed by quality methods for the automatic drawing of metro maps from abstract data.

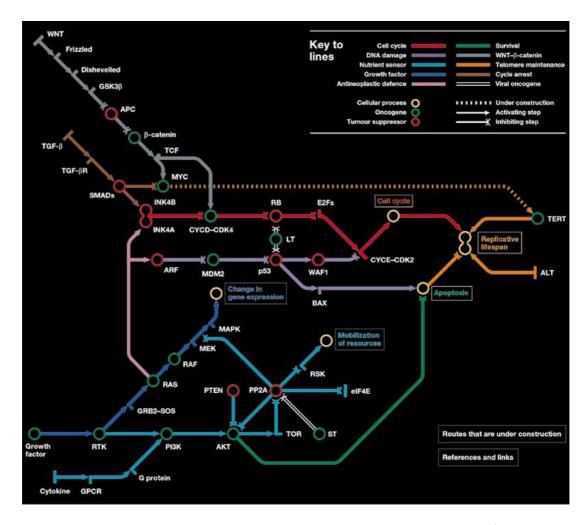


Figure 2.23: A subway map of cancer pathways by Hahn and Weinberg (designed by Claudia Bentley) [HW02].

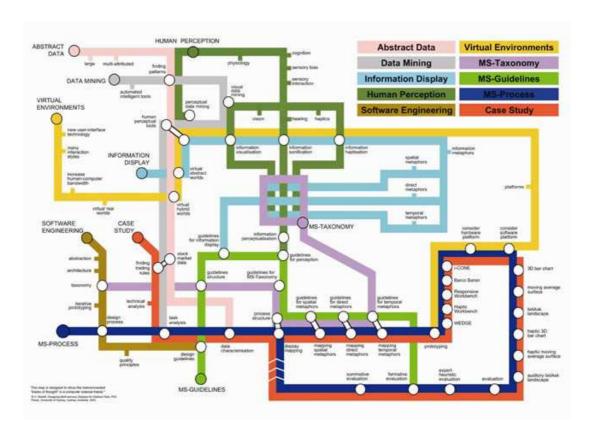


Figure 2.24: Ph.D. thesis map by Nesbitt showing abstract trains of thought that run through the thesis. [Nes04].  $\bigstar$  Unreadable - figure needs to be larger  $\bigstar$ 

### 2.8 Existing Automatic Metro Map Layout Research

This section introduces and describes existing research that directly tackles the metro map layout problem. Existing approaches include the use of a force-based method (Section 2.8.1), a mixed-integer programming method (Section 2.8.2), path simplification (Section 2.8.3) and centrality-based scaling (Section 2.8.4).

★ None of this research has performed any user tests to validate the quality of their results ★

### 2.8.1 Force Directed Metro Map Layout

Hong et al. use a force-directed approach to laying out metro maps [HMdN04, HMdN06]. In their paper, they put forward five different layout methods which use combinations of spring-based algorithms. The algorithms used are the GEM algorithm [FLM95], the PrEd algorithm [Ber99] and a magnetic spring algorithm [SM95]. Some of their methods include a preprocessing step which involves simplifying the graph by removing all nodes with only two incident edges.  $\bigstar$  What's the advantage of this? Reduces a number of segments to a simple line but there needs to be room to put in the intermediate stations  $\bigstar$  They also include a final step of labelling the graph which uses a combinatorial approach to try to achieve a labelling with as few overlaps as possible.

They analyse their methods with regards to a set of four criteria:

- that each line should be drawn as straight as possible  $\bigstar$  Using what criteria?  $\bigstar$
- that there should be as few edge crossings as possible
- that labels should not overlap
- that edges should be drawn orthogonally or at 45°

As their methods were progressively refined to the application, they produced increasingly better graphs. However, the results were hampered as the geographic topology of the maps was not considered at all. This is because they started with a random embedding of the graph, as is common with many other force-directed graph drawing algorithms. They argue that people using metro systems are not concerned with the real topology of the system, but this is clearly not the case, especially when one is using a "northbound" train and the map shows the line going from top to bottom (it seems reasonable to assume that people perceive north as being "up" and south as being "down").

The results they present generally satisfy the four criteria. Figure 2.25 shows their result of drawing the Sydney CityRail map and Figure 2.26 shows their London Underground result. Their best method is their fifth method which seems to produce the most aesthetically pleasing graph. However, the graphs suffer from a number of flaws, most notably the irregular spacing of nodes—some are very close together (so close that you can not discern any edge between them) and others are very far apart. Also, they do not consider drawing whole train lines; each pair of connected nodes are connected by only a single edge when in many real-world examples, many edges (different train lines) might need to be drawn between nodes. Their labelling step produces an acceptable labelling, but many metro maps tend not to use diagonal labels as horizontal labels are most likely easier to read.  $\bigstar$  Link to definition of metro map metaphor earlier  $\bigstar$  The main problem is that their resulting graphs generally have very few features (if any) in common with existing metro maps. This is evidently a problem if the maps were to be used as a replacement for the existing maps—people would most likely have issues if the mental map of the map is too greatly changed.

### 2.8.2 Mixed-Integer Program for Metro Map Layout

Nöllenburg describes a method of drawing metro maps using mixed-integer linear programming [Nö5]. Linear programming (LP) is a combinatorial optimisation method which aims to minimise or maximise a particular linear objective function subject to a set of linear constraints [CR99a, CR99b]. Mixed-integer linear programming (MIP) extends linear programming and introduces the notion of restricting variables to be within certain discrete integer ranges.

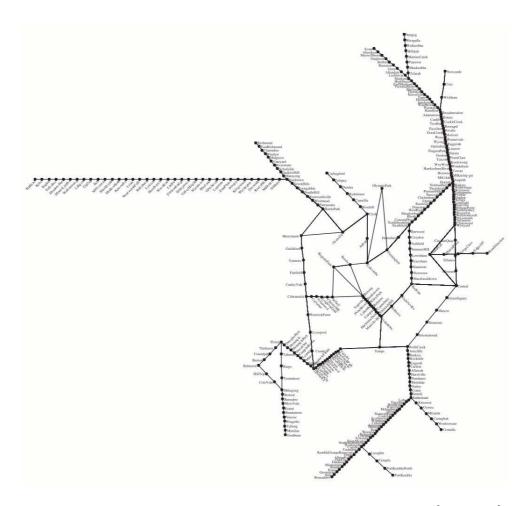


Figure 2.25: The Sydney CityRail map as drawn by Hong  $\operatorname{\it et}$   $\operatorname{\it al.}$  [HMdN06].

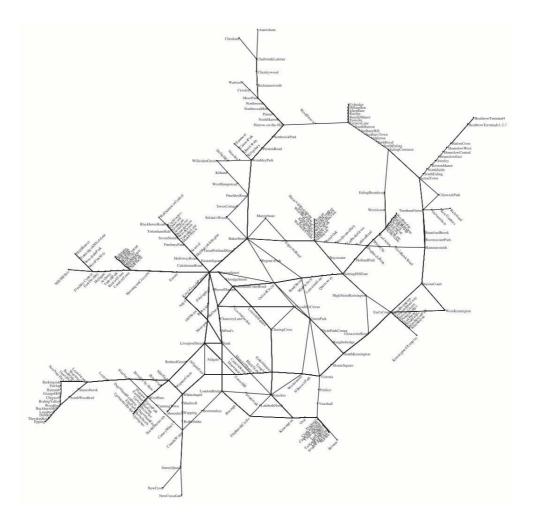


Figure 2.26: The London Underground map as drawn by Hong et al. [HMdN06].

As with other metro map layout methods, Nöllenburg models a metro map as a graph. Four *hard* constraints and three *soft* constraints are defined. The hard constraints relate to the linear constraints of a MIP and the soft constraints relate to the linear objective function. The set of hard constraints is as follows:

- The output drawing must preserve the topology of the input drawing.
- Every edge in the drawing must be composed of octilinear segments (horizontal, vertical or 45° diagonal).
- Every edge in the drawing has a minimum length.
- Every edge in the drawing has a minimum distance from non-incident edges.

The set of soft constraints is as follows:

- Each line should have as few bends as possible.
- The total length of all edges in the drawing should be small.
- The relative position of neighbouring nodes should be preserved as much as possible.

These constraints are combined in a mixed-integer linear program. As many real-world examples of metro maps are fairly large, the length of the resulting MIP can be enormous.  $\star$  How is the enormous length measured?  $\star$  To counter this, Nöllenburg describes using two heuristics to reduce the size of the MIP. The first of these is a 3-link heuristic and replaces long lines of nodes with degree two with a line containing at most two intermediate nodes. This extends the ideas introduced in [HMdN06] and [SR04] which replaced the entire line of degree two nodes with a single straight-line edge. The second heuristic combines a number of techniques for reducing the complexity of ensuring planarity.  $\star$  How? Needs more detail  $\star$ 

Finally, Nöllenburg explains how the requirement of labelling stations with their names can be incorporated into the MIP. The case when labelling nodes along lines that were collapsed by the 3-link heuristic is handled in some detail by considering a parallelogram-shaped area that fits around all the labels along a straight segment of a line.

There are several examples of the output of the MIP for real-world metro maps. Figure 2.27 shows the finished layout of the Sydney CityRail network and Figure 2.28 shows the finished layout for the London Underground map. The Sydney map highlights the effectiveness of their handling of long lines of degree-two nodes using the 3-link heuristic as the spacing of nodes is very regular. The London map is clearly more challenging by way of the highly interconnected central area. This seems to cause the spacing between adjacent nodes to become irregular when taking the whole map into account. Interestingly, some prominent features of the published map (Figure 1.3) are retained, including the characteristic sideways-bottle shape of the Circle (yellow) Line and the horizontal Central (red) Line. There are also some acute angles in some of the lines, particularly in the Victoria (light blue) Line around Euston in the north of the map. Neither of these examples include station labels. This may be because both of these maps are relatively large and the resulting MIP including labelling constraints would be too large to optimise. Nöllenburg shows the results for labelling the S-Bahn RheinNeckar map but state that the layout took 20 hours to produce, rather than 43 seconds for the equivalent unlabelled map.

#### 2.8.3 Path Simplification for Metro Map Layout

# ★ Colour of lines is omitted here? Are the figures as the authors produced them? If so they are pretty useless! ★

Merrick and Gudmundsson describe a method of path simplification which restricts the number of directions that segments of lines can take [MG07]. They call their method C-directed path simplification where C is the set of possible directions for line segments. Their method draws on previous line generalisation work discussed in Section 2.5.3. The simplification method imposes the restriction that the simplified line must intersect (stab) the  $\epsilon$ -circle for each point on the input path. The  $\epsilon$ -circle is analogous to the error criterion used in other line generalisation algorithms.  $\bigstar$  Figure here to illustrate what the  $\epsilon$ -circle looks like in context  $\bigstar$  The difference here is that the  $\epsilon$ -circle is

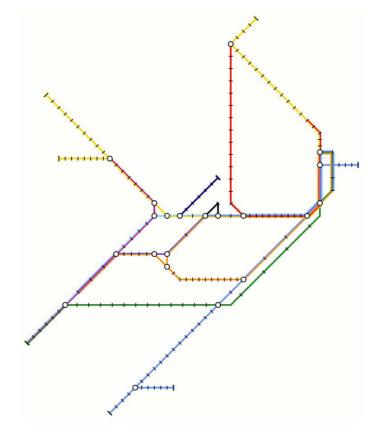


Figure 2.27: The Sydney CityRail map as drawn by Nöllenburg [N $\ddot{0}5$ ].

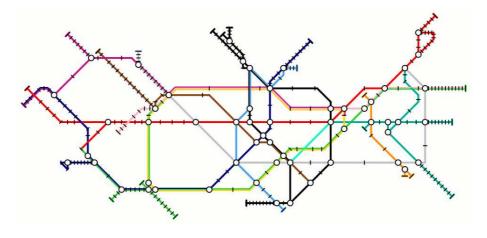


Figure 2.28: The London Underground map as drawn by Nöllenburg [NÖ5].

a closed disc with radius  $\epsilon$  centred on each point of the input path. Their algorithm runs in  $O\left(|C|^3 n^2\right)$  time.

Merrick and Gudmundsson explain how their C-directed path simplification algorithm can be extended to the layout of metro maps. As with other metro map layout methods, they use a graph model. Their path simplification algorithm must be extended to handle multiple intersecting lines. To do this, they consider the *importance* of each of the lines in the graph. The importance could be manually defined, but in this case, they use a heuristic function based on the number of nodes on the line that intersect other lines (interchange stations). They then proceed with the most important line and simplify each line in order of decreasing importance. When a line intersects other lines that have already been simplified, they split it into a set of lines so that each has at most one fixed node. To even out the spacing of nodes along lines, they are prepared to allow errors greater than  $\epsilon$  if this produces an even spacing between nodes. They also introduce two further modifications to their algorithm: the maximum angle that a line is allowed to bend and a minimum link length in the simplification. However, they do not consider the important task of labelling each node with its name.

They illustrate their algorithm by showing results for the Sydney CityRail map and the London Underground map (Figures 2.29 and 2.30 respectively). In the case of their London result, they first used a centrality-based scaling algorithm to reduce the relative density of the centre of the map [MG06]. Both the results satisfy the restriction of using only a limit of eight directions, meaning that every edge is octilinear. However, there are several examples of the topology of the original map not being maintained, particularly in the London map.  $\star$  Do they claim to preserve topology? Under what circumstances does it fail?  $\star$  The lack of any labelling does reduce the usefulness of the maps. However, the key advantage of their algorithm is that it produces results in a relatively quick time.

# 2.8.4 Centrality-based Scaling

One common characteristic of metro maps is that the central area of the map tends to be highly dense with a lot of interconnections, while the extremities of the map become

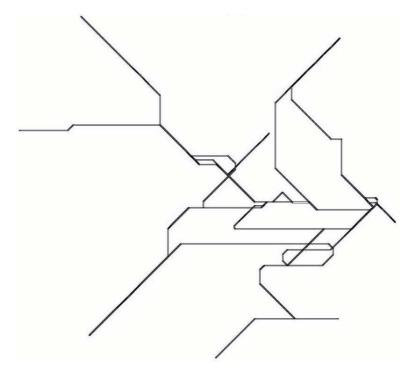


Figure 2.29: The Sydney CityRail map as drawn by Merrick and Gudmundsson [MG07].

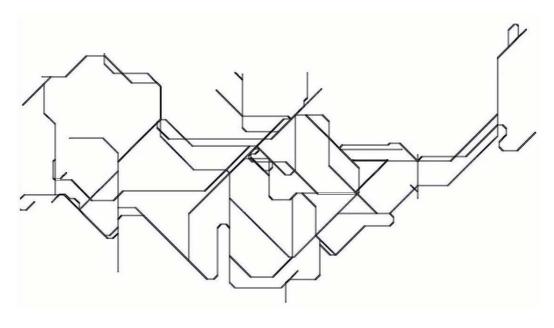


Figure 2.30: The London Underground map as drawn by Merrick and Gudmundsson  $[\mathrm{MG07}].$ 

less dense. This is due to the metro systems being focused around the centre of the town or city which they serve. Merrick and Gudmundsson use a method of centrality-based scaling to apply an increased scale to the centre of the map relative to the extremities [MG06]. This is analogous to the "convex effect" described in Section 2.2.1.

The scaling algorithm uses the concepts of time-distance mapping [SI03] and centrality [WF94]. Time-distance mapping scales a map of a transport network such that the Euclidean distance between nodes is determined by the time taken to travel between those nodes. Centrality assigns a measure for each node in the graph of the localised density of the graph at that point.  $\star$  How is centrality calculated?  $\star$  A number of measures of centrality are detailed by Merrick and Gudmundsson and include degree, betweenness and hubness.  $\star$  Define degree, betweenness and hubness  $\star$  A number of these measures of centrality are taken from research on social networks where it is necessary to have measures of the relative importance of each node in the social network. The scaling algorithm then adapts the time-distance mapping method of Shimizu and Inoue [SI03] to incorporate the measures of centrality.

The centrality-based scaling algorithm is illustrated by the Sydney CityRail and London Underground maps. The London Underground map in Figure 2.30 used centrality-based scaling before applying Merrick and Gudmundsson's path simplification algorithm.

# 2.9 Conclusion

★ Summary of the important concepts of that chapter and those that will be carried forward into the rest of the thesis. ★

# Chapter 3

# Metro Map Layout

This chapter describes our metro map layout algorithm which draws on the observations made of existing metro maps discussed in Section 2.2. We describe how the features that we intend to model can be broken down into a set of criteria and how those criteria can be combined using a multicriteria optimisation method. This chapter is only concerned with moving individual nodes, but there are occasions when the movement of several nodes simultaneously is required. Moving clusters of nodes is described in Chapter 4.

# 3.1 Aims

There are a large number of characteristics that are apparent on existing metro maps. These characteristics are discussed in detail in Section 2.2. We will take the following characteristics into consideration as part of our layout method:

# ★ Split aims up and give in terms of criteria ★

• Lines. Lines will be drawn to take advantage of parallel lines and lines of common angle using angle generalisation—more specifically, lines should be drawn horizontally, vertically or with 45° diagonals ★ octilinear? ★. In using lines of common angle, the shape ★ what does shape mean? ★ of lines will also be implicitly generalised. Lines should have stations spaced at regular intervals and the scale of the map should be generalised to allow for this.

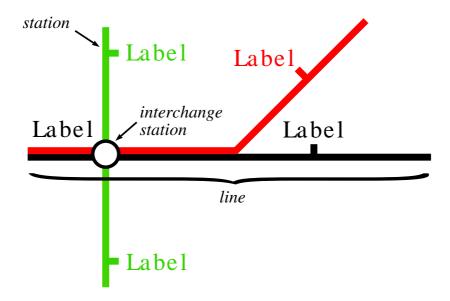


Figure 3.1: Metro map features.

• Labelling. The maps will have station labels and these labels will have an effect on the overall layout of the map.

# 3.2 Definitions

We use a graph as an abstract representation of a metro map. In this case, a graph, G, is a set of nodes, V, with connections between pairs of nodes represented by a set of edges, E. When drawing metro maps, we use the nodes to represent stations on the network and an edge to represent a single connection between two stations. In some cases, there may be several edges connecting two nodes where two or more metro lines run together. We use the term  $metro\ line$  to represent a subset of edges that form a particular line on the network (such as the Central or Northern Lines on the London Underground map). Edges also have metadata in the form of a colour that identifies which line they are part of. These features are illustrated in Figure 3.1.

We decided to use a graph model to represent metro maps, mainly because a metro map is clearly equivalent to a graph. A graph gives us the programmatic flexibility to make tasks such as finding neighbouring nodes or incident edges relatively straightforward.

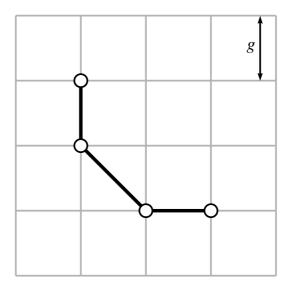


Figure 3.2: Grid used for embedding the metro map graph where g represents the grid spacing.

The graph is embedded on an integer square grid, as shown in Figure 3.2. This means that nodes can only be centred on grid intersections, but there is no explicit requirement for edges to follow grid lines in an orthogonal manner. The spacing between adjacent intersections in the grid is denoted by g. Making the search space discrete in this manner allows us to dramatically reduce the number of potential locations for nodes. Another significant advantage is that producing orthogonal graphs is much easier as nodes are more likely to be in line with one other.

# 3.3 Hill Climbing Multicriteria Optimisation

★ EXPAND SECTION! More detailed explanation of method. WHAT IS

MULTICRITERIA OPTIMISATION?! Relate to background. Pseudocode. ★

★ There is no complete description of multicriteria optimisation. This concept is too crucial to be given such sparse treatment. The explanation is inadequate: where does the 'five to ten iterations' come from? What exactly are simulated annealing and genetic algorithms? The reader cannot gauge the veracity of your argument unless he/she knows something about the alternatives. The hill climbing process and its disadvantages need much

more thorough treatment. The statement "Movement of the nodes depends on the calculation of several criteria" does not make sense unless we know more about the iterative optimisation method used. Similarly the discussion on cooling is unclear without knowing about the optimisation process. \*\pm\$

Our process is built around a hill climbing multicriteria optimization method that combines a number of criteria that are judged to affect the quality of the graph with some method to change the layout of the graph. We are only ever making improvements to the graph with each change in the layout. We considered a number of other optimization methods such as simulated annealing and genetic algorithms. However, we found that the simpler method using hill climbing was more appropriate. Simulated annealing adds an element of non-determinism in order to escape from local minima in the search space. However, for this to be effective, a large number of iterations are required  $\star$  define large - more than 5 to 10? (see next sentence)  $\star$ —certainly many more than the five to ten iterations that were needed for hill climbing. It was quicker and easier to cater for specific local minima as and when they became apparent. Genetic algorithms converge much more slowly than a hill climber or simulated annealing and would have significantly greater requirements for memory and computation owing to the need to maintain a population to evolve new generations of solutions.

Multicriteria optimization is an iterative process. For each iteration an attempt is made to move each node, followed by clusters of nodes, followed by each node label. The number of iterations can either be fixed beforehand or the process can be stopped if no further improvement can be made. We use a fixed number of iterations for each map  $\bigstar$  Why? Can't it be determined? No real reason!  $\bigstar$ , the number of which is indicated for the examples presented in Chapter 6. Care is made to ensure that the method is deterministic, which allows us to see how subtle changes in the criteria weightings affect the finished maps without randomness becoming a factor.

Procedure 2 illustrates the method. Here, a graph, G, consists of a set of nodes, V, a set of edges, E, and a set of labels, L. The section from line 6 to line 12 handles moving nodes (see Sections 3.5 to 3.8), the section from line 14 to line 20 handles moving

#### Procedure 1 Multicriteria Optimisation

```
1: c_0 \Leftarrow \text{calculateCriteria}()
 2: running \Leftarrow true
 3: while running do
        attempt to make some improvement
        c \Leftarrow \text{calculateCriteria}()
 5:
       if c \not< c_0 then
 6:
 7:
          running \Leftarrow \mathbf{false}
 8:
          c_0 \Leftarrow c
 9:
        end if
10:
11: end while
```

clusters of nodes (see Chapter 4) and the section from line 21 to line 27 handles moving labels (see Sections 3.9 and 3.10).

# 3.4 Initial Embedding

An important consideration which greatly affects the finished maps is that of the initial layout of the map. There are two main ways of generating an initial layout for the map. We could start with a completely random layout or with some other topologically correct layout such as the actual geographic layout of the map.

Starting with a random layout is going to make producing a map that corresponds in any way to the geographic layout very difficult. There is no explicit concept of the topology of the map making it impossible to say whether a certain line should be oriented in a particular direction (north-to-south or east-to-west) or to place nodes that are close together in reality close together on the final map. Even if the topology of the map was known and we could reason about the relative positions of nodes (that is to say that one node should be north of another or that a group of nodes are close together), it would be very likely that a large number of iterations and massive fluctuation in the starting layout would be required. Therefore, it becomes very difficult to predict and reason about potential movements of nodes.

Starting with an actual geographic layout is much more likely to produce better results than a random layout. The entire method then becomes a process of iterative

# Procedure 2 Metro Map Layout

```
1: G \Leftarrow (V, E, L)
 2: \operatorname{snapNodes}(V)
 3: m_{T0} \Leftarrow \text{calculateNodeCriteria}(V) + \text{calculateLabelCriteria}(L)
 4: running \Leftarrow true
 5: while running do
 6:
       for v \in V do
          m_{N0} \Leftarrow \text{calculateNodeCriteria}(v)
 7:
          m_N \Leftarrow \text{findLowestNodeCriteria}(v)
 8:
          if m_N < m_{N0} then
 9:
             moveNode(v)
10:
          end if
11:
       end for
12:
       P \Leftarrow \text{clusterOverlengthEdges}(V, E) \cup \text{clusterBends}(V, E) \cup \text{clusterPartitions}(V, E)
13:
14:
       for p \in P do
          m_{N0} \Leftarrow \text{calculateClusterCriteria}(p)
15:
          m_N \Leftarrow \text{findLowestClusterCriteria}(p)
16:
          if m_N < m_{N0} then
17:
             moveCluster(p)
18:
19:
          end if
20:
       end for
       for l \in L do
21:
          m_{L0} \Leftarrow \text{calculateLabelCriteria}(l)
22:
          m_L \Leftarrow \text{findLowestLabelCriteria}(l)
23:
          if m_L < m_{L0} then
24:
             moveLabel(l)
25:
          end if
26:
       end for
27:
       m_T \Leftarrow \text{calculateNodeCriteria}(V) + \text{calculateLabelCriteria}(L)
28:
       if m_T \not< m_{T0} then
29:
          running \Leftarrow \mathbf{false}
30:
       else
31:
32:
          m_{T0} \Leftarrow m_T
       end if
34: end while
```

refinement—fewer and smaller node movements would theoretically be required to produce finished maps of acceptable quality than starting with a random layout. Finding the geographic locations of stations in order to position the nodes becomes a time-consuming process, but precise accuracy is not necessarily required. A simple freehand sketch based on knowledge of the geography and topography of the metro system might suffice, particularly for simple maps.

One implicit advantage of metro maps is that they are nearly always planar. In this context, that means that edges are unlikely to intersect other than at nodes—actual intersections usually mean something specific to the geography of the map such as where one line physically passes over or under another line. In fact, it may be essential that this intersection is preserved in the final map. Therefore, an optional step is recommended where each intersection between lines is replaced by a dummy node. The dummy node is treated just like any other node in the graph, except that it does not have a label and is not drawn on the finished map. This ensures that the topography of the edge crossing is retained in the finished layout.

#### 3.4.1 Embedding on the Grid

The first stage of the metro map layout method is to ensure that all nodes are centred on grid intersections (line 2 of Procedure 2). We will be starting with a geographic layout (or a close alternative using a topologically-correct sketch of the map [BEKW02]) so some process of *snapping* nodes to the nearest grid intersections is needed.

Finding the nearest grid intersection to a node is simple, but care must be taken to ensure that more than one node does not share the same grid intersection. In the case of contention for a particular intersection, the node being snapped should be moved to the nearest grid intersection that is vacant. In other words, each node is snapped to the nearest grid intersection that is not already occupied by another node.

Nodes are processed in order of the distance that they are from a grid intersection such that those nodes closest to a grid intersection are moved first and those nodes furthest from a grid intersection are moved last. In the case where two ore more nodes are an equal distance from the same grid intersection, an arbitrary choice is made as to

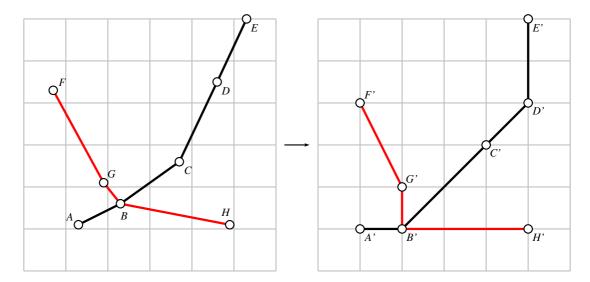


Figure 3.3: Moving nodes to grid intersections. The left-hand graph is the initial layout; the right-hand graph is the same graph but with nodes moved to grid intersections. In the cases where there is already a node occupying a grid intersection, the next nearest grid intersection is used—this is the case with nodes B and G in this example.

which one to move first. If the spacing between grid intersections, g, is too large then particularly dense parts of graphs (especially areas where the average length of an edge is less than 0.5g) may make it difficult to find points close to the starting point for the node. In these cases, the value of g should be reduced.

Figure 3.3 illustrates how the process of moving nodes to grid intersections works for a simple example. The graph on the left is the initial layout and the graph on the right is the new layout after nodes (A, B, etc.) have been moved to their new positions on grid intersections (A', B', etc.). In this case two nodes (B and G) could potentially both move to the same grid intersection, but to avoid this, G' is positioned on the nearest grid intersection first (as it is closest) and B' is positioned on the next nearest grid intersection.

# 3.5 Node Movement

★ Need to tie in with examples in background. Probably not in this section but certainly in the sections that describe each criterion—then say here that they are described later. Where did these criteria come from? Are there

# others? Relate to aims. Need to say that the purpose of each of the functions describing the criteria is that they should be minimised. ★

Movement of nodes depends on the calculation of several criteria which are judged to affect the aesthetic quality of the map. We have implemented a total of six different node movement criteria:

- Angular Resolution Criterion,  $c_{N1}$ . The angular resolution of incident edges at each node is maximised. See Section 3.6.1.
- Edge Length Criterion,  $c_{N2}$ . The edge lengths across the whole map should be approximately equal. See Section 3.6.2.
- Balanced Edge Length Criterion,  $c_{N3}$ . The length of edges incident to a particular node should be approximately equal. See Section 3.6.3.
- Edge Crossings Criterion,  $c_{N4}$ . The number of unnecessary edge crossings should be reduced. See Section 3.6.4.
- Line Straightness Criterion,  $c_{N5}$ . Edges that form part of a line should, where possible, be collinear either side of each node that the line passes through. See Section 3.6.5.
- Octilinearity Criterion,  $c_{N6}$ . Each edge should be drawn horizontally, vertically or diagonally at 45°. See Section 3.6.6.

As well as these six criteria, we have implemented four node movement rules which are strictly enforced during the layout process. We used rules in addition to criteria as that there are some cases in which the criteria are unable to guard against introducing undesirable node movements that would otherwise be difficult (or inefficient) to handle with additional criteria. The four node movement rules are:

• Bounding Area Restriction Rule. Restrict the movement of nodes to be within a certain bounding area. See Section 3.7.1.

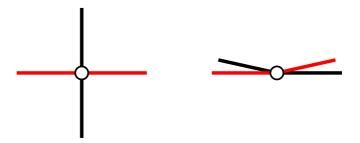


Figure 3.4: Examples of optimal angular resolution (left) and poor angular resolution (right).

- Geographic Relationships Rule. Enforce the geographic relationships between pairs of nodes (for example, that one node should be north of another node). See Section 3.7.2.
- Occlusions Rule. Avoid the introduction of occlusions of other edges and nodes. See Section 3.7.3.
- Edge Ordering Rule. Preserve the ordering of edges incident to a node. See Section 3.7.4.

These rules and criteria, as well as the strategy by which they are used to determine how nodes are to be moved are discussed in more detail in the following sections.

# 3.6 Node Movement Criteria

 $\bigstar$  Need to ensure that the actual layout problem that these criteria are trying to solve are explained. For example, the balanced edge length criterion doesn't say what the problem is before it explains how to solve it!  $\bigstar$ 

# 3.6.1 Angular Resolution Criterion, $c_{N1}$

In some metro maps there are occasions where many lines pass through a single node creating the situation where that node has many incident edges. If the edges are drawn such that there is only a small angle between any two adjacent edges then it can become difficult to distinguish between them (particularly if the edges are similarly coloured). Figure 3.4 illustrates this point—the left-hand example shows four incident edges which

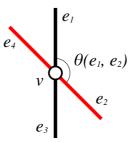


Figure 3.5: Calculating the angular resolution criterion.

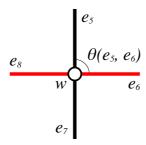


Figure 3.6: Calculating the angular resolution criterion.

are arranged so that the angle between each adjacent pair of edges is maximised; the right-hand example has only small angles between two pairs of edges but very large angles between the other two pairs of edges. The angular resolution criterion,  $c_{N1}$ , ensures that there is as large an angle as possible between adjacent edges incident to a node. The criterion is calculated using

$$c_{N1} = \sum_{v \in V} \sum_{\{e_1, e_2\} \in E_v} \left| \frac{2\pi}{\rho(v)} - \theta(e_1, e_2) \right|$$
(3.1)

where  $\rho(v)$  is the degree of the node v and  $\theta(e_1, e_2)$  is the angle in radians between two adjacent edges  $e_1$  and  $e_2$  incident to v.

Figure 3.5 shows an example of calculating the angular resolution criterion for a node v. In this example,  $\rho(v)$  is 4, so the ideal spacing between each adjacent pair of edges is  $\frac{2\pi}{\rho(v)}$  or  $\frac{\pi}{2}$  (90°). The angle  $\theta$  between edges  $e_1$  and  $e_2$  is  $\frac{3\pi}{4}$  (135°), so for that particular pair of edges, the absolute difference between the actual and ideal angles is  $\left|\frac{2\pi}{4} - \frac{3\pi}{4}\right| = \frac{\pi}{4}$  (45°). For the other pairs of edges, the difference is also  $\frac{\pi}{4}$ , so for the area around v,  $c_{N1} = \frac{4\pi}{4} = \pi$  (180°).

Figure 3.6 shows another example for a node, w. This time, the lines pass through w at right angles. Again,  $\rho(w) = 4$  therefore the ideal spacing between adjacent pairs of edges is  $\frac{\pi}{2}$ .  $\theta(e_5, e_6) = \frac{\pi}{2}$ , so the difference between the ideal angle and the actual angle is  $\frac{\pi}{2} - \frac{\pi}{2} = 0$ . The angles between each of the other three pairs of adjacent edges around w are equal, so the total value of  $c_{N1}$  for the area around w is 0.

#### 3.6.2 Edge Length Criterion, $c_{N2}$

A common feature of metro maps is that stations should be spaced evenly along lines and that the spacing should be reasonably consistent across the entire map. This comes about because the map is drawn to a irregular scale such that the scale decreases towards the extremities of the map (this scale generalisation was explored in Section 2.2.2).

The edge length criterion,  $c_{N2}$ , is based on some preferred multiple, l, of the grid spacing g. The purpose of the criterion is to penalize edges that are longer than or shorter than lg. The edge length criterion is calculated by

$$c_{N2} = \sum_{e \in E} \left| \frac{|e|}{lg} - 1 \right| \tag{3.2}$$

where |e| is the length of edge e. l should always be greater than or equal to 1 as it is impossible to have two nodes less than one grid spacing apart. In the case where |e| = lg, the length of edge e is exactly the length that we prefer and the criterion for e evaluates to zero. If |e| < lg or |e| > lg then the value of  $c_{N2}$  for that edge will be greater than zero.

To illustrate how the edge length criterion works, consider the example shown in Figure 3.7. There are three edges in this small graph, AB, BC and BD, with lengths 2g,  $\sqrt{18}g$  and 6g respectively. If we assume that g=1, we can see how  $c_{N2}$  would vary for different values of l by looking at the example values in Table 3.1. Notice how the criterion evaluates to give significantly higher values for edges which are longer than lg than those that are shorter. This has the effect of providing greater pressure on the graph to compress than to expand. This makes sense when we remember that the value of g and l should be chosen to suit the densest parts of the starting layout of the graph,

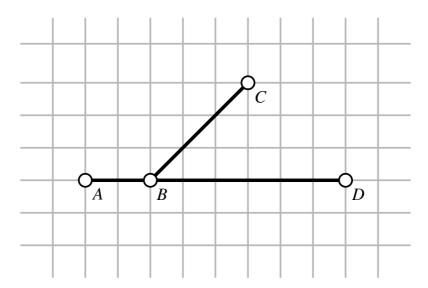


Figure 3.7: Example used to illustrate the edge length criterion.

meaning that most edges will initially be longer than lg. As l tends towards  $\infty$  the value for  $c_{N2}$  tends towards the number of edges in the graph.

While this works fine for horizontal or vertical edges, diagonal edges pose an interesting problem, as may have been noticed for edge BC in Figure 3.7. Where diagonal edges are concerned, it is very likely that a position on the grid where the edge length criterion can be zero simply does not exist. Figure 3.8 shows a trivial example of such a case. If l is 1, then the edge AB can be drawn to have length of exactly 1g, whereas the diagonal edge AC can only have a minimum length of  $g\sqrt{2}$ . Rather than making specific allowances for diagonal edges to break from the grid, we use a combination of the edge length criterion and other criteria (specifically the octilinearity criterion) to counteract this.

#### 3.6.3 Balanced Edge Length Criterion, $c_{N3}$

One of the characteristics of metro maps is that there are many nodes with two incident edges (degree two). This is very often the case when a line passes through a sequence of several stations. Unfortunately, the edge length criterion as discussed in Section 3.6.2 does not handle this particular feature of the maps effectively. The balanced edge length criterion overcomes this particular limitation.

Table 3.1: Examples of how the edge length criterion varies with different values of l (l is the preferred multiple of grid spacings for the length of an edge). The edges AB, BC and BD are shown on Figure 3.7 and have lengths 2,  $\sqrt{18}$  and 6 respectively. In calculating  $c_{N2}$  for each of these edge lengths, we have assumed that g = 1.  $c_{N2}^{XY}$  means the calculation of  $c_{N2}$  for just the part of the graph between nodes X and Y.

l	$c_{N2}^{AB}$	$c_{N2}^{BC}$	$c_{N2}^{BD}$	$c_{N2}$
1	1	3.24	5	9.24
2	0	1.12	2	3.12
3	0.33	0.41	1	1.74
4	0.5	0.06	0.5	1.06
5	0.6	0.15	0.2	0.95
6	0.67	0.29	0	0.96
7	0.72	0.39	0.14	1.24
8	0.75	0.47	0.25	1.47
:	:	:	:	:
$\infty$	1	1	1	3

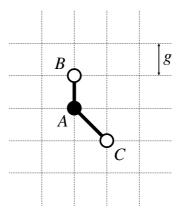


Figure 3.8: Problem with diagonal edge lengths and the edge length criterion.

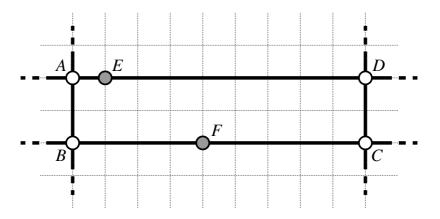


Figure 3.9: Balanced edge lengths.

Figure 3.9 shows an example whereby there are two nodes, E and F, with degree two. If we are only considering the edge length criterion for these two nodes, we can show that the criterion evaluates to the same value for both nodes. First assume that the preferred edge length, l, is 1g. In the case of node E, the edge length criterion evaluates to

$$\left| \frac{|e_{AE}|}{lg} - 1 \right| + \left| \frac{|e_{ED}|}{lg} - 1 \right| = \left| \frac{1}{1} - 1 \right| + \left| \frac{8}{1} - 1 \right| = 7 \tag{3.3}$$

and for node F

$$\left| \frac{|e_{BF}|}{lg} - 1 \right| + \left| \frac{|e_{FC}|}{lg} - 1 \right| = \left| \frac{4}{1} - 1 \right| + \left| \frac{5}{1} - 1 \right| = 7$$
 (3.4)

Clearly all the potential locations for E between A and D will result in the same value for the edge length criterion. However, we want to ensure that the edge lengths are approximately equal—this was after all the original intention of the edge length criterion. In these cases, the balanced edge length criterion can help by penalising nodes with degree two that have incident edges with unbalanced lengths.

Calculation of the balanced edge length criterion,  $c_{N3}$ , is simply the sum of the absolute difference between the lengths of the two incident edges of every degree-two node in the graph, i.e.

$$c_{N3} = \sum_{v \in V, \rho(v) = 2} ||e_1| - |e_2|| \tag{3.5}$$

where  $e_1$  and  $e_2$  are the incident edges of node v which has degree  $\rho(v) = 2$ .

Using the nodes E and F from Figure 3.9 as an example, the balanced edge length criterion for E is |1 - 8| = 7 and for F is |4 - 5| = 1.

The optimal position is therefore where F is (or indeed the grid intersection immediately to the right of F). If the lengths of the two incident edges to a node are equal, then the balanced edge criterion for that node evaluates to zero: the edge lengths are balanced perfectly.

# 3.6.4 Edge Crossings Criterion, $c_{N4}$

In the case of metro maps, edge crossings imply some kind of topographic feature such as two unconnected lines crossing. If an edge crossing is intentional, then a dummy node can be inserted at the crossing point and the map drawn with that node in place. The dummy node can then be removed after the map is drawn or simply left in place and not rendered. However, unintentional edge crossings can adversely affect the readability of the map [PCJ95].

Calculation of the edge crossings criterion,  $c_{N4}$ , is straightforward.  $\bigstar$  Really? What's the cost of this - it's significant for large graphs!  $\bigstar$  Each edge is checked to see whether any other edges cross it:  $c_{N4}$  is then the total number of crossings in the graph.

Special care has to be taken in the case where more than one line runs in parallel between two adjacent nodes. In this case, even though several edges are actually being crossed, only a single crossing is counted for the purposes of this criterion.

Two efficiency enhancements can be made for this criterion:

- As we are using a rule that stops new edge crossings being introduced (see Section 3.7.3), we only need check edges that we already know cross other edges.
- If the graph is already planar (so that there are no edge crossings) or can be made planar (by the introduction of dummy nodes), then there is clearly no need to calculate this criterion. (The weighting for the criterion can be set to zero to indicate that we do not want to calculate the value for that criterion.)

# 3.6.5 Line Straightness Criterion, $c_{N5}$

One of the important features of metro maps is that lines appear to pass through nodes so that the entry edge is more-or-less directly opposite the exit edge. It is not desirable for the line to turn sharply as it passes through a node (i.e., it makes a 90° or 135° turn). This is made all the more important if there are two or more lines passing through a node—if both of the entry edges are opposite each other and both of the lines make a 90° turn so that the exit edges are opposite, the readability of the map is degraded

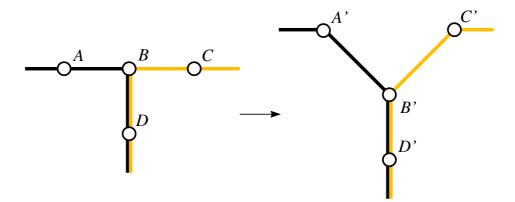


Figure 3.10: Examples of poor line straightness (left) and improved line straightness (right).

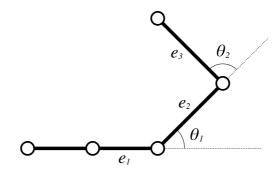


Figure 3.11: Example of the calculation of the line straightness criterion.

(especially if the colour of the lines are similar). Figure 3.10 shows examples where the graph on the left can have the line straightness improved by moving nodes.

To ensure this type of improvement, we introduce the line straightness criterion,  $c_{N5}$ , calcuated by

$$c_{N5} = \sum_{(v \in V)} \left( \sum_{e_1, e_2 \in E} \theta(e_1, e_2) \right)$$
 (3.6)

where  $\theta\left(e_{1},e_{2}\right)$  is the angle between adjacent edges  $e_{1}$  and  $e_{2}$  and  $e_{1}$  and  $e_{2}$  are the only two incident edges of the same line incident to the node v. If the end points of  $e_{1}$  and  $e_{2}$  are collinear,  $\theta\left(e_{1},e_{2}\right)=0^{\circ}$ ; if the two edges are at right angles, then  $\theta\left(e_{1},e_{2}\right)=90^{\circ}$ . The intuitive effect of this is to penalise turns in edges where  $\theta\left(e_{1},e_{2}\right)$  is large more than turns where  $\theta\left(e_{1},e_{2}\right)$  is small or zero.

In the example in Figure 3.11, the line in question includes three edges,  $e_1$ ,  $e_2$  and  $e_3$ . The line makes two turns between each two pairs of adjacent edges. To calcuate

the line straightness criterion for this example, we simply sum the angles between edges  $\bigstar$  could this be ambiguous? what about  $\theta$  and  $180 - \theta$ ?  $\bigstar$   $e_1$  and  $e_2$  ( $\theta_1$ ) and  $e_2$  and  $e_3$  ( $\theta_2$ ):  $\theta_1 + \theta_2 = 45^\circ + 90^\circ = 135^\circ$ .

# 3.6.6 Octilinearity Criterion, $c_{N6}$

The purpose of this criterion is to ensure that edges are drawn at some multiple of  $45^{\circ}$ , either orthogonally (vertically or horizontally) or diagonally with respect to the grid. The octilinearity criterion,  $c_{N6}$ , has the effect of penalizing edges that are not some multiple of  $45^{\circ}$  and is calculated using

$$c_{N6} = \sum_{\{u,v\} \in E} \left| \sin 4 \left( \tan^{-1} \frac{|y(u) - y(v)|}{|x(u) - x(v)|} \right) \right|$$
(3.7)

where  $\{u, v\}$  is an edge between nodes u and v, and y(v) and x(v) are the y-and xcoordinate of node v respectively.

Figure 3.12 shows an example of a graph that we will use to illustrate the calculation of the octilinearity criterion. The result of calculating the criterion  $c_{N6}^e$  for each edge  $e = \{u, v\}$  in this example graph is shown in Table 3.2. As is expected, edges which are already at an angle of some multiple of 45° (AB and FG) evaluate to zero, whereas edges which are at angles furthest from multiples of 45° evaluate to the highest values (edges BC and BF are at about 26.57°).

#### 3.6.7 Node Movement Criteria Weightings

# $\bigstar$ How are the weightings determined in practice? $\bigstar$

The intention of the weightings for each of the criteria is twofold. First, the functions generate values which can vary by an order of magnitude or more between each of the criteria. The weightings allow the values of each criterion to be brought within the same magnitude of each other. This is important to ensure that one criterion does not completely overwhelm the other criteria. Second, by using a higher weighting, a preference can be placed on a particular criterion if the effects of that criterion are required to be more prominent. Conversely, a lower weighting can be used to reduce the effect

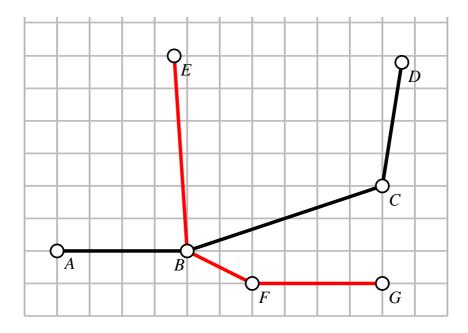


Figure 3.12: Example of the calculation of the octilinearity criterion.

Table 3.2: Examples of octilinearity criterion calculations with reference to Figure 3.12.  $\bigstar$  Explain why  $\{B,C\}$  and  $\{B,F\}$  evaluate to the same value even though the gradient of the lines is different.  $\bigstar$ 

Edge, $e = \{u, v\}$	$c^e_{N6}$
$\{A,B\}$	$\left \sin 4 \left(\tan^{-1} \frac{0}{4}\right)\right  = 0$
$\{B,C\}$	$\left \sin 4 \left(\tan^{-1} \frac{2}{6}\right)\right  = 0.96$
$\{C,D\}$	$\left  \sin 4 \left( \tan^{-1} \frac{3.8}{0.6} \right) \right  = 0.586$
$\{B,E\}$	$\left \sin 4 \left(\tan^{-1} \frac{6}{0.6}\right)\right  = 0.388$
$\{B,F\}$	$\left \sin 4 \left(\tan^{-1} \frac{1}{2}\right)\right  = 0.96$
$\{F,G\}$	$\left \sin 4\left(\tan^{-1}\frac{0}{4}\right)\right  = 0$

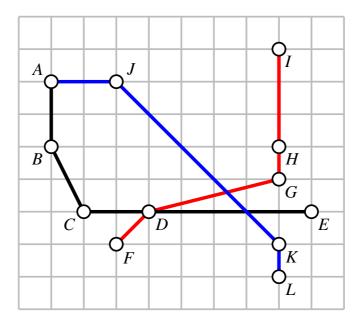


Figure 3.13: Simple example of node movement criteria for a complete graph. The values for each criterion for this graph are shown in Table 3.3.

of a particular criterion. If an application of the method does not require a particular criterion, the weighting can be set to zero. This allows flexibility in deciding which criteria should contribute to the characteristics of the map for various applications.

The sum of the weighted criteria for node movement,  $m_N$ , is given by

$$m_{N} = w_{N1}c_{N1} + w_{N2}c_{N2} + w_{N3}c_{N3} + w_{N4}c_{N4} + w_{N5}c_{N5} + w_{N6}c_{N6}$$

$$= \sum_{i=1}^{6} w_{Ni}c_{Ni}$$
(3.8)

The values for  $w_{Ni}$  can be modified by the user depending on the characteristics of the particular metro map being drawn.

To illustrate how the weighted criteria combine and to give some idea of the magnitudes of each of the weightings, an example of a complete graph is shown in Figure 3.13, together with the weighting and weighted value for each criterion shown in Table 3.3.

Criterion	$c_{Ni}$	Weighting, $w_{Ni}$	$w_{Ni}c_{Ni}$ for graph
			in Figure 3.13
$c_{N1}$	19.2962	1.0000	19.2962
$c_{N2}$	7.0080	3.0000	21.0240
$c_{N3}$	16.7375	1.5000	25.1063
$c_{N4}$	2.0000	20.0000	40.0000
$c_{N5}$	3.9270	6.0000	23.5620
$c_{N6}$	1.7904	12.0000	21.4848

Table 3.3: Typical node movement criteria weightings.

The total of the weighted criteria in Figure 3.13 is:

$$m_N = 19.2962 + 21.0240 + 25.1063 + 40.0000 + 23.5620 + 21.4848$$
  
= 150.4733 (3.9)

# 3.7 Node Movement Rules

As well as all the weighted criteria, a number of rules were devised which serve the purpose of restricting the potential movement of a node. These rules essentially forbid movement of a node to a new location if any of the rules are broken. Specific examples justifying the reason for each rule that we used are detailed later in this Section.

We created rules to restrict the movement of a node to stay within the boundaries of a drawing area; to enforce the geographic relationships between nodes; to cope with occasions when other nodes or edges may be occluded; and to preserve the ordering of the edges around a node. This set of rules are described in the following sections.

#### 3.7.1 Restrict Movement to Boundary of Drawing Area

When drawing maps on a finite drawing area, it is sometimes a reqirement that any node movement does not cause any node to move outside the drawing area. This is not always necessary—in practice, it should be possible to ensure that the drawing area is always large enough to contain the graph and any potential growth of the overall dimensions of the graph.

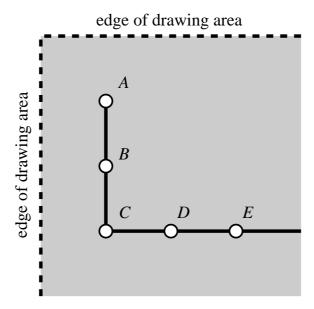


Figure 3.14: Example of the restriction of node movements to stay within the drawing area boundary. Nodes C, D and E form part of a longer straight line and ideally nodes A and B would move to be parallel to them, but the edge of the drawing area restricts this movement.

To illustrate a case where this rule applies, consider the graph shown in Figure 3.14. In this example, nodes C, D and E form part of a longer straight line and ideally nodes A and B should move so that they would be collinear with them. However, moving nodes A and B in this way would put them either wholly or partially outside the drawing area.

#### 3.7.2 Enforcement of Geographic Relationships

Although metro maps are a generalization of the real geography of the network, relationships such as one node being north of another node still indicate some general meaning in the drawn map. This rule allows these geographic relationships to be enforced so that the relative positions between nodes do not change.

Figure 3.15 illustrates the effect of enforcing geographic relationships. In the left-hand diagram, node A is being moved but its initial position with respect to B means that it is only free to move in the area that is both above and to the left of B (the grey shaded area). Notice that we still allow a movement so that A is directly to the left

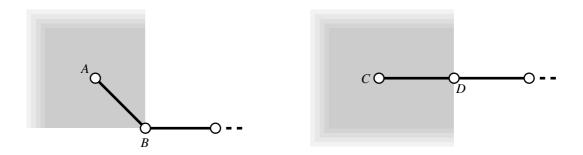


Figure 3.15: Example of the enforcement of the geographic relationships when moving a node. The grey shaded area shows the degree of freedom afforded node A (left) and node C (right).

or above B—if this was not the case then it would not be possible to move A into an optimal octilinear position with respect to B.

In the right-hand figure node C is being moved but is already immediately to the left of D. This allows C freedom to move anywhere that is to the left of—or directly above or below—D.

One of the side-effects of allowing nodes to move so that they are vertically or horizontally aligned with another connecting node (as C and D are initially) is that on the second iteration of moving a node it may move into a position that breaks the initial geographic relationships. For example, if we started with a graph as in the left-hand diagram of Figure 3.15 and node A moved to a position immediately to the left of B, the graph would become equivalent to the right-hand graph. At the next movement of A, the freedom of movement is the same as that for C, allowing A to move below B. In practice, this tended not to be the case—for it to happen, a new bend in a line is likely to be introduced which would make that position less optimal when calculating the weighted criteria, especially with regard to the line straightness criterion.  $\bigstar$  Not appropriate to dismiss this problem in this way. At this stage, the criteria are being defined and they should be done so independently of the particular examples that are used later in the thesis.  $\bigstar$ 

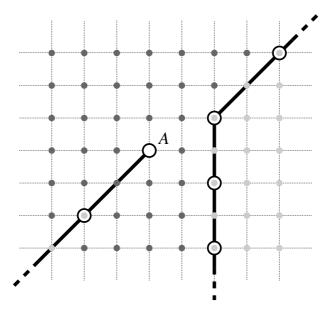


Figure 3.16: How node and edge occlusions can restrict the search space. Light grey dots show positions where node A cannot be moved to because either a node or edge occlusion would result. Dark grey dots show valid positions where no occlusions would happen.

# 3.7.3 Handling Node and Edge Occlusions

In Section 3.6.4 we introduced a criterion which penalises edge crossings in the graph. While this criterion is useful for removing unwanted edge crossings, it is usually the case that we do not want new edge crossings to be introduced. We also want to make sure that any node movement does not cause another edge or node to be occluded. The node and edge occlusions rule takes care of this.

The node and edge occlusions rule specifically stops the introduction of any of these situations by moving node A:

- any edge incident to A crossing or lying on top of any other edge
- any edge incident to A crossing any other node
- node A moving such that it will be on top of any other edge
- node A moving such that it will be on top of any other node

The second and third of these restrictions are especially important. If an unconnected node is moved so that it looks as though it is connected to another edge, someone reading the graph may interpret it as representing a genuine connection.

Figure 3.16 shows an example of the restrictions imposed by the node and edge occlusions rule. In this figure, node A is being moved and the grey spots represent possible new locations for A. In locations where any of the above four situations occur the spot is shown with light grey, meaning that that location would invalidate the node and edge occlusions rule. Node A cannot move to any of these locations. The spots coloured with dark grey do not introduce any node or edge occlusions so these are valid new locations for A. Out of the 49 possible locations for A, 13 would invalidate the rule, meaning that the node movement criteria need only be calculated for the remaining 36 locations.

#### 3.7.4 Preservation of Edge Ordering

★ Is this rule necessary in situations where the geographic requirements are not provided or strong enough? Seems that this is exactly the situation where geographic consideration should be supplied! How does geographic requirements affect topology? ★

The geographic relationships rule (Section 3.7.2) allows us to restrict the geographic relationships between two nodes. However, there are limitations to this rule that mean that the topology of the graph could be changed by the movement of a node. Figure 3.17 shows such a problem. In this example, node C is being moved. If we assume that the most optimal new position for C is as shown in the right-hand diagram, we can see that the topology of this graph has been changed. This is easy to show by considering the order of the incident edges to A. In the left-hand diagram the clockwise ordering of edges starting with  $e_1$  is  $\{e_1, e_2, e_3\}$ . In the right-hand diagram, the change in position of C has changed the order of the edges to  $\{e_1, e_3, e_2\}$ .

To implement this rule we need to find the clockwise ordering of edges around the node being moved and any neighbouring node in the graph. Using the example of the left-hand diagram in Figure 3.17 as a starting point, the ordering around the node being

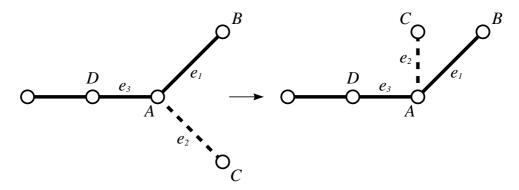


Figure 3.17: Preservation of edge ordering. Without preserving the ordering of edges, node C would be able to move as shown, changing the topography of the map.

moved, C is  $\{e_2\}$  and the neighbouring node, A is  $\{e_1, e_2, e_3\}$ . We can move node C to any position that maintains these edge orderings, so the edge orderings at each potential new location for C must be checked and the location disregarded if the orderings change.

# 3.8 Moving Nodes

The way that nodes are moved greatly affects the outcome of the final drawing of the metro map. There are a number of points that need to be considered when selecting a position to move a node to: the sum of the weighted criteria,  $m_N$  (Equation 3.8); whether or not another node occupies that grid intersection; whether moving the node would occlude other nodes or edges; how far to move the node; whether the distance to move the node is reduced with each iteration (cooling); and whether the cyclic ordering of edges incident to a node would change.

Our approach is to specify a maximum radius within which a node can move. This is given as some multiple, r, of the grid spacing g. As the whole process effectively refines a sketch of the map or the geographic layout of the map, the value of r is usually fairly small (generally less than  $10 \bigstar not much use without some idea of how big the whole map area is for some practical examples <math>\bigstar$ ). Larger values of r would allow movements that could alter the map so that it differs too greatly from reality. A larger value of r is chosen for maps with small values of g so that nodes can move greater distances if there are many grid intersections between connected nodes. In the case of a large map, such as the London Underground map, g has to be small to

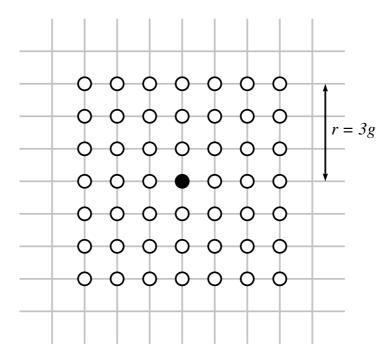


Figure 3.18: The node movement radius. The black node in the centre is being moved and r = 3g: all the potential new locations in this radius are shown with white nodes.

allow for enough grid intersections for nodes in the dense centre of the map, while the extremities are relatively sparse. An example of the potential movements for a node when r = 3g is shown in Figure 3.18.

# 3.8.1 Cooling

#### ★ Cooling has not been defined ★

# $\bigstar$ This section needs rewriting - paragraph structure makes the description confusing. $\bigstar$

A cooling process is used whereby the maximum radius that a node can move, r, is reduced to 1 by the last iteration, using a linear cooling schedule. The purpose of cooling is to allow large movements to be made during early iterations, with later iterations moving nodes less and less to allow for finer refinement. We experimented with a number of different cooling schedules such as a logarithmic schedule or an irregular schedule. However, the difference between results using each different schedule was negligible. It therefore made sense to use the least computationally expensive linear cooling schedule.

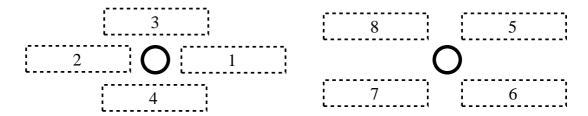


Figure 3.19: Search space for labelling the metro map.

When considering potential movements for a node, the initial value of  $m_0$ , is first calculated using Equation 3.8. Each grid intersection up to r intersections from the initial node location is tested by moving the node there and recalculating m. A set of locations, T, is remembered for each potential movement where the new value of m is less than the initial value of  $m_0$ . When there is a single location in T with the smallest value of m, the node is moved to that location. In the case of no potential movements being discovered, the node is left in its original location; if there is more than one location with the same smallest value of m, we select the first improved location that was found.

# 3.9 Labelling

Labelling is an integral part of metro maps and hence it should form an integral part of our multicriteria optimization method. To this end, a number of criteria are introduced for label placement. In order to reduce the number of potential locations for labels and to allow a preference for one position over another, we limit the number of positions using a labelling space. Figure 3.19 shows our chosen labelling space, which allows eight different label positions.

Occasionally a label might contain a large amount of text with several words, which is why we also allow long, many-word labels to span multiple lines. We decide whether to split a long label over multiple lines by causing a line break in the label if the length of the label exceeded 0.75lg. This value was chosen because the distance between nodes tends towards lg: it would be better, if labels were to fit between other labels, to avoid a label for one node appearing too close to another node. No attempt was made to split

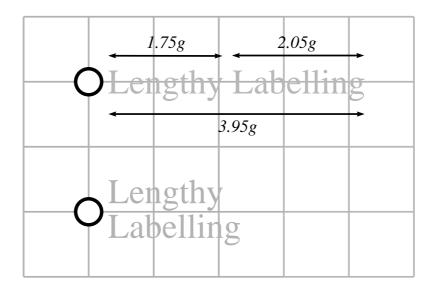


Figure 3.20: Splitting a long, multi-word label of multiple lines. The word "Lengthy" is about 1.75g and "Labelling" is about 2.05g causing the label to be about 3.95g in total. The bottom label shows how the label "Lengthy Labelling" can be split over two lines.

single words that were longer than 0.75lg. Also, labels were still split over several lines even in the case where there is plenty of space for it in the graph. Figure 3.20 shows an example where a long label with two words can be split over two lines.

# 3.9.1 Labelling Criteria

We have implemented a total of seven labelling criteria:

- Label Occlusion Criteria,  $c_{L1}$ ,  $c_{L2}$ ,  $c_{L3}$ . Three criteria that take into account the number of nodes, edges and other labels that intersect labels. See Section 3.9.2.
- Label Position Criterion,  $c_{L4}$ . Places a preference on label positions in the labelling space by weighting each position. See Section 3.9.3.
- Label Position Consistency Criterion,  $c_{L5}$ . Gives preference to labels along a line in the map that consistently appear on the same side of the line. See Section 3.9.4.
- Node Proximity Criterion,  $c_{L6}$ . Considers labels as they come into close proximity to unrelated nodes with the intention of discouraging labels from being positioned too close to other unrelated nodes. See Section 3.9.5.

• Perpendicular Tick Criterion,  $c_{L7}$ . Attempts to ensure that the tick (and therefore the position of the label) for a particular node is perpendicular to the line. See Section 3.9.6.

As with the node movement criteria, the labelling criteria are weighted with individual weightings. The criteria are described in more detail in the following sections.

# 3.9.2 Label Occlusion Criteria, $c_{L1}$ , $c_{L2}$ , $c_{L3}$

These are a set of three criteria which take into account occasions when a label intersects or occludes any other label, node or edge in the graph. As these intersections drastically reduce the readability of the map, it is highly desirable to ensure that they happen as infrequently as possible. However, there may be occasions where the readability of the graph would be improved if a label were allowed to occlude an edge. In these cases, the label may not fit in any position around a node such that it does not occlude an edge without first significantly altering the layout of that part of the graph. As this could have a further impact on the rest of the graph, it would be very difficult to make space for the label  $\bigstar$  why does it follow that it is difficult to make space for a label if it doesn't fit in any position around a node without occlusions? Needs more detail/description to provide a more convincing argument  $\bigstar$ .

The unweighted value of these occlusion criteria is the sum of the number of occlusions. For example, when counting node occlusions, if one label in the graph occluded two nodes, the unweighted value of the node occlusion criterion would be two.

Figure 3.21 shows an example of a graph that has been poorly labelled. In this case, there are seven nodes which have labels and all except one of these labels (label d) occlude other labels, nodes and edges. Table 3.4 shows how many occlusions of each type are present in this example and which labels exhibit occlusions. Notice that, although only there are only two examples of occluding labels, the count for the label-label occlusion is four.

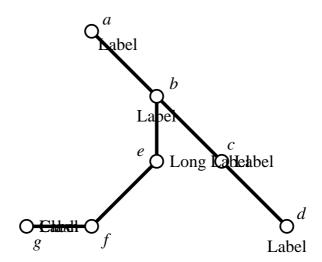


Figure 3.21: Illustration of the label occlusion criteria.

Table 3.4: Number and type of label occlusions for each node in Figure 3.21.

0 1	Number of Occlusions	Nodes with Occlusions
Label-label $(c_{L1})$	4	c,e,f,g
Label-node $(c_{L2})$	1	e
Label-edge $(c_{L3})$	7	a, b  (twice), e  (twice), f, g

#### 3.9.3 Label Position Criterion, $c_{L4}$

★ Is this really a 'weighting'? Aren't you just giving each position a value?

It's slightly confusing to use weighting in this conext as it has been using in Eq 3.8 as a user controllable parameter which multiplies a criterion value. ★

As discussed in Section 3.9, the positions of labels can have a significant impact on the readability of maps. We use a position criterion with a limited search space to allow us to allocate a preferred position to each node label in the graph. Just as with the limitation of the search space for moving nodes (by the restriction of only allowing nodes to be positioned at the intersection of grid lines), we limit the search space for labelling to just a small selection of positions. This is not necessarily an unreasonable limitation as many existing metro maps also restrict themselves to a small number of positions for labels. Another restriction on our search space is that we only consider labels drawn horizontally and not vertically or diagonally. While there are maps which use vertical or—more commonly—diagonal labels, they do tend to be a minority with most maps

preferring to use only horizontal labels. The reasoning behind this is that it is easier to read the text of the labels if they are all drawn in the same orientation—frequent switching of the orientation of labels makes maps harder to read.

A label can occupy any one of the eight locations in the labelling space shown in Figure 3.19. These positions will be referred to by their position with respect to a compass oriented so that north is to the top. Therefore, a label in position 1 is said to be to the east of the node and a label in position 7 is said to be to the south-west of the node. Some label positions are more preferential than others, so we use a set of weightings for each different position in the labelling space. Table 3.5 shows the set of weightings for each position in the labelling space. The label position criterion is then defined as the sum of the position weightings for each label in the graph.

By setting the weightings, it is possible to say which label position is the most preferential and which is least preferential. So for example, a label that is directly to the east of a node (position 1) has a weighting of 1.0 and is therefore the most preferred location, while a label which is oriented to the north-west of a node has weighting 1.8 and is the least preferred location. The values chosen for each of the weightings were determined through experimentation  $\bigstar$  HOW? Not really experimentation  $\bigstar$  with a number of scenarios. Existing metro maps were also inspected to see whether there was a preference for any particular label position. The preference that designers place on label positions is reflected in our weightings. Where there was not a particular preference, the weightings were chosen so that they were similar or even identical (for example, there is no difference between the weightings for north and south labels<sup>1</sup>). This then allows other labelling criteria to have a greater effect on the label position.

#### 3.9.4 Label Position Consistency Criterion, $c_{L5}$

# ★ Does this criterion apply to horizontal and vertical lines? It doesn't say! ★

<sup>&</sup>lt;sup>1</sup>This is because we sometimes want labels to alternate either side of a horizontal line where there is not enough horizontal space between adjacent labels to fit each label. This is discussed with examples in Section 3.9.

Table 3.5: Label Position Weightings. The positions refer to the positions shown in Figure 3.19.

position		weighting
1	east	1.0
2	west	1.1
3	$\operatorname{north}$	1.4
4	south	1.4
5	north-east	1.5
6	south-east	1.6
7	south-west	1.7
8	north-west	1.8

The label position consistency criterion attempts to ensure that all the labels along a line appear on the same side of the line. In other words, the effect of the criterion is to prefer labels which have the same position as the labels of their neighbouring nodes. This has the effect of preferring labels that follow the same side of a line and therefore improving readability. Readability is improved because the labels appear as a list which can be read easily rather than having to switch from one side of the line to the other with your eyes. The criterion is only calculated for labels with exactly two neighbouring nodes—nodes with more than two neighbours would be complex to deal with as the label would have to be consistent with more than one line.

The calculation is fairly simple: for each node in the graph with degree not equal to two, a count is kept of the number of times the position of the label of an adjacent node (if that node has degree less than three) differs to the position of the current node. Figure 3.22 shows an example of poor label position consistency where the unweighted value of the label position criterion would be three (there are differences in label position between Label 1 and Label 2, Label 2 and Label 3, and Label 3 and Label 4).

#### 3.9.5 Node Proximity Criterion, $c_{L6}$

The node proximity criterion addresses the problem whereby a label for an individual node could be positioned such that the node to which it belongs is uncertain. For example, Figure 3.23 shows a label ("Bad Labelling") which may belong to either the

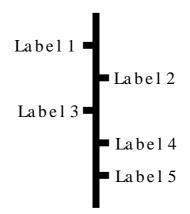


Figure 3.22: Label position consistency.

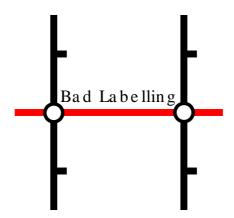


Figure 3.23: An example of ambiguous labelling.

left-hand or the right-hand node. To counter this problem, we introduce a node proximity criterion which works by forcing a label away from a node to which it does not belong.

The node proximity criterion,  $c_{L6}$ , is given by

$$c_{L6} = \sum_{k \in L} \sum_{n \in V, k_n \neq k} \frac{1}{d(k, n)^2}$$
(3.10)

where d(k, n) is a function giving the distance from the closest point on the bounding box of label k to node n. Notice that we are interested in  $n \in V, k_n \neq k$ , that is all nodes in the graph except the one for which the label  $(k_n)$  is the label we are considering (k). In other words, we do not take into account the distance between a label and node that that label belongs to.

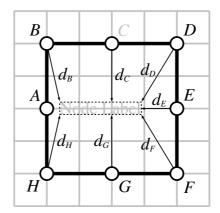


Figure 3.24: Calculation of the node proximity criterion.

In practice, because most nodes in the graph will be some way from the label in question, they will contribute very little to  $c_{L6}$ . We can therefore approximate the contribution of nodes with d(k, n) > x to zero. We use a value of 1 for x.

The example in Figure 3.24 illustrates how the criterion is calculated for a particular node. In particular, it shows how the distance function d(k, n) relates to the bounding box for the label.

#### 3.9.6 Perpendicular Tick Criterion, $c_{L7}$

One of the disadvantages of using a combination of criteria to decide on the position of a label is that it may choose positions that do not fit with the characteristics of many existing metro maps. One such characteristic relates to the labelling of nodes with degree two which form part of a line. As we have seen earlier in Section 3.9.4, in these cases we prefer the labels to follow the same side of the line. If we are drawing the icon that represents a station as a tick or a bar, then that icon should be perpendicular to the line. Figure 3.25 illustrates this point. The left-hand diagram shows a line where the ticks showing stations have been drawn perpendicular to the line. The right-hand diagram shows ticks always drawn straight to the right (labels are positioned to the east). The difference is quite clear—while the labels and ticks for the vertical part of the line remain the same, the perpendicular ticks on the diagonal part of the line are more prominent. The minimum distance between the line and the labels on the diagonal part is also greater when the labels are drawn to the south-east, but the association with

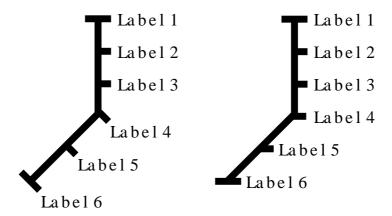


Figure 3.25: Examples of perpendicular tick labels (left) and non-perpendicular tick labels (right)

the relevant tick is not lost. This criterion does not become irrelevant if we use a different device for representing nodes (such as a ring) as the criterion will force the closest part of the label to the line to be furthest from the line but still close to the node.

The example shown in Figure 3.25 also illustrates a problem with regards to labelling nodes on corners of lines. Label 4 is positioned at a point where the line changes direction and as such it is impossible to draw the tick so that it is perpendicular to both parts of the line. It would be possible to draw the tick so that it is as close as possible to being perpendicular to both parts of the line. However, this would require greater flexibility in the positioning of the label. In practice, existing maps avoid this problem either by ensuring that stations are only ever drawn on straight parts of lines (as on the London Underground map), or by not restricting the label position as much as we are, or by not using ticks as station icons.

Although we have called this criterion the perpendicular tick criterion, it still has merit when drawing maps that use different types of station icons. In the case of using a circle as the station icon, the orientation of the icon is irrelevant. However, the position of the label is still important—it is desirable to keep the label as close to the icon as possible but still a reasonable distance from the line.

Calculation of the perpendicular tick criterion is fairly straightforward. Figure 3.26 shows two examples of nodes being labelled—the left-hand example has a label drawn to the east and the right-hand example has a label being drawn to the south-east. In

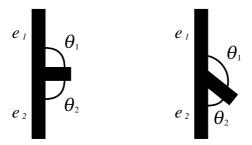


Figure 3.26: Calculating the perpendicular tick criterion

both examples,  $e_1$  and  $e_2$  are the edges and  $\theta_1$  and  $\theta_2$  are the angles between the tick and  $e_1$  and  $e_2$  respectively. The unweighted value of this criterion for a single node, v, is the absolute difference between the two angles:

$$c_v = |\theta_1 - \theta_2| \tag{3.11}$$

The total value for all nodes, V, in the graph is therefore

$$c_{L7} = \sum_{v \in V} |\theta_1 - \theta_2| \tag{3.12}$$

In the examples in Figure 3.26, the value of  $c_{L7}$  for the left-hand example will be zero as both  $\theta_1$  and  $\theta_2$  are equal at 90°. This indicates that the tick is positioned to be perpendicular to the edges  $e_1$  and  $e_2$ . In the right-hand example, the value of  $c_{L7}$  is  $135^{\circ} - 45^{\circ} = 90^{\circ}$ . This shows that the label is not perpendicular to either  $e_1$  or  $e_2$ .

#### 3.9.7 Labelling Criteria Weightings

Each criterion is independently weighted depending on the importance associated with that criterion. The values for weightings were determined through experimentation with various examples.  $\bigstar$  More detail required on how the weightings were chosen. Why are some of the weightings in the example the same?  $\bigstar$ 

Criterion	$c_{Li}$	Weighting $w_{Li}$	$w_{Li}c_{Li}$ for graph
			in Figure 3.27
$c_{L1}$	2.0000	100.0000	200.0000
$c_{L2}$	0.0000	100.0000	0.0000
$c_{L3}$	2.0000	100.0000	200.0000
$c_{L4}$	24.0000	3.0000	72.0000
$c_{L5}$	10.0000	12.0000	120.0000
$c_{L6}$	70.1923	1.0000	70.1923
$c_{L7}$	7.8540	10.0000	78.5400

Table 3.6: Typical label position criteria weightings.

The sum of the weighted criteria for labelling,  $m_L$ , is given by

$$m_{L} = w_{L1}c_{L1} + w_{L2}c_{L2} + w_{L3}c_{L3} + w_{L4}c_{L4} + w_{L5}c_{L5} + w_{L6}c_{L6} + w_{L7}c_{L7}$$

$$= \sum_{i=1}^{7} w_{Li}c_{Li}$$
(3.13)

As with the node movement critera weightings, the values for the  $w_{Li}$  can be modified by the user depending on the characteristics of the particular metro map being drawn.

Table 3.6 shows a typical set of weightings for each of the labelling criteria and the values as applied to the example in Figure 3.27. This example has several labels in sub-optimal positions (such as "Label B" and "Label E") in order to illustrate the calculation of the labelling criteria. As with the node criteria, it is very difficult to find a set of weightings that balance out in many cases. It is an inherent problem with optimization algorithms that local minima exist in the search space, so no one set of weightings would be applicable in every single case.

The total of the weighted criteria in Figure 3.13 is

$$m_L = 200.0000 + 0.0000 + 200.0000 + 72.0000 + 120.0000 + 70.1923 + 78.5400$$
  
= 740.7323 (3.14)

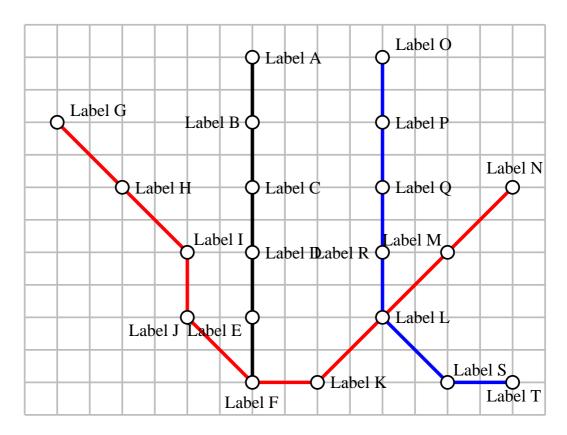


Figure 3.27: Simple example of label criteria for a complete graph. The labels are shown in a sub-optimal position to illustrate the calculation of the criteria. The values for each criterion for this graph are shown in Table 3.6.

# 3.10 Moving Labels

Labelling for all the nodes in the graph was performed once during each iteration, after all of the individual nodes had been moved. We could have attempted to label the map at the same time as nodes were being moved, but due to the large amount of computation required this proved to be excessively slow. Potential label positions are tested in the same way that potential node locations were tested when moving nodes; the sum of the weighted labelling criteria is calculated for each position and the one with the best improvement to the initial label position is chosen.

### 3.11 Combining Node Movement and Labelling Criteria

★ This section doesn't make sense if it not in context. Is this the way the formulae were actually summed together? There is no discussion of possible weighting ★

The total of the weighted node movement criteria,  $m_N$ , and labelling criteria,  $m_L$ , can be combined to produce a total sum for the entire graph. This total,  $m_T$ , is given by

$$m_T = m_N + m_L$$

$$= \sum_{i=1}^{6} w_{Ni} c_{Ni} + \sum_{j=1}^{7} w_{Lj} c_{Lj}$$
(3.15)

### 3.12 Conclusion

★ Chapter needs a firm conclusion ★

# Chapter 4

# Clustering and Partitioning

- ★ This chapter is not well motivated enough. It is not clear why this additional process is needed—what problems are there if this process is not done? The reader needs several examples of problematic situations. ★
- ★ Is there any milage in splitting the nodes into subsets according to geography and optimising the subgraphs and then using this as a "good" initial setup? ★

When experimenting with our initial set of criteria for moving individual nodes we encountered many situations where we would ideally like to be able to move several nodes together at the same time. We needed to find some way of clustering the graph into groups of nodes that could be moved together so that we had some chance of improving the graph with respect to the weighted node movement criteria. We used three different methods to find clusters of nodes:

- clustering based on overlength (or underlength) edges
- clustering based on bends in lines
- clustering based on partitioning the graph into two parts that can be moved closer together

Once clusters have been identified, they were moved in exactly the same way that individual nodes were moved  $\bigstar$  Vague: individual nodes are moved to neighbouring grid intersections—what happens with clusters?  $\bigstar$  with the only difference being that rather than moving one node at a time, all the nodes in the cluster are moved. Indeed, it was actually possible to treat moving single nodes as moving clusters with only one node.

- $\bigstar$  What is the difference between clustering and partitioning? (Partitioning == specific type of clustering)  $\bigstar$ 
  - $\bigstar$  What is the difference between a cluster and a partition? There isn't  $\bigstar$

This chapter will discuss some of the problems that caused us to investigate moving groups of nodes and then detail the process by which we identified clusters in the graphs using the three methods listed above.

### 4.1 Clustering Overlength Edges

### 4.1.1 Why Cluster Overlength Edges?

The biggest problem that we encountered when experimenting with our node movement criteria was that of long edges that refused to reduce in length. We define overlength edges as being edges which are longer than lg where g is the grid spacing and l is the preferred multiple of grid spacings for an edge. Figure 4.1(a) shows such an example where edge AB is too long. The optimal position for nodes B, C and D is as shown in (d). However, if we only allow one node to move at a time, it is impossible for this optimum to be found. Take the case of trying to move node B towards A as shown in (b): while this is a slightly more optimal location with regards to the length of edges AB and BC, it introduces another turn in the line ABCD and edge BC is no longer at AB0 diagonal. This would increase the line straightness and octilinearity criteria in this new position for B.

One way to approach this problem without requiring any particular modifications to our method would be to increase the weighting for the edge length criterion so that the new position for B is more optimal even considering the increase in line straightness

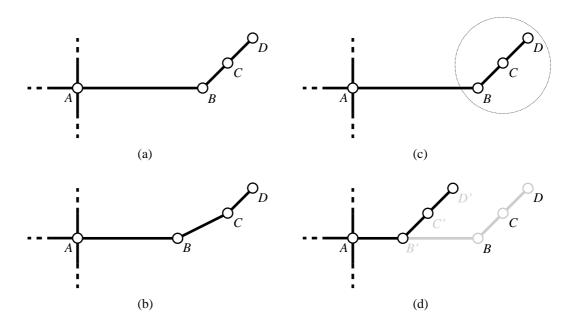


Figure 4.1: Clustering overlength edges. The edge AB is too long (a), but it is not possible to reduce the length of this edge (b) without moving nodes B, C and D (c) at the same time (d).

and octilinearity criteria. However, for this to be the case, the edge length criterion weighting must be increased by several orders of magnitude to the point where the length of edges completely overwhelms all of the other criteria.

Another idea would be to impose some other kind of weighting based on the graph theoretic distance that a node is from the "centre" of the graph. This extra weighting would apply to individual nodes and edges and would decrease as the graph theoretic distance from the centre of the graph increased. In the example in Figure 4.1, the length of edge AB would contribute to the edge length criterion to a greater extent than the edge BC. It would then be possible to potentially move B closer to A without having to increase the overall weighting for the edge length criterion to such a great extent.

However, both of these solutions still rely on the movement of individual nodes. In our example, we could well need three iterations to move all three nodes B, C and D to their optimal positions as shown in (d). It is also undesirable to have to increase criteria weightings significantly in order to overcome local minima as this could well have a detrimental effect on the rest of the graph.

#### 4.1.2 Identifying Clusters Based On Overlength Edges

Our first implementation of an algorithm to find clusters of nodes only considered clusters separated by a single overlength edge. We later extended our algorithm to handle clusters separated by multiple overlength edges—this enhancement is discussed in the following section.

Our algorithm works by finding the partitions in the graph that would be created if a single overlength edge was cut. It can be summarised as follows:

- 1. Find the set of edges  $\Phi \subset E$  such that the length of each edge  $e \in \Phi > lg$ .
- 2. For each edge  $e \in \Phi$  being connected to nodes X and Y:
  - (a) Perform a breadth-first search of the graph starting from X but not following e. We can determine if there is a cluster of nodes separated by the single edge e if we have exhausted the search for nodes without encountering Y. If at any point in the search from X we find Y, then the search should be terminated.
  - (b) Repeat the previous step starting from Y and terminating the search if we encounter X.

For each cut made in the graph we only need to remember the partitions with fewest nodes (or an arbitrary partition if both partitions are of equal size). The smaller of the two paritions would require fewer criteria calculations when potential locations for the nodes are being tested.

To illustrate this algorithm, consider the graph in Figure 4.2. In this graph just one edge, CE, is overlength and we wish to find the two clusters of nodes created by cutting the graph at this edge. Performing the breadth-first search from node C will find the four nodes to the left of CE giving the cluster  $\{A, B, C, D\}$ . Similarly searching from E will find the cluster  $\{E, F, G, H\}$ .

Now consider the graph shown in Figure 4.3. In this case, an extra loop has been added between nodes D and H. As before, we wish to try cutting the graph at the overlength edge CE. Attempting the depth-first search from C results in potentially

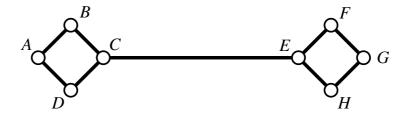


Figure 4.2: Example of clustering with a single overlength edge. An attempt to cut the graph based on the edge CE will produce two clusters of nodes:  $\{A, B, C, D\}$  and  $\{E, F, G, H\}$ 

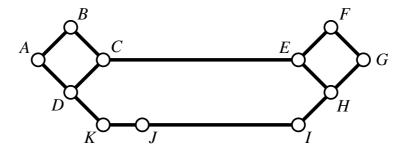


Figure 4.3: Example of failed clustering with a single overlength edge. In this case, cutting the graph based on the edge CE will not find two clusters of nodes.

eight iterations (shown in Table 4.1), but our algorithm would stop at the seventh iteration as we would be adding E, the node at the opposite end of the edge CE, to the cluster. The loop from D to H causes this to happen, and as can be seen from the eighth iteration, would result in the entire graph being treated as a single cluster. This would clearly not achieve our aim of attempting to cluster the graph based on a single multiple edge.

Graphs as in Figure 4.3 can be clustered, but we must take into account cases of multiple overlength edges. This is discussed in the next section.

#### 4.1.3 Multiple Overlength Edges

There are a number of cases when finding single overlength edges with which to partition the graph is not enough. Figure 4.4(a) shows such an example where the cluster of C, D and E is separated from the rest of the graph by the overlength edges AE and BC. It would be better if the cluster could be moved to the new positions as shown by C', D' and E' in (b).

Search	Next	Cluster	Remaining
Iteration	Node(s)		Nodes
	{}	{}	$\{A, B, C, D, E, F,$
			G, H, I, J, K
1	$\{C\}$	$\{C\}$	$\{A, B, D, E, F, G, \}$
			H, I, J, K
2	$\{B,D\}$	$\{B,C,D\}$	$\{A, E, F, G, H, I,$
			$J,K\}$
3	$\{A,K\}$	$\{A, B, C, D, K\}$	$\{E, F, G, H, I, J\}$
4	$\{J\}$	$\{A, B, C, D, J, K\}$	$\{E, F, G, H, I\}$
5	$\{I\}$	$\{A, B, C, D, I, J, K\}$	$\{E, F, G, H\}$
6	$\{H\}$	$\{A, B, C, D, H, I,$	$\{E, F, G\}$
		$J,K\}$	
7	$\{E,G\}$	$\{A, B, C, D, E, G, H,$	$\{F\}$
		$I,J,K\}$	
8	$\{F\}$	$\{A, B, C, D, E, F, G,$	{}
		HIJK	

Table 4.1: Breadth-first search of the graph in Figure 4.3, starting at node C.

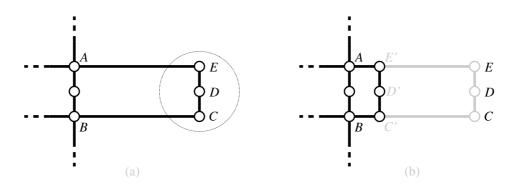


Figure 4.4: Clustering multiple overlength edges. The edges AE and BC are too long (a) and it is only possible to reduce the length of these edges by moving nodes C, D and E at the same time (b).

Search	Next	Cluster	Remaining
Iteration	Node(s)		Nodes
	{}	{}	$\{A,B,C,D,E,F,G,H,I,J,K\}$
1	$\{C\}$	$\{C\}$	$\{A,B,D,E,F,G,H,I,J,K\}$
2	$\{B,D\}$	$\{B,C,D\}$	$\{A, E, F, G, H, I, J, K\}$
3	$\{A,K\}$	$\{A, B, C, D, K\}$	$\{E, F, G, H, I, J\}$
4	$\{J\}$	$\{A, B, C, D, J, K\}$	$\{E,F,G,H,I\}$
Search stops here as edge $IJ$ is overlength.			

Table 4.2: Breadth-first search of the graph in Figure 4.3 taking multiple overlength edges into account, starting at node C.

The algorithm presented in the previous section is unable to handle such circumstances. It relies on the overlength edge being the only edge to be cut in order to partition the graph into two disjoint subgraphs. This was illustrated using the example in Figure 4.3. We now need to extend this to take account of having multiple edges to cut to create two disjoint subgraphs.

To extend our existing algorithm, first recall that the existing termination condition was whether we found the node at the opposite end of the edge to the one we started from. If it was not for this termination condition, we could end up searching the entire graph. We now add a new condition to the algorithm:

• If at any point during the search another overlength edge is encountered, do not search beyond that edge.

Illustrating this with the example in Figure 4.3 and trying to cut the graph along the edge CE now produces a different result. The iterations of the search starting at node C are shown in Table 4.2. The search starts off as before but will not search beyond node J as the edge IJ is overlength.

A similar search can be performed starting at the other end of edge CE, E. This results in a cluster of nodes consisting of  $\{E, F, G, H, I\}$ . Note that an attempt to cluster the graph by cutting edge IJ will, in this case, produce exactly the same two clusters.

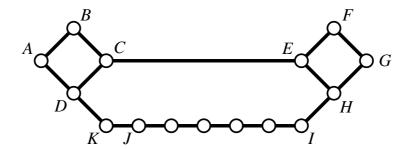


Figure 4.5: Example of failed clustering with overlength edges.

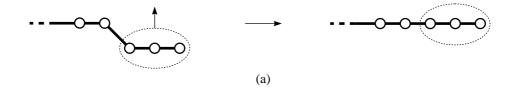
However, circumstances exist where it remains impossible to find two distinct clusters. An example of such a scenario is shown in Figure 4.5. This graph is much the same as the one in Figure 4.3 except that the overlength edge IJ has been replaced by nodes and edges such that no overlength exists in this part of the graph. The overlength edge CE remains, but it should be fairly obvious that trying to cut this edge will result in a similar outcome as before other multiple overlength edges were added as a condition to the search algorithm. The search will follow the loop from D to H before encountering the other end of the edge CE. Having said this, in this example, it would not be possible to make an improvement such that the edge CE could be reduced in length. This is therefore an example of a local minimum that can not be improved using our multicriteria optimisation method.

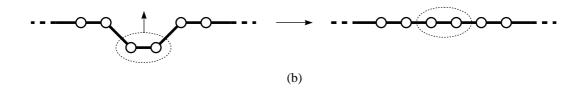
# 4.2 Clustering Non-straight Lines

#### 4.2.1 Why Cluster Non-straight Lines?

Related to the problem of overlength edges (discussed in the previous section) is that of lines which contain short deviations or kinks. Figure 4.6 shows examples of the kinds of common scenarios where these deviations become apparent. There are two main explanations as to why these deviations are so common:

1. The deviations become manifest when fitting a slightly off-straight line to the grid (as shown in (a) and (b) in Figure 4.6).





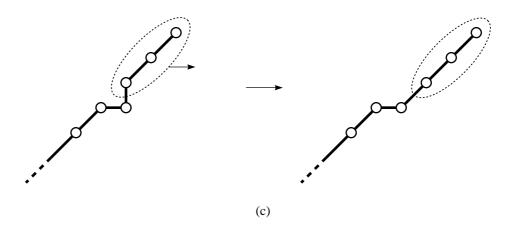


Figure 4.6: Finding non-straight lines. Potential clusters are highlighted by dashed areas. The right-hand side of (a), (b) and (c) show the result of moving the indicated clusters of nodes to a more optimal location. In the case of (c), the resulting movement of the cluster to the right introduces the possibility of reclustering in a similar manner to (a).

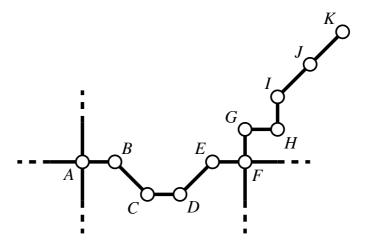


Figure 4.7: Clustering nodes to find non-straight lines. Ultimately, six clusters will be identified in this graph:  $\{BC\}$ ,  $\{CD\}$ ,  $\{DE\}$ ,  $\{GH\}$ ,  $\{HI\}$  and  $\{IJK\}$ .

2. Three nodes are too close together to fit onto the grid without the middle node being offset relative to the rest of the line (as shown in (c) in Figure 4.6).

The line straightness criterion (Section 3.6.5) helps in the case where a single node can be moved into a vacant grid intersection. For example, if the cluster indicated in Figure 4.6(a) contained only one node, this scenario can be catered for in our single-node-movement method. We would like to be able to apply the line straightness criterion to scenarios where more than one node should be moved simultaneously. As such, we need some way of being able to identify the cluster of nodes to be moved as one.

#### 4.2.2 Identifying Clusters Based on Non-straight Lines

Our method for identifying clusters of nodes that can be moved to improve non-straight lines is very straightforward. Figure 4.7 shows a simple example that will be used to illustrate how these clusters are found. First off, as we are only considering improving deviations in lines, we only need to look at nodes which have exactly one or two neighbours. This means that nodes A and F are discounted from forming part of a cluster from the outset (and could even be removed from the graph while we are searching for clusters). Clusters are then identified by finding the minimum set of connected nodes which are collinear. In the example, this produces five clusters of two nodes and one cluster of three nodes:  $\{BC\}$ ,  $\{CD\}$ ,  $\{DE\}$ ,  $\{GH\}$ ,  $\{HI\}$  and  $\{IJK\}$ .

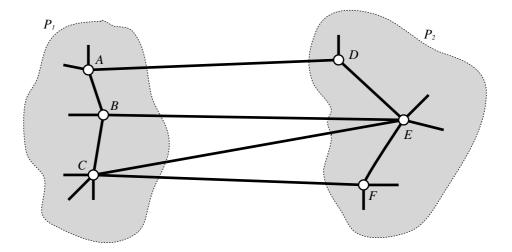


Figure 4.8: Partitioning a graph into two. The edges AD, BE, CE and CF are all too long but the only way of shortening them is to move either the left-hand partition  $P_1$  or the right-hand partition  $P_2$ . Both  $P_1$  and  $P_2$  contain other overlength edges that would stop the method for clustering based on overlength edges from finding these partitions.

# 4.3 Partitioning

Clustering overlength edges works quite well for simple scenarios and certainly in cases where there is just a single overlength edge separating clusters of nodes. However, more complex situations can arise where a visual inspection of the graph would highlight cases where improvements can be made. These improvements to the visual aesthetics of the graph can be made by partitioning it into two parts and moving one of these partitions. These partitions usually cannot be discovered using the method for finding clusters by multiple overlength edges (Section 4.1.3) as the partition itself will more than likely contain more overlength edges. It would therefore be highly unlikely that the graph would be partitioned into exactly two clusters using the multiple overlength edges clustering method.

Figure 4.8 shows a simple example of partitioning a graph into two halves,  $P_1$  and  $P_2$ . There are four edges in the graph that we would like to identify as separating the graph into two partitions: AD, BE, CE and CF. However, the method for finding clusters based on overlength edges is not likely to find  $P_1$  and  $P_2$  as both of these partitions contain other edges that are overlength. If we are able to identify  $P_1$  and  $P_2$ , we can

move one of these partitions as a cluster, causing the edges AD, BE, CE and CF to shorten.

Our approach to finding partitions in the graph can be summarised as follows:

- 1. Preprocess the graph to remove edge crossings, unwanted multiple edges and trailing lines. Edge crossings must be removed as the graph must be planar in order to find the faces. If there are any edge crossings, they can be replaced by dummy nodes for the duration of this process. Multiple edges (where there are several edges between two nodes) should be removed or at least considered as a single edge as this would otherwise affect the ability to identify faces. Trailing lines do not necessarily have to be removed at a preprocessing stage as they can be discovered and ignored as a by-product of the process of finding faces.
- 2. Find the set of faces (discussed in Section 4.3.1).
- 3. Use the set of faces to derive the dual graph (discussed in Section 4.3.2).
- 4. Diminish the dual graph to remove unnecessary edges (discussed in Section 4.3.3).
- 5. Cut the dual graph by finding edges which are *most opposite* each other (discussed in Section 4.3.4).

The following sections as indicated above detail the process of finding all the available partitions in a planar graph.

#### 4.3.1 Finding Faces

When finding faces in a planar graph, we are essentially looking for minimal cycles that enclose a region of the graph. To do this, we must first ignore any multiple edges between any two nodes (i.e. where more than one line passes between two adjacent stations) by treating them as a single edge. We describe the method that we use to find the set of faces in the graph as we will reuse the idea of traversing edges in Section 4.3.4 when finding cuts in the graph.

We define a traversal of an edge as being either to-from or from-to. For example, an edge between nodes A and B can be traversed to A from B (to-from: denoted as  $\overleftarrow{AB}$ )

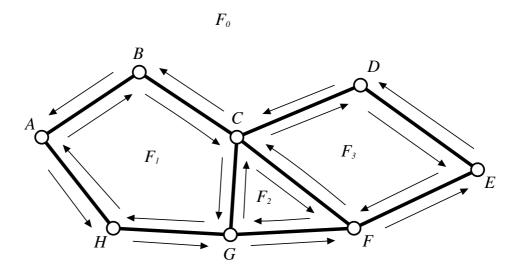


Figure 4.9: Finding faces. Arrows indicate the direction of traversal of each edge, for example, an arrow from A to B represents the from-to traversal  $\overrightarrow{AB}$ .

or from A to B (from-to:  $\overrightarrow{AB}$ ). When an edge has been traversed in both directions, we describe that edge as being *fully traversed*. The ultimate aim of our face-finding algorithm is to ensure that every edge has been fully traversed.

To find a face F, take any edge,  $e_0$ , that has not yet been fully traversed. We then follow that edge either to-from or from-to by consistently taking the next edge in an anticlockwise direction,  $e_n$ , at each node adding each edge to F. When  $e_n = e_0$  we have found the minimal cycle that encloses F.

To illustrate this with an example, consider the graph shown in Figure 4.9. This example shows the direction of traversal of each edge in order to find each of the faces. For example, face  $F_1$  can be found by starting at edge AB and following it through edges BC, CG, GH and AH. We can define each face in the graph by also giving the direction of traversal of each edge:

$$F_{0} = \{\overrightarrow{AB}, \overrightarrow{AH}, \overleftarrow{GH}, \overleftarrow{FG}, \overleftarrow{EF}, \overleftarrow{DE}, \overleftarrow{CD}, \overleftarrow{BC}\}$$

$$F_{1} = \{\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CG}, \overrightarrow{GH}, \overleftarrow{AH}\}$$

$$F_{2} = \{\overrightarrow{CF}, \overrightarrow{FG}, \overleftarrow{CG}\}$$

$$F_{3} = \{\overrightarrow{CD}, \overrightarrow{DE}, \overrightarrow{EF}, \overleftarrow{CF}\}$$

Notice that  $F_0$  is the outside face. Every edge in  $F_0$  is therefore also an *outside edge*.

In the previous section, we mentioned that trailing edges can be removed implicitly during the face finding algorithm. A trailing line is a connected subset of edges forming a line containing only nodes with degree one or two. To remove these trailing edges from the graph during the face finding algorithm, we should remove edges  $e_n$  and  $e_{n+1}$  if  $e_n = e_{n+1}$ . In other words, if the node at the to-end of  $e_n$  has degree one.

#### 4.3.2 Deriving the Dual Graph

The complete dual graph,  $G^*$ , of a planar graph, G, is the graph with nodes representing faces and with nodes being adjacent iff the corresponding faces in G are adjacent (i.e. the faces share an edge). The number of edges between any two adjacent nodes  $F_1$  and  $F_2$  in  $G^*$  indicates the number of edges along the interface between  $F_1$  and  $F_2$ . Outside edges are those edges in  $G^*$  that are connected to the node that represents the outside face,  $F_0$ .

We can derive the dual graph from the sets of faces. An example of the derivation of a dual graph is shown in Figure 4.10.

#### 4.3.3 Diminishing the Dual Graph

When finding appropriate cuts in the graph, it is not desirable to include in the dual graph those edges which are already at the preferred length, lg. The reason for this is that in finding cuts we are trying to reduce the length of many edges at the same time—if any edge is already at the preferred length then it will not be possible to move either partition without shortening the edge with preferred length. In diminishing the dual graph,  $G^*$ , we remove any edges that represent adjacent faces in G by way of an edge with length less than or equal to lg.

#### 4.3.4 Finding Partitions

We considered a number of different strategies for partitioning the graph into two halves. All of these methods involved finding a path through the dual graph,  $G^*$ . A cycle in  $G^*$  can be considered as a cut through the planar graph G. In cutting G, we clearly need to find a path in  $G^*$  that joins two outside edges in G—in other words, a cycle

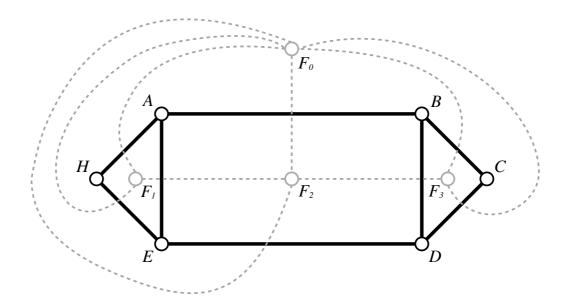


Figure 4.10: Example of a planar graph, G, and the complete dual graph,  $G^*$ . The planar graph  $G = \{A, B, C, D, E, H\}$  is shown in black and the dual graph,  $G^* = \{F_0, F_1, F_2, F_3\}$ , is shown in grey with dotted edges.

in  $G^*$  that starts at the outside face  $F_0$ , passes through some other nodes in  $G^*$  and returns to  $F_0$ . Figure 4.10 shows an example of a graph,  $G = \{A, B, C, D, E, H\}$ , and its complete dual graph,  $G^* = \{F_0, F_1, F_2, F_3\}$ . An illustration of a potential cut in this graph is shown in Figure 4.11, starting at  $F_0$ , passing through  $F_1$  (cutting edge AB) and back to  $F_0$  (cutting edge DE). In this case,  $P = \{AB, DE\}$ .

The cut must be chosen with regard to how the partitions will respond to the movement criteria. If a cut is chosen which includes many edges of differing orientation, it is likely to be very difficult for either partition of the graph to move relative to one another. This becomes clear when you consider that when a partition is moved, some edges of the cut may end up being longer than before and other edges may end up becoming shorter. A more reliable approach would be to ensure that the cut contains as many edges as possible that can be reduced in length. So, rather than have edges of differing orientation, we try to ensure that the cut includes edges which ideally have the same orientation. This means finding a set of edges for a cut which are parallel to each other—or at least as parallel as possible. For example, a cut should include only parallel edges but where this isn't possible, the cut should contain edges as close as parallel as possible. In this case, a cut may contain, say, some number of horizontal

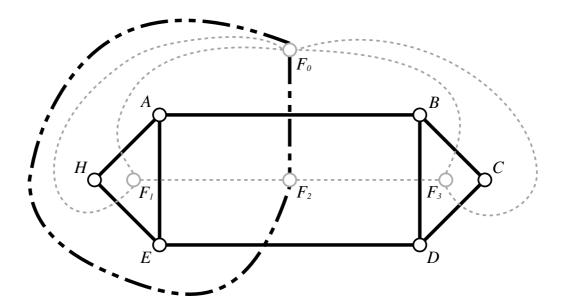


Figure 4.11: Example of a cycle in the dual graph equating to a cut in the planar graph represented by the dual graph. The cycle  $\{F_0, F_2, F_0\}$  in the dual graph  $G^*$  (shown in bold) equates to the cut  $P = \{AB, DE\}$  in the planar graph G.

edges and one diagonal edge. Figure 4.12 illustrates this: the cut in (a) is not likely to lead to partitions that can be improved using the optimiser while the cut in (b) consists of edges which are roughly parallel (although not exactly as it is not possible to cut the graph using just parallel edges other than horizontally through the middle) which means that the partitions are more likely to be improved by the optimiser.

#### Entry and Exit Edges

In the process of finding a cut through a graph, we use the terms *entry edge* and *exit* edge to refer to the outside edges where the cut starts and finishes. Entry and exit edges also apply during the process of finding a cut, but in this case we are trying to find the entry and exit edges for each face that the cut passes through. This section is concerned with finding the entry and exit edges for a particular face.

There are a number of naïve approaches that can be taken to find an exit edge (such as taking the edge which is furthest away from the entry edge either geometrically or by graph theoretic distance). However, the whole purpose of finding an exit edge is that a suitable cut can be formed in the graph to allow the graph to be partitioned into two. As discussed earlier, this means that it is necessary for the cut to contain edges which

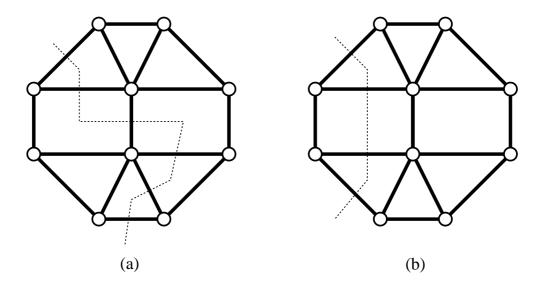


Figure 4.12: Example of cuts which are likely to lead to poor and good partitions. The example in (a) is likely to lead to poor partition selection while the example in (b) is likely to lead to a better partition selection.

are parallel or nearly parallel to each other. A naïve approach will not take this into account thereby leading to cuts containing edges with many different orientations and with little likelihood of success.

Our approach is more considered in that it takes into account the direction of each edge in a cycle as well as the orientation of each edge in relation to the entry edge. It is, therefore, more likely that the set of edges forming the cut are not only parallel but also more likely to form a straight cut through the graph. In the task of finding an exit edge, we start with the entry edge,  $e_0$ , and use the set of edges that describe the cycle forming the edge of the face, F.

We have to consider each edge in F as being directed such that the direction should follow around the cycle. The choice whether to follow the direction around a face clockwise or anticlockwise is arbitrary, but that choice must be consistent. Figure 4.13 shows a cycle where the clockwise direction of each edge has been marked with an arrow.

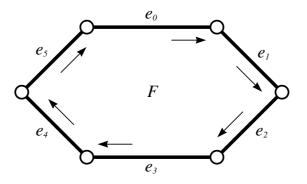


Figure 4.13: Finding the exit edge.

Once we have determined the direction of each edge in F, we calculate the normalised unit vector,  $\mathbf{E}_n$ , that represents the direction of each edge,  $e_n$ :

$$e_0: m{E}_0 = egin{bmatrix} 1 \ 0 \end{bmatrix}$$
  $e_1: m{E}_1 = egin{bmatrix} 1 \ -1 \end{bmatrix}$   $e_2: m{E}_2 = egin{bmatrix} -1 \ -1 \end{bmatrix}$   $e_3: m{E}_3 = egin{bmatrix} -1 \ 0 \end{bmatrix}$   $e_4: m{E}_4 = egin{bmatrix} -1 \ 1 \end{bmatrix}$   $e_5: m{E}_5 = egin{bmatrix} 1 \ 1 \end{bmatrix}$ 

We then calculate the sum of the entry edge vector,  $E_0$ , and each other edge vector and find its magnitude. The exit edge is determined by selecting the minimum of these magnitudes. Comparing the edges in this manner means that any edge which is on the

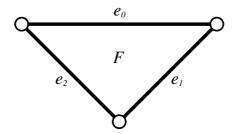


Figure 4.14: Finding the exit edge in a triangular cycle. Both edges  $e_1$  and  $e_2$  are candidates for being the exit edge but only one can be selected.

"other side" of F and also parallel to  $e_0$  is considered as an exit edge.

$$|\mathbf{E}_0 + \mathbf{E}_1| = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \sqrt{5}$$

$$|\mathbf{E}_0 + \mathbf{E}_2| = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} = 1$$

$$|\mathbf{E}_0 + \mathbf{E}_3| = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0$$

$$|\mathbf{E}_0 + \mathbf{E}_4| = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 1$$

$$|\mathbf{E}_0 + \mathbf{E}_5| = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \sqrt{5}$$

As can be seen from these calculations, if the entry edge in Figure 4.13 is  $e_0$ , the corresponding exit edge is  $e_3$  as the magnitude of the resulting vector is 0. This can clearly be seen in Figure 4.13 as  $e_3$  is both parallel and has opposite direction to the entry edge  $e_0$ .

In the case where there is more than one potential exit edge, a choice has to be made as to which one should be used. The triangular cycle shown in Figure 4.14 poses such a dilemma.

$$e_0: m{E}_0 = egin{bmatrix} 1 \ 0 \end{bmatrix}$$
  $e_1: m{E}_1 = egin{bmatrix} -1 \ -1 \end{bmatrix}$   $e_2: m{E}_2 = egin{bmatrix} -1 \ 1 \end{bmatrix}$ 

It is obvious by inspecting the cycle that taking edge  $e_0$  as the entry edge will result in both edges  $e_1$  and  $e_2$  being candidates for the exit edge. This visual inspection is reinforced by calculating the magnitude of the sum of the vectors as follows:

$$|\mathbf{E}_0 + \mathbf{E}_1| = \begin{vmatrix} 1 \\ 0 \end{vmatrix} + \begin{vmatrix} -1 \\ -1 \end{vmatrix} = \begin{vmatrix} 0 \\ -1 \end{vmatrix} = 1$$
 $|\mathbf{E}_0 + \mathbf{E}_2| = \begin{vmatrix} 1 \\ 0 \end{vmatrix} + \begin{vmatrix} -1 \\ 1 \end{vmatrix} = \begin{vmatrix} 0 \\ 1 \end{vmatrix} = 1$ 

We have three potential strategies for resolving this dilemma:

- 1. Make an arbitrary decision. It does not matter which candidate exit edge we choose as both edges are equally preferable.
- 2. Take the edge which is furthest away geometrically. The geometric distance between the midpoints of the entry edge and each candidate exit edge is calculated and the one which is furthest away from the entry edge is selected as the exit edge.
- 3. Take the edge which has greater graph-theoretic distance from the entry edge. The minimum graph theoretic distance from the entry edge (remembering to follow the cycle both ways) to each candidate exit edge is found and the edge which is furthest away from the entry edge is selected as the exit edge.

Each of these strategies will fail and succeed in differing circumstances. Hence we used the least computationally expensive option and just made an arbitrary selection of the exit edge from the candidates.

#### **Cutting Many Faces**

We can now apply the method for finding entry and exit edges of a face discussed in the previous section to a graph containing several faces. Recall that in finding a cut in the graph we are trying to find a cycle in the dual graph, starting from the node that represents the outside face,  $F_0$ , passing through some inside faces and returning to  $F_0$ .

The cutting algorithm to find the set of edges in a cut,  $\Phi$ , for a graph  $G = \{V, E\}$  proceeds as follows:

#### ★ formalise this algorithm using algorithmic ★

- 1. Find the set of outside edges  $E_{\text{out}} \subset E$ .
- 2. For each outside edge  $e_0 \in E_{\text{out}}$ :
  - (a) Add  $e_0$  to  $\Phi$ .
  - (b) Treat  $e_0$  as the entry edge into the next inside face F.
  - (c) Find the exit edge,  $e_{\text{exit}}$  of F.
  - (d) If  $e_{\text{exit}} \in E_{\text{out}}$ , add  $e_{\text{exit}}$  to  $\Phi$  and terminate as we have reached the outside face again.
  - (e) Otherwise, treat  $e_{\text{exit}}$  as  $e_0$  and repeat step 2.

However, this algorithm may never terminate. The way in which the exit edge is determined when cutting the graph can lead to situations where an infinite loop can occur. Such a situation is illustrated in Figure 4.15. In this example, the initial starting edge is  $e_0$  and the cut continues through faces  $F_1$ ,  $F_2$  through to  $F_8$ . At this point, the cutting algorithm would take us back through  $F_1$  and on in to  $F_2$ —thereby resulting in a loop that never finds another outside edge. To resolve this, an extra terminating condition can be added to the cutting algorithm which causes it to stop if  $e_{\text{exit}} \in E_{\text{out}}$  or  $e_{\text{exit}} \in \Phi$ .

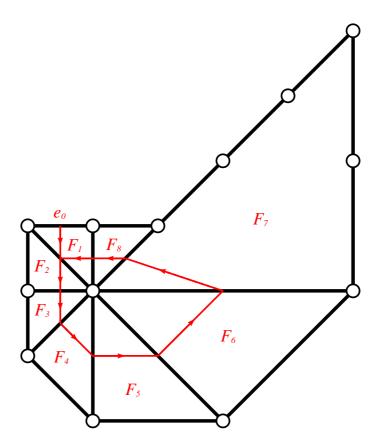


Figure 4.15: Non-terminating cutting algorithm. If the initial entry edge is  $e_0$ , the cut as indicated by the red lines with arrows will result in a perpetual loop in the cutting algorithm.

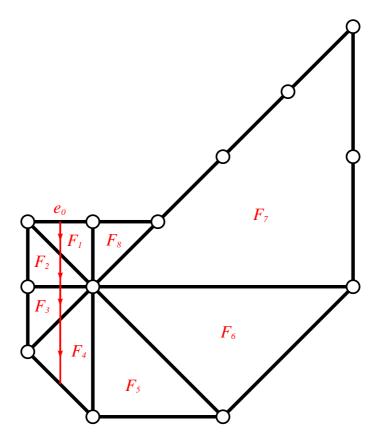


Figure 4.16: Modified cutting algorithm to remove the risk of an infinite loop in the cutting algorithm. The cut is indicated by the red lines with arrows.

As well as a non-terminating algorithm, Figure 4.15 also illustrates a case where the cut may not include edges which are parallel or mostly parallel. In fact, the cut illustrated includes almost no two edges with the same orientation. This comes about because each face we are not taking into account the orientation of the outside edge where the cut was started; just the current entry edge to that face is considered when finding the exit edge. We can modify the cutting algorithm subtly to improve this: instead of comparing each edge around a face with the vector of the entry edge to that face, we compare each edge with the vector of the outside edge where the cut was started. This gives the benefit of preferring much straighter cuts through the graph, as illustrated in Figure 4.16. Also notice that the cycle that previously stopped the algorithm from terminating is no longer apparent.

# 4.4 Conclusion

★ Chapter ends rather abruptly—needs a conclusion and discussion as to how this clustering and partitioning was used. ★

# Chapter 5

# Implementation

This chapter explains how we implemented a system to automatically lay out metro maps according to the methods introduced in the previous two chapters. The software that was written to support our method is described, including the format used for data to represent metro maps. We then provide a discussion regarding how appropriate criteria weightings were selected for each diagram as well as an analysis on particular local minima.

# 5.1 Software Implementation

We implemented our method using software written by us using the Java programming laguage [Jav08]. Java is an object-oriented programming language which makes it easy and logical to encapsulate functionality regarding each of the "objects" in a metro map in individual classes. We modelled graph as a Graph object which contained a collection of Nodes and a collection of Edges. Nodes contained references to the incident Edges and Edges contained references to the two endpoint Nodes. A separate HillClimber class encapsulated the bulk of the functionality for each part of our method, including methods to move nodes and to determine clusters and partitions. Other classes represented station labels (Label) and faces (Face) used during the clustering stage (Section 4.3).

A Java Swing [JS008] graphical user interface allowed us to visualise the current embedding of a graph interactively. The user interface also allowed us to interact with



Figure 5.1: Screenshot of our metro map layout software.

the graph, allowing us to move nodes and create or delete nodes and edges. We were also able to specify settings for running our layout method including configuration of all of the criteria weightings and setting other parameters such as the grid spacing or number of iterations. We were able to interactively watch the progress while laying out a metro map.

Additional features were implemented to allow us to investigate the values of the criteria for the immediate area around a particular node. This allowed us to direct our selection of criteria and the exploration of local minima.

Finally, we were able to import or export the maps using our own data format (described in Section 5.2) and to capture the current map using encapsulated PostScript (EPS) or as a PNG image file.

Figures 5.1 and 5.2 show screenshots of our software in action.



Figure 5.2: Another screen shot of our metro map layout software.  $\,$ 

 $\bigstar$  Information on the efficiency of the software (how long did it take to draw certain maps? comparison between different maps of different sizes)  $\bigstar$ 

#### 5.1.1 Calculation of Criteria in Practice

★ Explain how our software actually calculated the criteria. How were the criteria represented in the software? ★

## 5.2 Metro Map Data Format

We created a custom data format to abstractly represent metro maps and to allow us to load and save maps from our software. The format consists of a list of NODE definitions and a list of EDGE definitions. Appendix B provides the data in this format as necessary for each of the example maps illustrated in Chapter 6.

A NODE definition looks like this:

NODE: label=Five Points: x=-84.3916: y=33.7539

where the label field contains the station name and the x and y fields contain the xand y-coordinates for the node. In this case, the coordinates used are the latitude and longitude of the station.

An EDGE definition describes either a single edge or a sequence of edges to form a metro line. An example EDGE definition looks like this:

EDGE:label=Central:color=255,0,0:adjlist=Ealing Broadway, West Acton,
North Acton

where the label field contains the name of the metro line, color is the colour of the line (red, green and blue values from zero through 255) and adjlist contains a comma-separated list of station names. In this example, a metro line starting at Ealing Broadway and ending at North Acton and passing through West Acton is defined. A total of two edges would be created by this definition. The station names given in the adjlist must match exactly with the name given in the label field for that station.

## 5.3 Selection of Criteria Weightings

★ Describe how criteria weightings were derived. Explain the process by which I arrived at an appropriate selection of weightings. Explain why the same weightings seem to produce reasonable results for many maps (which is why the results in the next chapter mostly use the same weightings). When they are different what was the impetus for changing them?  $\bigstar$ 

 $\bigstar$  Discussion on the effect of individual weightings. What if the weighting for one criteria is cranked up? What if I draw the same map with different weightings? Explain how the differences come about.  $\bigstar$ 

## 5.4 Local Minima

★ Local minima are an innate part of optimisation algorithms. Ours is no different! Explain here how some of the more common types of local minima occur and why it's not possible to deal with them using global weightings.  $\bigstar$ 

## Chapter 6

# Results and Discussion

This chapter presents a number of examples of our method when applied to various real-world metro maps. The following maps are illustrated here:

- 1. Mexico City (Section 6.1): in-depth discussion of the method.
- 2. London Underground (Section 6.2).
- 3. Atlanta MARTA (Section 6.3).
- 4. Auckland (Section 6.4).
- 5. Bucharest (Section 6.5).
- 6. San Francisco BART (Section 6.6).
- 7. Stockholm T-bana (Section 6.7).
- 8. Sydney CityRail (Section 6.8).
- 9. Toronto (Section 6.9).
- 10. Washington D.C. WMATA (Section 6.10).

In the case of the Mexico City map, an in-depth discussion is presented which aims to illustrate how the method performs on a substantially-sized map  $\star$  What does "substantial" mean in this context?  $\star$ . For the Sydney CityRail map,

comparisons are given between the results produced by our method and those by Hong et al. [HMdN04, HMdN06], Nöllenburg [NÖ5] and Merrick and Gudmundsson [MG07].

★ More detail on how the weightings were determined needs to be mentioned. Most weightings are similar, so why change the few that were changed? ★

★ Examples of local minima need to be illustrated with large-scale "zooms" of the areas in question ★

## 6.1 Mexico City: Discussion

The Mexico City Sistema de Transporte Colectivo Red del Metro map [Mex08] is a complex, decentralised map. It has a relatively high number of lines and faces and a total of 175 stations (Table 6.1). The officially published map is shown in Figure 6.6. Unusually for a metro system in a large city, there is no obvious centre of the map and most lines intersect many other lines (the purple line 1 intersects all but one of the other lines). Also unusual is that that the officially published map lacks any clear use of the characteristics of metro maps described in Section 2.2. The geographic sketch map of the network is shown in Figure 6.1. The high number of faces in this map should provide an interesting illustration of our partitioning algorithm.

The map produced using our method is shown in Figure 6.7. We used the criteria weightings shown in Table 6.2 to produce this map. For this map, we have produced intermediate layouts showing the map at the end of the first iteration (Figure 6.2), the second iteration (Figure 6.3) and the third iteration (Figure 6.4). For the fourth iteration, the changes were negligible, so the intermediate layout is omitted.

To illustrate how the method optimises the layout, the charts in Figure 6.5 show how the total weighted node movement criteria,  $m_N$ , decreased during each iteration. These charts show that the majority of the optimisation occurred during the first iteration and by the fourth iteration there was only a small improvement. Table 6.3 shows the values of  $m_N$  after the node movement and cluster movement phases. The number of nodes identified in each cluster is also shown. As is to be expected, the number of clusters

identified at each iteration falls. By the fifth iteration, no improvement to  $m_N$  was made and the value of  $m_N$  was 0.156 times that of the initial value.

Our finished map shows a significant enhancement over both the geographic and official maps. The spacing between stations is much more regular and nearly all the edges have been drawn octilinearly. The labelling is also of high quality, particularly along long lines. The line to the top-right of the map shows an interesting meander where the line has been compressed horizontally in order to fit within the bounds of the drawing area. Due to the large number of faces in this map, the clustering by partitioning algorithm was very effective in straightening a number of lines and to compress some overlength edges (that would not have compressed using the overlength edges clustering algorithm).

Examples of local minima are notable in our map, particularly where several lines pass through a station (as at *Chabacano*) or where a triangular face exists (such as the one bounded by *Morelos*, *San Lazaro* and *Candelara*). In the first of these examples, two lines are passing through a single station from east to west. The line straightness criterion tends to force these lines to become horizonal thereby reducing the angle between them. This could be avoided by increasing the weighting for the angular resolution criterion, but in practice this tends to result in less optimal conditions elsewhere in the map. The second local minima example involves triangular faces. These faces can only be drawn with each edge being octilinear when two of the edges are of the same length (the face then forms an isoceles triangle).

#### ★ Why are some edges not octilinear? ★

Table 6.1: Characteristics of the Mexico City map.

Stations	175
Interchange Stations	24
Edges	165
Lines	11
Faces	19

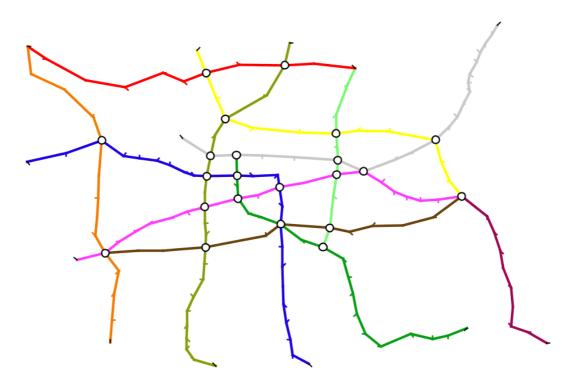


Figure 6.1: Mexico City map: geographic layout.

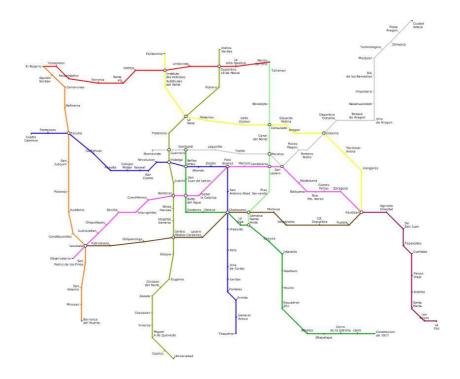


Figure 6.2: Mexico City map: after iteration 1.

Table 6.2: Criteria weightings and parameters used for the Mexico City map.

Node	lode movement		elling	Other Parameters	
$w_{N1}$	30000.0	$w_{L1}$	300.0	Iterations	5
$w_{N2}$	50.0	$w_{L2}$	80.0	Pref. grid spacing $l$	4
$w_{N3}$	45.0	$w_{L3}$	19.0	Grid Spacing $g$	40
$w_{N4}$	1000000.0	$w_{L4}$	15.0	Min. Cluster Distance	3
$w_{N5}$	220.0	$w_{L5}$	3.0	Max. Node Movement	8
$w_{N6}$	9250.0	$w_{L6}$	1.0		
		$w_{L7}$	30.0		

Table 6.3: Total weighted node criteria,  $m_N$ , over five iterations of drawing the Mexico City map.

Iteration	$m_N$ after	$m_N$ after	Clusters
	moving nodes	moving clusters	
Start		$4.408 \times 10^8$	
1	$1.764 \times 10^{8}$	$1.130 \times 10^{8}$	172
2	$0.941 \times 10^{8}$	$0.788 \times 10^{7}$	125
3	$0.741 \times 10^{7}$	$0.724 \times 10^{7}$	108
4	$0.691 \times 10^{7}$	$0.689 \times 10^{7}$	104
5	$0.689 \times 10^{7}$	$0.689 \times 10^{7}$	90

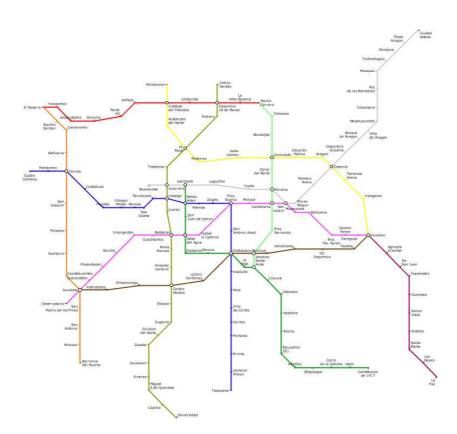


Figure 6.3: Mexico City map: after iteration 2.

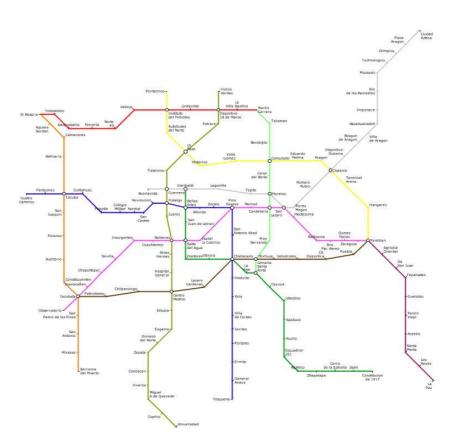


Figure 6.4: Mexico City map: after iteration 3.

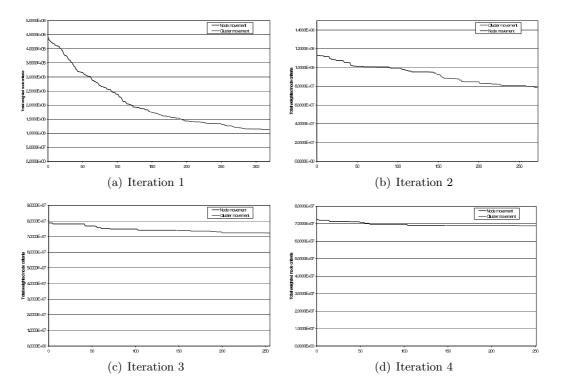


Figure 6.5: Charts showing the decrease in the value of the total weighted node and labelling criteria over five iterations of drawing the Mexico City map. The x-axis on each chart represents the progression of each iteration as each node or cluster is moved.  $\bigstar$  These figures need fixing—distinguish node/cluster movement parts. What do the figures show? Local minima?  $\bigstar$ 



Figure 6.6: Mexico City map: official layout.

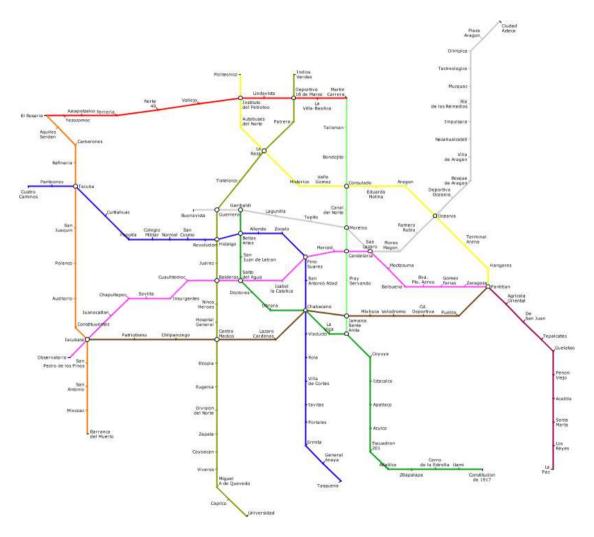


Figure 6.7: Mexico City map: our layout.

## 6.2 London

## $\bigstar$ Discussion of London maps $\bigstar$

Table 6.4: Characteristics of the London Underground map.

Stations	★ Fill in! ★
Interchange Stations	$\bigstar$ Fill in! $\bigstar$
Edges	$\bigstar$ Fill in! $\bigstar$
Lines	$\bigstar$ Fill in! $\bigstar$
Faces	★ Fill in! ★

Table 6.5: Criteria weightings and parameters used for the London Underground map.

No	de movement		Labelling	Other Parameters	
$w_{N1}$	★ Fill in! ★	$w_{L1}$	★ Fill in! ★	Iterations	★ Fill in! ★
$w_{N2}$	★ Fill in! ★	$w_{L2}$	$\bigstar$ Fill in! $\bigstar$	Pref. grid spacing $l$	★ Fill in! ★
$w_{N3}$	★ Fill in! ★	$w_{L3}$	$\bigstar$ Fill in! $\bigstar$	Grid Spacing $g$	★ Fill in! ★
$w_{N4}$	★ Fill in! ★	$w_{L4}$	$\bigstar$ Fill in! $\bigstar$	Min. Cluster Distance	★ Fill in! ★
$w_{N5}$	★ Fill in! ★	$w_{L5}$	$\bigstar$ Fill in! $\bigstar$	Max. Node Movement	★ Fill in! ★
$w_{N6}$	★ Fill in! ★	$w_{L6}$	$\bigstar$ Fill in! $\bigstar$		
		$w_{L7}$	$\bigstar$ Fill in! $\bigstar$		

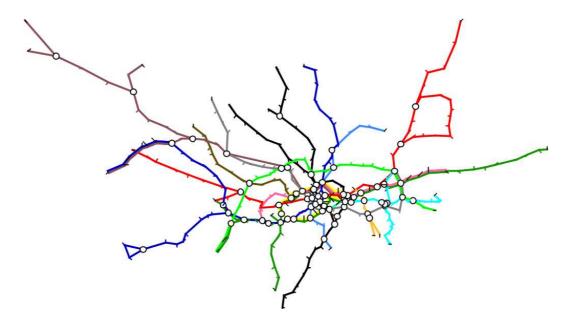


Figure 6.8: London Underground map: geographic layout.

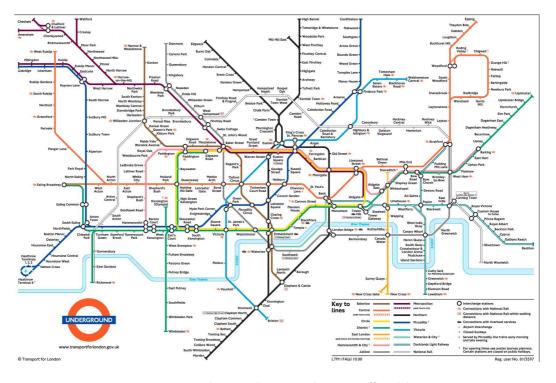


Figure 6.9: London Underground map: official layout.

Figure 6.10: London Underground map: our layout.

### 6.3 Atlanta

# ★ Newer official Atlanta map has four different lines shown. Either find the older version or explain this ★

The Atlanta MARTA metro map [MAR08] is a simple radial map with just two lines and a single face (the outside face). As such this map should show the effect of clustering overlength edges and non-straight lines. Other characteristics of the Atlanta map are shown in Table 6.6. The official published map is shown in Figure 6.12 and the geographic sketch map is shown in Figure 6.11. The published map shows a small amount of metadata in the form of some highways.

The map produced with our method is shown in Figure 6.13 which was produced using the criteria weightings shown in Table 6.7.

Our layout method clearly demonstrates the efficacy of our clustering algorithms for overlength edges and non-straight lines. The spacing of stations along each line is much more regular than both the geographic map and the official published map. The three edges between Inman Park/Reynoldstown and Decatur were not picked up by any of the clustering algorithms, but they might have been drawn more octilinearly if the octilinearity criterion was increased. Labelling is generally of a high standard, except the area just to the left of Five Points. In this case, a very long label (Dome/Philips Arena/GWCC/CNN) would require much more space when placing it horizontally and therefore the label obscures one other station just below Five Points. The published map handles the labelling along the horizontal east-west lines using diagonal labels.

Table 6.6: Characteristics of the Atlanta MARTA map.

Stations	39
Interchange Stations	3
Edges	36
Lines	2
Faces	1

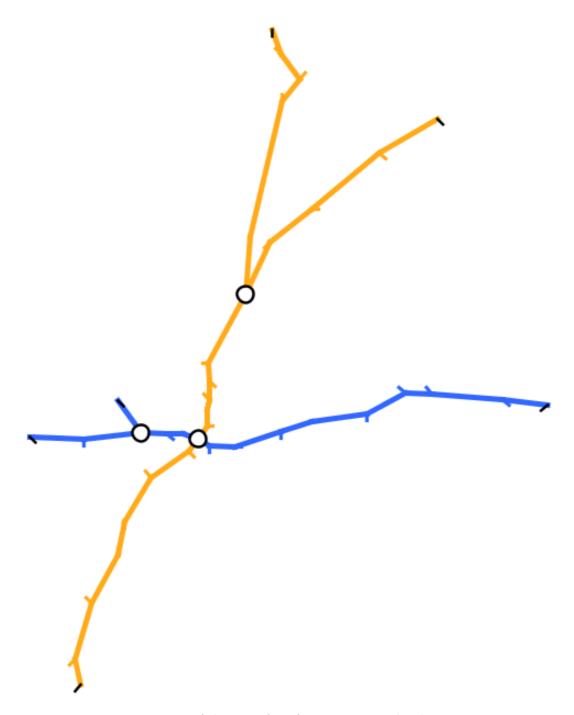


Figure 6.11: Atlanta MARTA map: geographic layout.

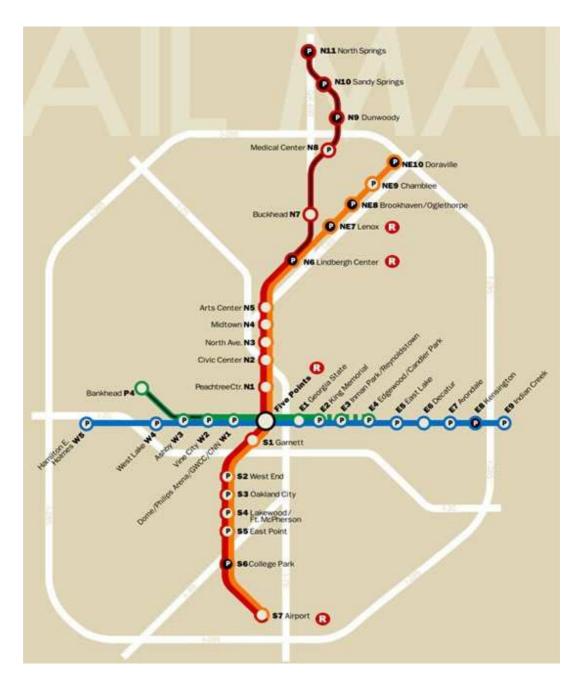


Figure 6.12: Atlanta MARTA map: official layout.

Table 6.7: Criteria weightings and parameters used for the Atlanta MARTA map.

Node	Node movement Labelling		ng Other Parameters		
$w_{N1}$	30000.0	$w_{L1}$	300.0	Iterations	5
$w_{N2}$	50.0	$w_{L2}$	80.0	Pref. grid spacing $l$	4
$w_{N3}$	45.0	$w_{L3}$	19.0	Grid Spacing $g$	50
$w_{N4}$	1000000.0	$w_{L4}$	15.0	Min. Cluster Distance	3
$w_{N5}$	2200.0	$w_{L5}$	3.0	Max. Node Movement	8
$w_{N6}$	9250.0	$w_{L6}$	1.0		
		$w_{L7}$	30.0		

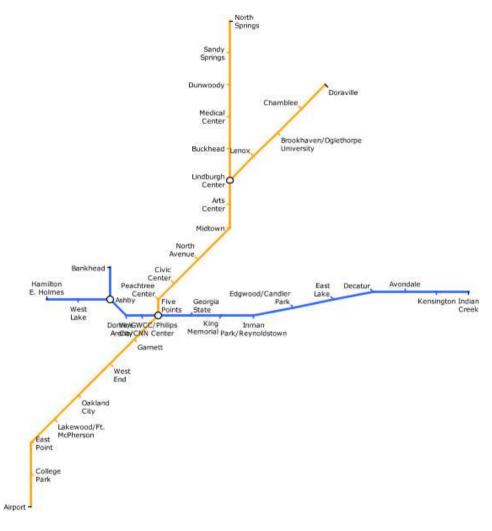


Figure 6.13: Atlanta MARTA map: our layout.

## 6.4 Auckland

The Auckland map [Auc07] is a simple radial map with a small loop in the city centre. The official published map 6.15 has a strong horizontal bias, probably so that the map can fit easily in train carriages. The geographic map 6.14 shows that the map really features an east-west line and two north-south lines which run in parallel for much of the way. The characteristics of the Auckland map are shown in Table 6.8.

The version of the Auckland map drawn using our method is shown in Figure 6.16 and uses the criteria weightings given in Table 6.9.

Our finished map clearly follows the topography of the network more than the published map. As such, the relative positions of stations are more obvious. The loop in the centre of the map is quite effective—the two lines heading south of the city centre pass through Westfield in as straight a line as possible without octilinearity of angular resolution being sacrificed. The spacing of stations along the lines is also very regular, especially considering that the spacing of stations on the geographic map is quite irregular. Labelling is generally good, but the bends in the long lines have forced some unattractive position inconsistencies. One label (Glen Eden) occludes an edge, but in this case the position consistency and position criteria preferred the occlusion than have the label on the other side of the line.

#### ★ What was Avondale moved but not Glen Eden? ★

Table 6.8: Characteristics of the Auckland map.

Stations	39
Interchange Stations	1
Edges	52
Lines	3
Faces	2

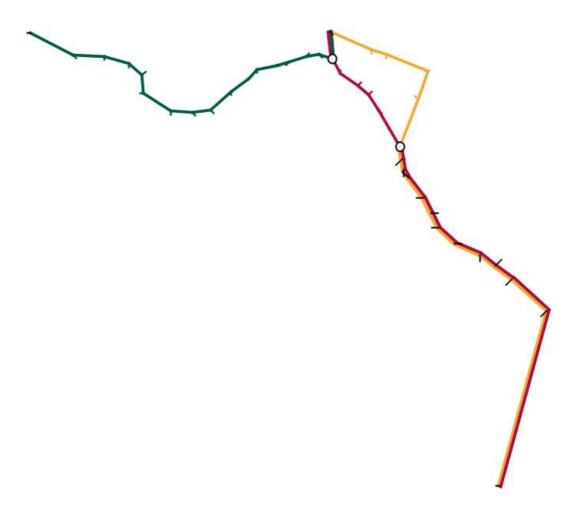


Figure 6.14: Auckland map: geographic layout.



Figure 6.15: Auckland map: official layout.

Table 6.9: Criteria weightings and parameters used for the Auckland map.  $\,$ 

Node	Node movement		elling	Other Parameters	
$w_{N1}$	30000.0	$w_{L1}$	300.0	Iterations	4
$w_{N2}$	50.0	$w_{L2}$	80.0	Pref. grid spacing $l$	4
$w_{N3}$	45.0	$w_{L3}$	19.0	Grid Spacing $g$	50
$w_{N4}$	1000000.0	$w_{L4}$	15.0	Min. Cluster Distance	3
$w_{N5}$	220.0	$w_{L5}$	3.0	Max. Node Movement	8
$w_{N6}$	9250.0	$w_{L6}$	1.0		
		$w_{L7}$	30.0		

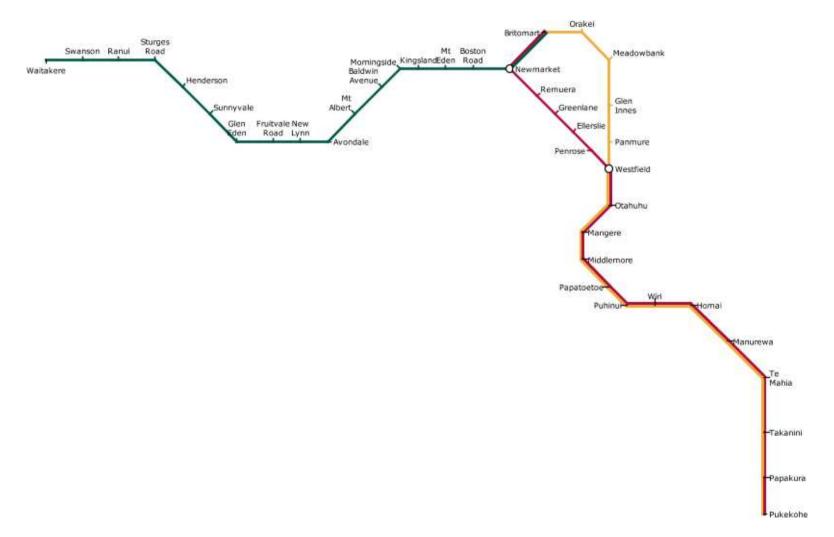


Figure 6.16: Auckland map: our layout.

## 6.5 Bucharest

The Bucharest metro map [Met08] is an example of a centralised, radial map but with an interesting circular line in the middle (the red line). The official published map is shown in Figure 6.18 and the geographic sketch map is shown in Figure 6.17. Characteristics of the map are given in Table 6.10. The official map features two notable curiosities: an unorthodox kink in the red line on the right-hand side of the map and a similar kink in the same line on the left-hand side of the map.

The map drawn using our method is shown in Figure 6.19 and uses the criteria weightings given in Table 6.11.

Our map clearly demonstrates the application of many of the characteristics of metro maps. Most of the edges are drawn octilinearly and the circular loop remains prominent. The map labelling is reasonably good. There are a handful of examples where labels occlude lines—in these cases, a local minimum was reached where the weighting on the occlusion criteria did not quite outweigh a combination of the other labelling criteria.

#### ★ Explain the two label occlusions ★

Table 6.10: Characteristics of the Bucharest map.

Stations	45
Interchange Stations	6
Edges	45
Lines	3
Faces	3

Table 6.11: Criteria weightings and parameters used for the Bucharest map.

Node	Node movement		elling	Other Parameters	
$w_{N1}$	30000.0	$w_{L1}$	300.0	Iterations	5
$w_{N2}$	50.0	$w_{L2}$	80.0	Pref. grid spacing $l$	4
$w_{N3}$	45.0	$w_{L3}$	19.0	Grid Spacing $g$	50
$w_{N4}$	1000000.0	$w_{L4}$	15.0	Min. Cluster Distance	3
$w_{N5}$	220.0	$w_{L5}$	3.0	Max. Node Movement	8
$w_{N6}$	9250.0	$w_{L6}$	1.0		
		$w_{L7}$	30.0		

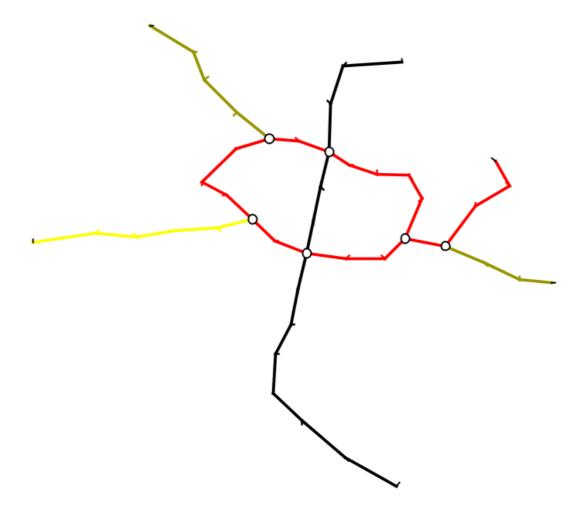


Figure 6.17: Bucharest map: geographic layout.



Figure 6.18: Bucharest map: official layout.



Figure 6.19: Bucharest map: our layout.

#### 6.6 San Francisco BART

The San Francisco Bay Area Rapid Transit (BART) map [BAR07] features five lines radiating from the San Francisco Bay area of California. The network covers a sizeable area: around 60km from north to south and 70km from east to west. It also features some rather long lines between neighbouring stations, for example, Castro Valley to Dublin/Pleasanton is 16km. Not all the stations are as far apart though. The distance from Oakland City Center/12th St to 19th St/Oakland is only 550m. However, the official published map (shown in Figure 6.21) still uses a geographic layout style, as can be seen when compared with the geographic map (shown in Figure 6.20). The use of a geographic map for network that covers such a large area means that some stations are drawn very far apart while others are drawn very close together—the lines between some pairs of neighbouring stations are barely discernable. The density of stations in some areas of the map means that some labels cannot be positioned immediately next to the nodes. This is particularly prevalent in the Oakland/Berkeley area where three labels are positioned some way from their nodes and linked with a line. Characteristics of the San Francisco BART map are given in Table 6.12.

The version of the San Francisco BART map drawn using our method is shown in Figure 6.22 and uses the criteria weightings given in Table 6.13.

Our final map shows regular node spacing and all except one edge has been drawn octilinearly. Labelling is also good, with only one label occluding edges. The meandering of the lines from West Oakland to Daly City is curious and may have been caused when the nodes were aligned with grid intersections and there not being enough space on the grid for the nodes. The area around Oakland City Center/12th St also suffers from a sharp angle in one of the lines that pass through that node.

 $\bigstar$  Why the 'squiggle' in the SW corner?  $\bigstar$ 

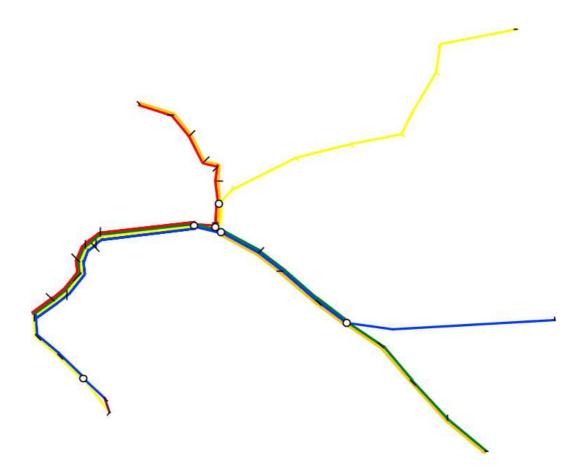


Figure 6.20: San Francisco BART map: geographic layout.

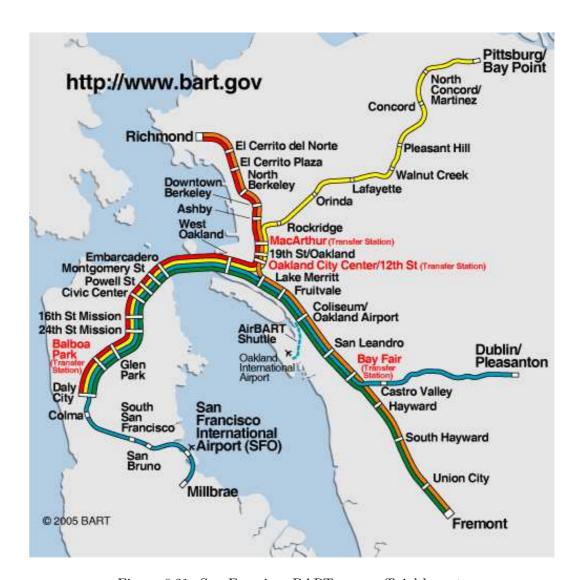


Figure 6.21: San Francisco BART map: official layout.

Table 6.12: Characteristics of the San Francisco map.

Stations	43
Interchange Stations	6
Edges	94
Lines	5
Faces	1

Table 6.13: Criteria weightings and parameters used for the San Francisco map.

Node movement		Labelling		Other Parameters	
$w_{N1}$	30000.0	$w_{L1}$	300.0	Iterations	7
$w_{N2}$	50.0	$w_{L2}$	80.0	Pref. grid spacing $l$	4
$w_{N3}$	45.0	$w_{L3}$	19.0	Grid Spacing $g$	50
$w_{N4}$	1000000.0	$w_{L4}$	15.0	Min. Cluster Distance	3
$w_{N5}$	220.0	$w_{L5}$	3.0	Max. Node Movement	8
$w_{N6}$	9250.0	$w_{L6}$	1.0		
		$w_{L7}$	30.0		

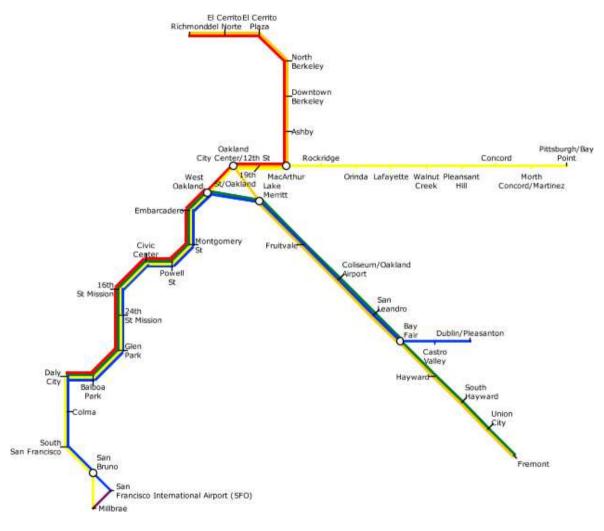


Figure 6.22: San Francisco BART map: our layout.

## 6.7 Stockholm

The Stockholm map [SL008] is a larger version of a centralised radial map. The official map (shown in Figure 6.24) is characterised by the starburst effect with all lines radiating from the *T-Centralen* station. Characteristics of the Stockholm map are given in Table 6.14. As with other radial maps, there are very few faces—only two in this case. The geographic sketch of the Stockholm map is shown in Figure 6.23.

The version of the Stockholm map drawn using our method is shown in Figure 6.25 and uses the criteria weightings given in Table 6.15.

Our map retains the characteristic radial features of the official map but also retains some of the geographic topography (particularly the way the two lines on the left-hand side of the map meander). Two areas of our map are more compressed than would be ideal where two lines run almost parallel to each other. The junction of the lines at the bottom-right area of the map suffers from an over-dependence on the angular resolution criterion, particularly around *Gullmansplan* and *Skarmarbrink*—this area also highlights how the line straightness criterion breaks down for branches in lines. The line from *Skarmarbrink* to *Farsta Strand* in the south-east of the map has a curious zig-zag form, mainly due to having to fit in around the labels that had already been positioned along the two adjacent lines. The labelling is generally of good quality but occlusions are present mainly where two lines have been drawn too close together.

Table 6.14: Characteristics of the Stockholm map.

Stations	100
Interchange Stations	9
Edges	101
Lines	3
Faces	2

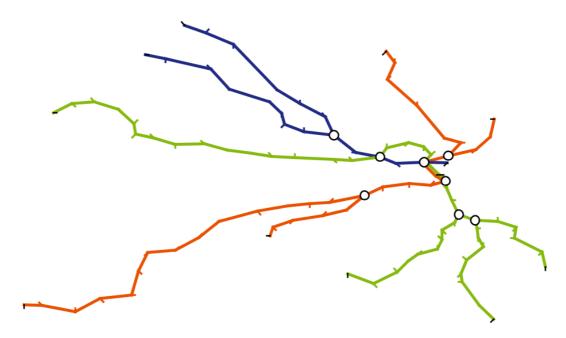


Figure 6.23: Stockholm map: geographic layout.



Figure 6.24: Stockholm map: official layout.

Table 6.15: Criteria weightings and parameters used for the Stockholm map.

Node movement		Labelling		Other Parameters		
$w_{N1}$	30000.0	$w_{L1}$   300.0		Iterations	6	
$w_{N2}$	50.0	$w_{L2}$ 80.0		Pref. grid spacing $l$	4	
$w_{N3}$	45.0	$w_{L3}$	19.0	Grid Spacing $g$	30	
$w_{N4}$	1000000.0	$w_{L4}$	15.0	Min. Cluster Distance	3	
$w_{N5}$	220.0	$w_{L5}$	3.0	Max. Node Movement	8	
$w_{N6}$	9250.0	$w_{L6}$	1.0			
		$w_{L7}$	30.0			

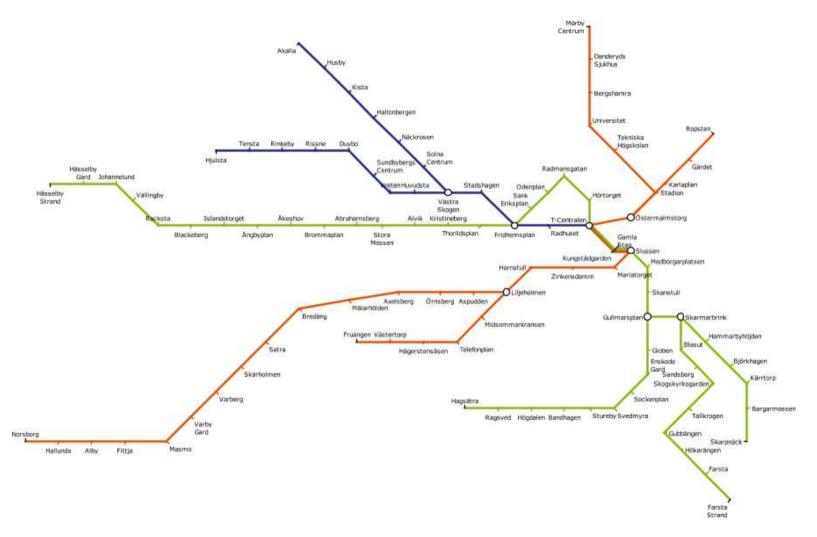


Figure 6.25: Stockholm map: our layout.

### 6.8 Sydney

The Sydney CityRail [SCR07] is a very large network covering a area of approximately  $3600 \,\mathrm{km^2}$  of metropolitan Sydney. The map makes use of most of the usual metro map design characteristics, including horizontal, vertical and  $45^\circ$ -diagonal lines, scale generalisation and line generalisation. The use of scale generalisation is very prominent in the central Sydney area where most of the lines converge in a tight loop around the city centre. Long horizontal lines have forced the use of diagonal labels, but all diagonal labels are of the same orientation. The official Sydney CityRail map is shown in Figure 6.27 and the geographic map with stations positioned according to their latitude and longitude is shown in Figure 6.26. Characteristics of the Sydney map are shown in Table 6.16. We have constrained that area of interest in the Sydney map to the main metropolitan area of Sydney, bounded by *Berowra* in the north, *Emu Plains* in the west and *Macarthur* and *Waterfall* in the south.

The version of the Sydney CityRail map drawn using our method is shown in Figure 6.28 and uses the criteria weightings given in Table 6.17.

The finished version of our map has succeeded in evening out node spacing and nearly all the edges are drawn octilinearly. Labelling is also of a good quality. One particular area posing a problem for our method is the central area at the right-hand side of the map. This section has up the seven lines passing through each station and features a very tight loop. The published map handles this area by significantly increasing the scale (possibly one of the most dramatic uses of scale generalisation in any of the maps presented in this chapter), but our method does not explicitly handle scale generalisation for such a small area of the map. A few edges are not drawn octilinearly, most notably between Glenfield and Wolf Creek (the bottom-most horizontal line in the map). In this case, a local minimum has been reached where none of the clustering algorithms will find the right cluster of nodes as the length of some of the edges is greater than the minimum cluster distance.

We can compare our result with those of Hong *et al.* (Figure 6.29), Nöllenburg (Figure 6.30) and Merrick and Gudmundsson (Figure 6.31). Our result is most similar

to Nöllenburg's, but the lines in our map are not as straight due to the use of the 3-link heuristic in Nöllenburg's method. However, our automatically-drawn map incorporated labelling as part of the method meaning that enough space was left for station labels. While it would be possible to achieve a reasonable labelling of Nöllenburg's map, there are areas where this may not be possible without causing label occlusions. Hong's map includes labelling and the use of diagonal labels helps significantly, particularly with long horizontal lines. The dense area around the centre of Sydney (where most of the lines converge) is handled well by Hong and Nöllenburg but less well by our method and Merrick and Gudmundsson's method.

Table 6.16: Characteristics of the Sydney map.

Stations	173
Interchange Stations	19
Edges	284
Lines	10
Faces	8

Table 6.17: Criteria weightings and parameters used for the Sydney CityRail map.

Node	Node movement		elling	Other Parameters		
$w_{N1}$	30000.0	$w_{L1}$	300.0	Iterations	10	
$w_{N2}$	50.0	$w_{L2}$	80.0	Pref. grid spacing $l$	4	
$w_{N3}$	45.0	$w_{L3}$	19.0	Grid Spacing $g$	15	
$w_{N4}$	1000000.0	$w_{L4}$	15.0	Min. Cluster Distance	3	
$w_{N5}$	220.0	$w_{L5}$	3.0	Max. Node Movement	8	
$w_{N6}$	9250.0	$w_{L6}$	1.0			
		$w_{L7}$	30.0			

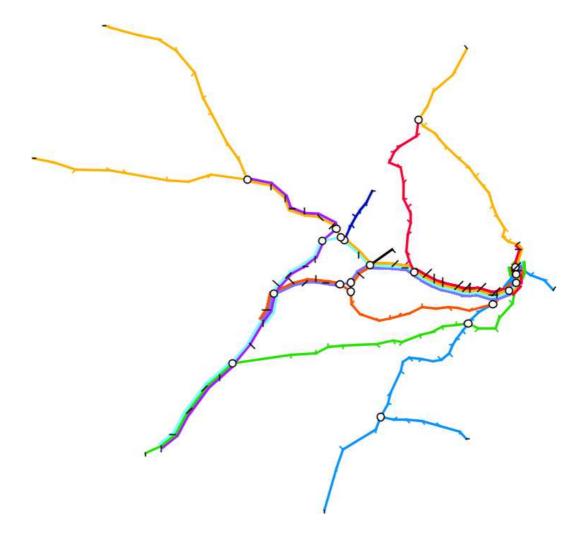


Figure 6.26: Sydney CityRail map: geographic layout.

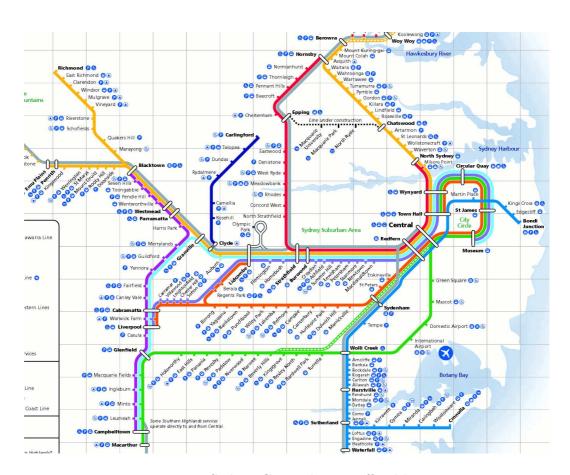


Figure 6.27: Sydney CityRail map: official layout.

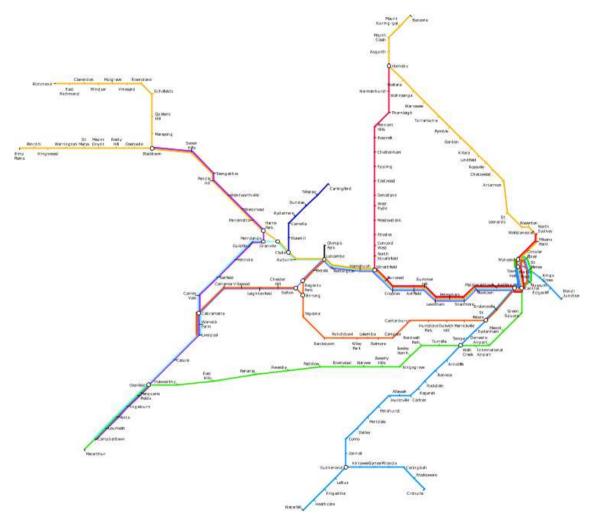


Figure 6.28: Sydney CityRail map: our layout.

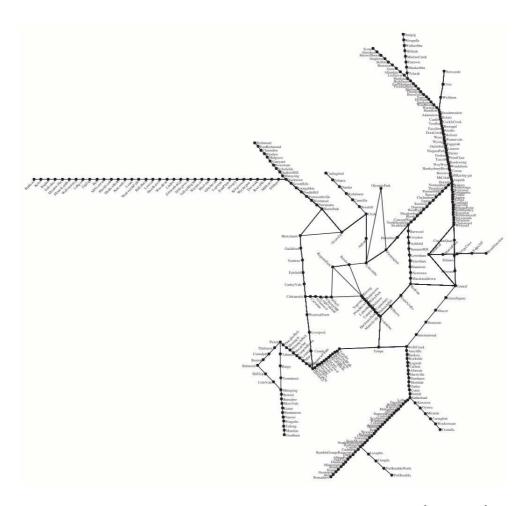


Figure 6.29: Sydney CityRail map: as drawn by Hong et al. [HMdN06].

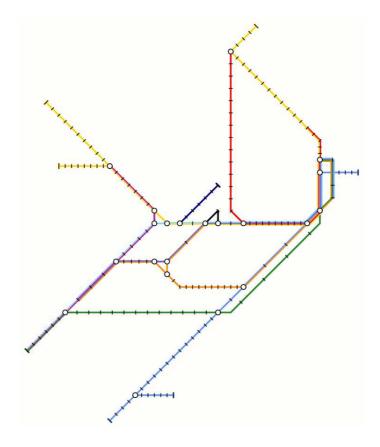


Figure 6.30: Sydney CityRail map: as drawn by Nöllenburg [NÖ5].

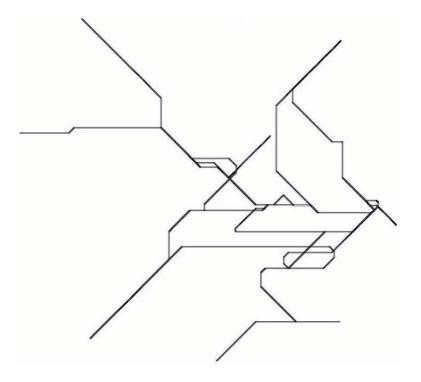


Figure 6.31: Sydney CityRail: as drawn by Merrick and Gudmundsson  $[\mathrm{MG07}].$ 

### 6.9 Toronto

The Toronto Transit Commission Subway map [TTC08] is a simple radial map. The characteristics of the Toronto map are given in Table 6.18. The most prominent feature of this map is the U-shaped line that passes through the centre and loops back again. The official published map is shown in Figure 6.33—the horizontal nature of the map comes about from being used in the train carriages. This causes the spacing between stations on vertical lines to be much less when compared with the spacing between stations on horizontal lines. The use of horizontal lines and horizontal labels means that the labels have to alternate from one side of the line to the other for each station. The geographic sketch of the Toronto subway map is shown in Figure 6.32.

The version of the Toronto map drawn using our method is shown in Figure 6.34 and uses the criteria weightings given in Table 6.19.

The map drawn with our automatic layout method emphasises the strong horizontal line from *Kipling* to *Woodbine* with just one change of direction. All except one edge is drawn octilinearly. Labelling is generally good but there a number of occlusions in the central area of the map. Some of the labels along the horizontal line alternate from one side to the other where there is not enough space, but this is not consistent. The only real problem with the map is not leaving enough space between both arms of the yellow looping line—this is particularly noticeable where the line loops at the bottom of the map.

Table 6.18: Characteristics of the Toronto map.

Stations	70
Interchange Stations	5
Edges	70
Lines	4
Faces	2

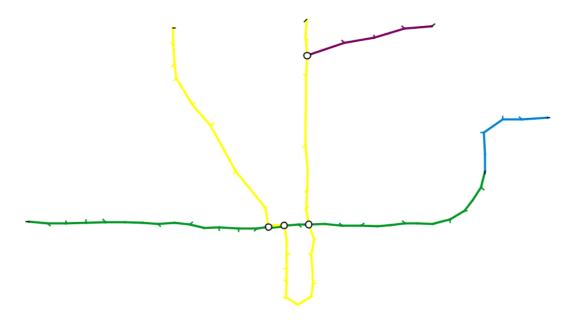


Figure 6.32: Toronto map: geographic layout.



Figure 6.33: Toronto map: official layout.

Table 6.19: Criteria weightings and parameters used for the Toronto map.  $\,$ 

Node movement		Labelling		Other Parameters		
$w_{N1}$	30000.0	$w_{L1}$	300.0	Iterations	5	
$w_{N2}$	50.0	$w_{L2}$	80.0	Pref. grid spacing $l$	4	
$w_{N3}$	45.0	$w_{L3}$	19.0	Grid Spacing $g$	50	
$w_{N4}$	1000000.0	$w_{L4}$	15.0	Min. Cluster Distance	3	
$w_{N5}$	220.0	$w_{L5}$	3.0	Max. Node Movement	8	
$w_{N6}$	9250.0	$w_{L6}$	1.0			
		$w_{L7}$	30.0			

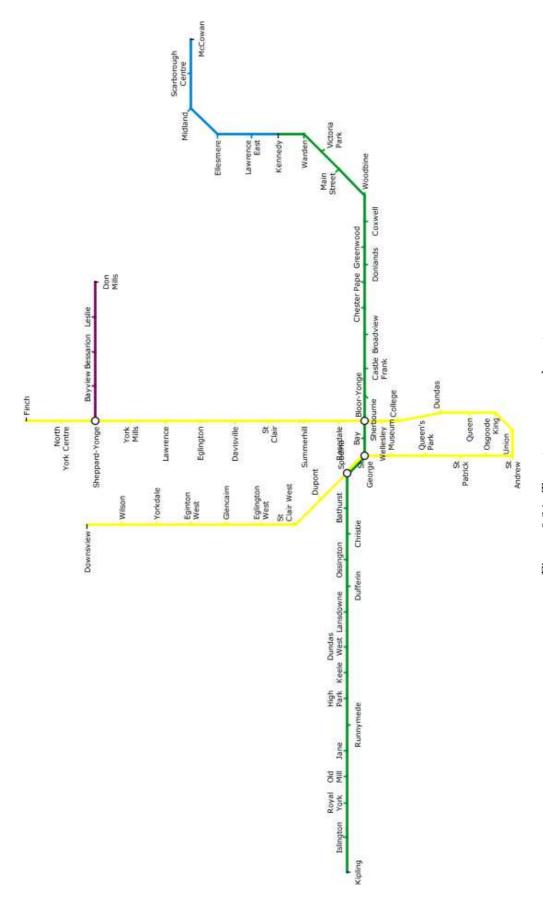


Figure 6.34: Toronto map: our layout.

## 6.10 Washington D.C.

### $\bigstar$ Problem with parallel lines being drawn on top of each other $\bigstar$

The Washington Metropolitan Area Transit Authority Metro System Map [WMA08] is from one of the newer metro systems in the world. The official published map is shown in Figure 6.36. The map is characterised by five lines radiating from a relatively compact central area. Characteristics for the Washington map are given in Table 6.20. The official map has been influenced by modern design concepts with very broad lines overlaid on topographic metadata showing main roads, the river and parkland. The map uses a combination of horizontal and two orientations of diagonal labels. There are many examples of cases where labels are allowed to overlap the edges. The text of the labels is also relatively small when compared with the symbol representing the station and the thickness of the line. In some cases, the size of the device used to represent stations causes there to be barely any space between adjacent stations. The geographic map of the Washington Metro with stations positioned according to their latitudes and longitudes is shown in Figure 6.35.

The version of the Washington map drawn using our method is shown in Figure 6.37 and uses the criteria weightings given in Table 6.21.

Our finished layout shows a map with very regular node spacing along lines and all edges have been drawn octilinearly. The line straightness criterion has clearly had an effect, particularly on the green, yellow and red lines. The central area of the map is still slightly too compressed and the edge crossing between *Metro Center* and *Farragut North* has caused an example of poor angular resolution between the red line and the blue/orange lines. Labelling is generally good, but in this case a significant number of labels suffer from occlusions. This is partially due to the labels being very long and partially due to there not being enough space to position the labels ideally. It might be that a modification of the criteria or other parameters might have found a more optimal solution that allows for a better labelling. The blue and orange lines are not drawn as straight as they might be—in one case (around *Metro Center*) this is due to the aforementioned edge crossing, and in another case (around *Federal Center SW* and

Capitol South) this is due to a local minimum caused by the position of the label for L'Enfant Plaza and the line to the right not being on quite the same horizontal level as L'Enfant Plaza.

Table 6.20: Characteristics of the Washington D.C. map.

Stations	86
Interchange Stations	9
Edges	108
Lines	5
Faces	5

Table 6.21: Criteria weightings and parameters used for the Washington D.C. map.

Node movement		Labelling		Other Parameters		
$w_{N1}$	30000.0	$w_{L1} = 300.0$		Iterations	20	
$w_{N2}$	50.0	$w_{L2}$	80.0	Pref. grid spacing $l$	4	
$w_{N3}$	45.0	$w_{L3}$	19.0	Grid Spacing $g$	20	
$w_{N4}$	1000000.0	$w_{L4}$	15.0	Min. Cluster Distance	3	
$w_{N5}$	220.0	$w_{L5}$	3.0	Max. Node Movement	8	
$w_{N6}$	15250.0	$w_{L6}$	1.0			
		$w_{L7}$	30.0			

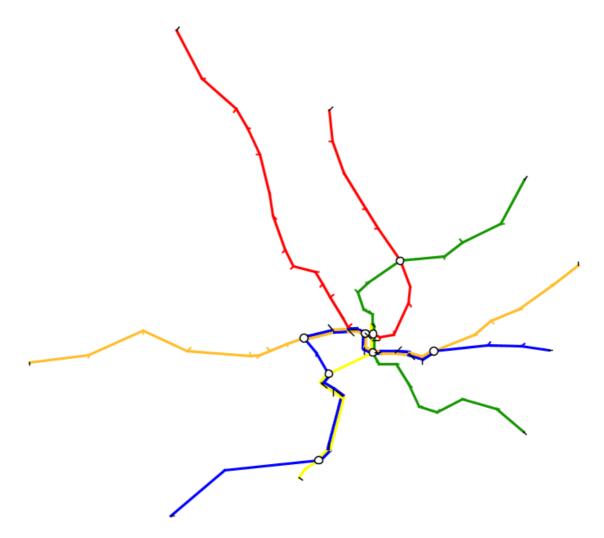


Figure 6.35: Washington D.C. map: geographic layout.



Figure 6.36: Washington D.C. map: official layout.

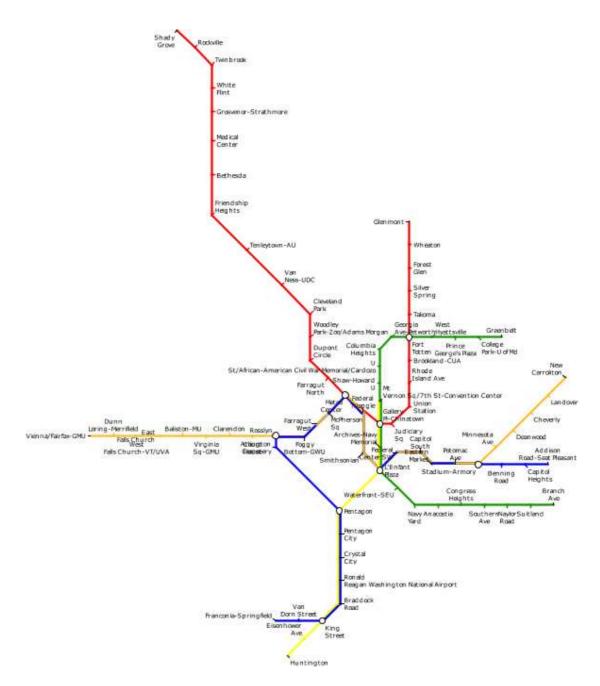


Figure 6.37: Washington D.C. map: our layout.

# Chapter 7

# **Empirical Evaluation**

### 7.1 Introduction

In the previous chapters, a number of aesthetic criteria for automatically drawing metro maps have been detailed, together with a method that combines these criteria in order to produce some automatically drawn metro maps. In order to show that the maps produced using this method are suitable, we devised an empirical experiment involving a number of human subjects performing route-planning tasks with a variety of different metro maps. In this experiment we measured the time and accuracy of finding optimal routes as well as the preference for each metro map.

## 7.2 Hypotheses

As introduced in Section 2.2, a number of design principles have evolved throughout the history of the schematic metro map. When combined, these principles are thought to provide a map that is of greater quality than a map that does not follow the principles (for example, a geographic map of the metro system). Direct measurement of the quality of a map directly is hard (and is essentially what the multicriteria optimisation method aims to achieve!), so we instead intended to indirectly measure the quality by asking human participants to perform common route-planning tasks and measuring their speed and accuracy.

We have four hypotheses:

**Hypothesis 1.** A map of a metro system drawn with our automated software is better for finding an optimal route than a geographic map of the system.

**Hypothesis 2.** A map of a metro system drawn with our automated software is better for finding an optimal route than the official published map of the system.

**Hypothesis 3.** A map of a metro system drawn with our automated software is preferred over a geographical map of the system.

**Hypothesis 4.** A map of a metro system drawn with our automated software is preferred over the official published map of the system.

In order to be able to test these hypotheses, we devised methods that will allow us to measure the accuracy and the time that it takes for someone to use a map for simple route-planning tasks, as well as a method to elicit the preference of one map over another. We chose two different types of questions that would be posed in order to ascertain the suitability of a map for route-planning:

- 1. How many changes are required to get from Station X to Station Y?
- 2. How many stations do you go through to get from Station X to Station Y?

These questions were chosen on the basis that they are likely to require the most common route-planning tasks when people use metro maps and because they generate easily quantifiable answers. This is in contrast to questions which are not easily quantifiable, such as "what is the shortest path from  $Station\ X$  to  $Station\ Y$ " or "which stations do you need to change at to get from  $Station\ X$  to  $Station\ Y$ ".

★ How were the questions for each map chosen? Why choose the stations that we did? How were the selection of answer options decided? ★

We used maps of six different real-life metro systems:

- 1. Atlanta Metropolitan Area Rapid Authority (MARTA) Rail Map [MAR08].
- 2. Bucharest Metro Map [Met08].

Map	Stations	Lines	Interchange	Edges	Faces
			Stations		
Atlanta	39	2	3	36	1
Bucharest	45	3	6	45	3
Mexico City	175	11	24	165	19
Stockholm	100	3	9	101	2
Toronto	70	4	5	70	2
Washington D.C.	86	5	9	108	5
Recife	28	9	9	28	1

Table 7.1: Characteristics of metro maps used in the empirical evaluation. The Recife map was used for training questions.

- 3. Mexico City Sistema de Transporte Colectivo Map [Mex08].
- 4. Stockholm Tunnelbana Map [SL008].
- 5. Toronto Transit Commission (TTC) Subway/RT Route Map [TTC08].
- Washington Metropolitan Area Transit Authority (WMATA) Metro System Map [WMA08].

These maps differ in characteristics and complexity from a fairly simple two-line, centralised network in the case of Atlanta through to a complex, highly-interconnected, decentralised map in the case of Mexico City. Table 7.1 gives details of the characteristics in terms of number of stations (nodes), lines, interchange stations, edges and faces of each map. Three variants of each map were used, one of which was a geographic representation of the stations, the second was a representation of the published map and the third was drawn using the automatic layout method described in Chapters 3 and 4. We also used the Recife metro map as a training map so that subjects had time to familiarise themselves with the procedure.

One of the problems in using published maps is that they all use different graphic design approaches. The fonts, line styles, icons used to represent stations and labelling all vary from one map to another. Also, some maps incorporate topographic metadata while others do not. These differences can be seen by looking at the examples in Section 2.2. We decided that all three variants of each map would be drawn using the

same graphic design to remove any question of the way that features such as stations and lines are drawn making a difference to the subjects' route planning tasks. To illustrate the three map variants, the geographic map (Figure 7.1), normalised published map (Figure 7.2) and automatically drawn map (Figure 7.3) are shown here. The official *Mapa de la Red* metro map for Mexico City is shown in Figure 7.4. All 18 maps as used in the evaluation are shown in Appendix A: Section A.1.1 (Atlanta), Section A.1.2 (Bucharest), Section A.1.3 (Mexico City), Section A.1.4 (Stockholm), Section A.1.5 (Toronto) and Section A.1.6 (Washington D.C.).

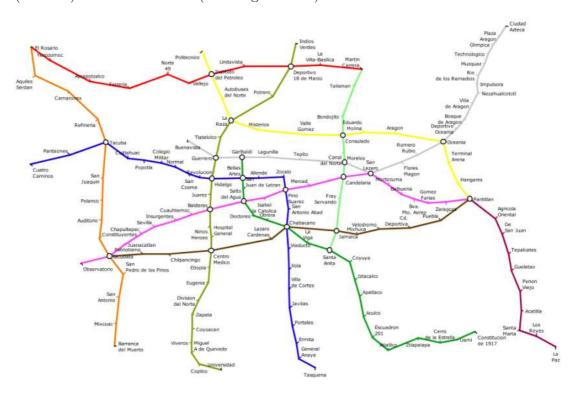


Figure 7.1: Mexico City geographic map.

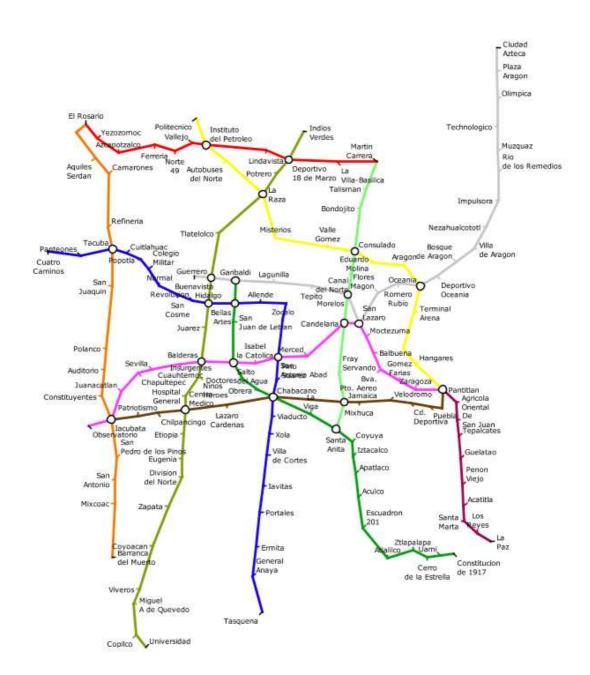


Figure 7.2: Mexico City normalised published map.

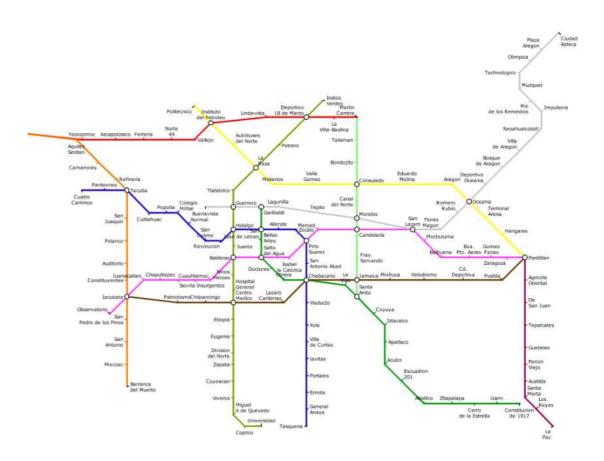


Figure 7.3: Mexico City automatically-drawn map.



Figure 7.4: Mexico City Mapa de la Red official metro map.

Group	Number of subjects
A	13
В	15
$^{\mathrm{C}}$	15

Table 7.2: Number of subjects in each experiment group.

### 7.3 Methodology

We conducted a number of experiments over a two-week period. A total of 43 subjects participated, nearly all of whom were Computer Science undergraduates. As a number of experiments were run, we attempted to ensure that each experiment was as identical as possible in order to reduce any potential external influences. We split the set of subjects into three groups which we called A, B and C and ensured that each group contained roughly the same number of subjects. The number of subjects in each group is shown in Table 7.2.

Each group would receive exactly the same questions and in exactly the same order but they would only see one variant of each map per group for each question. For example, for a question using the Atlanta map, all subjects in all groups are performing the same task but group A will be doing this with the normalised published map, group B will use the geographic map and group C will use the automatically drawn map. The purpose of this was to ensure that the possibilities of learning the map between questions—while not completely eliminated—can at least be reduced as much as possible.

In order to conduct the experiment, a software application was written which ensured a controlled environment when showing the maps. The application took over the entire screen of the computer, ensuring that nothing else on the computer would distract the subjects. Interaction with the software during the tests was entirely with the mouse. Using a software application in this manner allowed us to time each question in the experiment accurately and also to see automatically whether they answered correctly or incorrectly and to collate the results. It also allowed the environment to be more

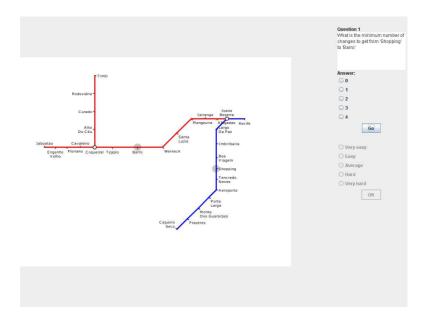


Figure 7.5: Screenshot of the software application used to conduct the empirical evaluation.

closely controlled and ensured uniformity in the way that the test was conducted for all subjects. A screenshot of the software application is shown in Figure 7.5.

The following procedure was used for each experiment:

- Subjects were directed to a specific computer. They were told not to touch anything until directed. Each computer in the room had previously been configured for a specific groups' questions.
- 2. A preliminary script was read out. This script introduced the subjects to the information that they needed in order to understand the questions asked of them in the experiment. To remove any potential ambiguity, explanations of terminology used were provided in the script, including *Metro Map*, station and line. The script also mentioned that any stations joined by a line in a single colour are connected by direct services and that where two or more lines pass through a single station it is possible to change from one line to another. The script included examples of questions that the subjects would be answering and introduced them to the application interface. To do this, two sample questions are worked through by demonstrating the application to the subjects using an overhead projector at

the front of the room. The script placed emphasis on the need to get questions correct and subjects were asked not to rush the questions. The script informed the subjects that the questions would be timed and they were told during which parts of each question that timing was measured. The full script is shown in Appendix A, Section A.2.1.

- 3. The subjects were told when they could commence using the software application. Everyone started at the same time and were presented with questions for 20 minutes. We decided to limit the total duration of the test so that everyone started and finished at the same time. This way we could avoid any anxiety or disturbance that could be caused by one subject finishing before another. We chose 20 minutes as being a more-than-ample time in which to answer the minimum number of questions that we needed. If a subject managed to complete these questions they continued to be presented with dummy questions until the 20 minutes was up. These dummy questions were discarded prior to analysis of the results. Each question proceeded in the following manner:
  - (a) The map is shown together with the question. Timing of the subjects' answers started as soon as the map, question and potential answers were shown.
  - (b) The subject selects their answer from a list of five potential answers by clicking on a radio button. They can change their mind at this point but they must have selected an answer before they could continue.
  - (c) When an answer is selected, the subject had to click on a button to continue.

    When this button was clicked, timing of that question stopped.
  - (d) The subject then selected their perception of the difficulty level of the task from a list of five options from "very hard" to "very easy". The subjects were able to rest at this point before continuing to the next question.
  - (e) The subject then clicks on another button in order to proceed to the next question.

- 4. After 20 minutes had elapsed, the subjects were presented with a screen that showed them how many of their answers were correct and incorrect. For any questions that they got incorrect, they were shown the correct answer as well as the answer that they gave.
- 5. A second script was then read out which introduced the subjects to the questionnaire. The questionnaire was in two parts: the first part involved viewing the three variants of each map on the overhead projector screen and them being asked to write down their preference for each variant from "most preferable" to "least preferable", the second part involved answering four questions:
  - (a) Have you seen any of the metro maps shown here before these tests? If so, which ones?
  - (b) Which features of the metro map layout did you find most helpful when completing the tests?
  - (c) Which features of the metro map layout did you find least helpful when completing the tests?
  - (d) Did you find any of the questions ambiguous?
- 6. The subjects were then rewarded with £5 for their time and were allowed to leave.

# 7.4 Results and Analysis

★ A difficult section! More explanation and discussion is needed. Why not use something more simple like ANOVA or t-test with grouped data? Explain how wrong answers are handled (censoring) ★

### $\bigstar$ Reference Hougaard [Hou00] $\bigstar$

This section describes how we analysed the data that we collected from our empirical experiment. The raw data is included in Appendix A. We were measuring the efficiency of the maps with regards to the time that it took to find a solution to the questions. Sections 7.4.1 and 7.4.2 detail how we used frailty models to analyse the route-finding

tasks. Section 7.4.3 explains our analysis of the preference task and Section 7.4.4 looks at any interesting comments made in the questionnaire.

### 7.4.1 Statistical Model for Analysing Route-Finding Tasks

The intention of modelling the results from our experiment is to produce three baseline functions—one for each map type—using independent frailty models. Frailty models are appropriate as we are handling the repeated observation of a question across the three groups of individuals in the study (as described in Section 7.3) using the three different map types.

We used frailty models (introduced by Clayton and Cuzick [CC85, The99, TGP03]) which are a generalisation of Cox's proportional hazards model [Cox72, CO84, EJJ80]. Survival models are more often used for analysing the survival of biological organisms (for example, after some form of medical treatment) or the failure of mechanical systems. The idea of survival analysis is to study time-event data (for example the survival time or the time taken to complete a task). In our case we are modelling the time, T, that is taken to answer a question.

T has the cumulative density function  $P(t) = Pr(T \le t)$  and probability density function  $p(t) = \frac{d}{dt}P(T)$  where t is some time. The survival function is then the complement of P: S(t) = 1 - P(t). The time that a question is answered can also be represented by the hazard function, h(t):

$$h(t) = \lim_{\Delta t \to 0} \frac{Pr\left(T \in [t, t + \Delta t) \mid T \ge t\right)}{\Delta t}$$
$$= \frac{p(t)}{S(t)}$$
(7.1)

In frailty models, the hazard function is described by

$$h_{ij}(t \mid \boldsymbol{x}_{i}, v_{i}) = h_{0}(t) \exp\left(\boldsymbol{x}_{ij}^{'}\boldsymbol{\beta}\right) v_{i}$$
(7.2)

where i indexes the group of individuals, j indexes individuals,  $x_{ij}$  is a p-dimensional covariate vector,  $\boldsymbol{\beta}$  is a p-dimensional unknown parameter vector,  $h_0$  is the baseline

hazard function and  $v_i$  is a multiplicative random effect. In our model,  $x_{ij}$  is a vector containing five dichotomous variables, using one variable for each map. Proportional hazards models are designed to analyse independent data. Since we are dealing with repeated observations for every subject in the study, we incorporated a frailty using this final multiplicative variable and this is used to allow the hazard function to be differentiated across subjects (derived by repeated or dependent observations). The multiplicative variable can be reparameterised as  $\theta_i = \log v_i$ .  $\theta_i$  can then be incorporated in the linear exponential component of the hazard function:

$$h_{ij}\left(t\mid\boldsymbol{x}_{i},v_{i}\right)=h_{0}\left(t\right)\exp\left(\boldsymbol{x}_{ij}^{\prime}\boldsymbol{\beta}+\theta_{i}\right)$$
(7.3)

### 7.4.2 Route-Finding Tasks

We analysed our data using the R statistical software [Tea06] using the survival and frailty packages [obTTpbTL07, GR05]. The R code for our analysis is shown in Appendix C.

Tables 7.3, 7.4 and 7.5 show the coefficients and significance for the model when applied to the official published, geographic and automatically-drawn maps respectively. Table 7.6 shows the frailty parameters,  $\theta$ , and log-likelihoods for the different map types. The frailty parameters show that times for answering questions across subjects in our experiment are more homogeneous  $\star$  What does "homogeneous" mean?  $\star$  for the official published map than the other two maps, with the geographic maps being least homogeneous. The log-likelihoods show that the fit of the models to the data is fairly similar across all three map types. Figure 7.6 shows the baseline survival functions for the different map types for comparison.

The baseline survival functions for the three map types can be compared visually using the chart in Figure 7.6. This chart shows that the performance of subjects using the official published maps and our automatically-drawn maps are roughly similar. The performance with the geographic map is notably worse with subjects taking longer to respond to the questions with this map type. This analysis can be used to confirm Hypothesis 1 that automatically-drawn maps are better for finding an optimal route

than a geographic map of a metro system. However, the analysis is inconclusive when attempting to confirm Hypothesis 2.

★ Explain more about how these results are interpreted. HOW DOES

THE ANALYSIS LEAD TO THESE CONCLUSIONS? ★

Table 7.3: Shared Gamma Frailty model parameter estimates using a Penalised Likelihood on the hazard function for the set of official published maps.

Γ	Variable	Coef	$\exp(\operatorname{coef})$	SE coef	${f z}$	р	95% CI
	Map 2	-0.367	0.693	0.205	-1.79	$7.3 \times 10^{-2}$	(0.46 - 1.03)
	Map 3	-1.015	0.362	0.227	-4.47	$7.9 \times 10^{-6}$	(0.23 - 0.57)
	Map 4	-0.976	0.377	0.220	-4.43	$9.4 \times 10^{-6}$	(0.24 - 0.58)
	Map 5	-1.441	0.237	0.258	-5.58	$2.5\times10^{-8}$	(0.14 - 0.39)
	Map 6	-1.104	0.331	0.229	-4.83	$1.4 \times 10^{-6}$	(0.21 - 0.52)

Table 7.4: Shared Gamma Frailty model parameter estimates using a Penalised Likelihood on the hazard function for the set of geographic maps.

Variable	Coef	$\exp(\operatorname{coef})$	SE coef	$\mathbf{Z}$	p	95% CI
Map 2	-0.1607	0.852	0.230	-0.699	$4.8 \times 10^{-1}$	(0.54 - 1.34)
Map 3	-0.0623	0.940	0.230	-0.271	$7.9 \times 10^{-1}$	(0.60 - 1.48)
Map 4	-0.1202	0.887	0.250	-0.481	$6.3 \times 10^{-1}$	(0.54 - 1.45)
Map 5	-1.2439	0.288	0.296	-4.197	$2.7 \times 10^{-5}$	(0.16 - 0.52)
Map 6	0.3622	1.436	0.222	1.629	$1.0 \times 10^{-1}$	(0.93 - 2.22)

Table 7.5: Shared Gamma Frailty model parameter estimates using a Penalised Likelihood on the hazard function for the set of automatically-drawn maps.

Variable	Coef	$\exp(\operatorname{coef})$	SE coef	$\mathbf{Z}$	p	95% CI
Map 2	-0.396	0.673	0.226	-1.750	$8.0 \times 10^{-2}$	(0.43 - 1.05)
Map 3	-0.865	0.421	0.229	-3.775	$1.6 \times 10^{-4}$	(0.27 - 0.66)
Map 4	-0.575	0.562	0.224	-2.571	$1.0 \times 10^{-2}$	(0.36 - 0.87)
Map 5	-0.241	0.786	0.244	-0.986	$3.2 \times 10^{-1}$	(0.49 - 1.27)
Map 6	0.139	1.150	0.211	0.663	$5.1 \times 10^{-1}$	(0.76 - 1.74)

Table 7.6: Frailty parameters and log-likelihoods for the different map types.

Map type	Frailty parameter, $\theta$	Penalised marginal		
		log-likelihood		
Official Published	0.151	-989.15		
Geographic	0.297	-860.41		
Automatically-drawn	0.243	-1014.3		

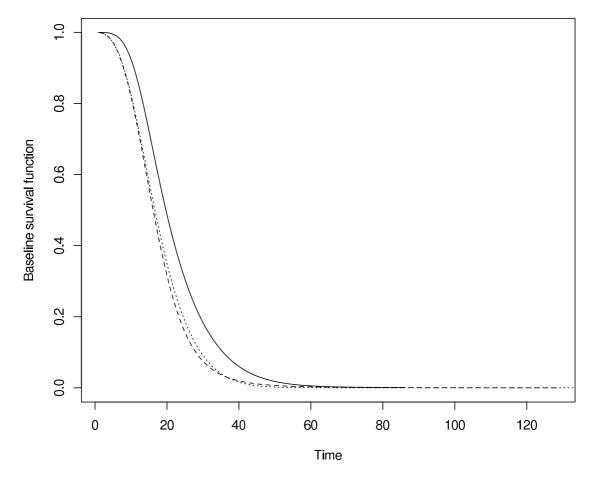


Figure 7.6: Baseline survival functions for the official published maps, automatically-drawn maps and the geographic maps. The survival function for the official published map is shown with a dashed line; the function for the automatically-drawn maps is shown with a dotted line and the function for the geographic maps is shown with a solid line.  $\bigstar$  key on chart  $\bigstar$ 

#### 7.4.3 Preference Tasks

We have analysed the preference data by considering the number and proportion of subjects that most preferred and least preferred each map. The numbers and proportions for the most preferable maps are shown in Table 7.7 and for the least preferable maps in Table 7.8.

Table 7.7: Number and proportion of subjects selecting a given map as their most preferable map.

Map	Number			Percent		
	Auto	Geo.	Pub.	Auto	Geo.	Pub.
	drawn			drawn		
Atlanta	21	10	12	48.84%	23.26%	27.91%
Bucharest	29	9	5	67.44%	20.93%	11.63%
Mexico	41	2	0	95.35%	4.65%	0.00%
Stockholm	5	2	36	11.63%	4.65%	83.72%
Toronto	24	5	14	55.81%	11.63%	32.56%
Washington	33	0	10	76.74%	0.00%	23.26%

Table 7.8: Number and proportion of subjects selecting a given map as their least preferable map.

Map	Number			Percent		
	Auto	Geo.	Pub.	Auto	Geo.	Pub.
	drawn			drawn		
Atlanta	10	23	10	23.26%	53.49%	23.26%
Bucharest	8	12	23	18.60%	27.91%	53.49%
Mexico	1	5	37	2.33%	11.63%	86.05%
Stockholm	10	32	1	23.26%	74.42%	2.33%
Toronto	2	18	23	4.65%	41.86%	53.49%
Washington	1	40	2	2.33%	93.02%	4.65%

### ★ Why do the Stockholm and Mexico preferences differ so greatly? ★

The proportions of individuals in the survey that prefer the automatically-drawn maps (shown in Table 7.7) were generally higher than the proportions of individuals that preferred either of the other two maps. The only map where this was not the case was the Stockholm map. The results for Mexico City are particularly notable as none of the 43 individuals preferred the official published map. The maps which were least

preferable (Table 7.8) also show a preference for the geographic maps to be the least preferable type, followed by the official published maps.

These observations support both of Hypothesis 3 and Hypothesis 4 that the automatically-drawn maps are generally more preferable amongst the group of individuals surveyed than either the geographic maps or the official published maps.  $\bigstar$  How does the analysis support the hypotheses?  $\bigstar$ 

#### 7.4.4 Questionnaire

The feedback from all subjects for the four questionnaire questions that they were asked about is shown in Appendix A.3.3. The questions in this questionnaire were mainly for our interest—it was interesting to note some of the responses about the features of the metro maps layouts from people who were using the maps.

### ★ Use bullet-points here? ★

The first question asked whether the subject had seen any of the metro maps shown here before the tests and if so, which ones. Seven people mentioned that they had seen metro maps that were not used in our experiment (mainly the London Underground map). Four people said that they had seen maps that were used in the experiment.

The next two questions asked which features of the metro map layouts were most helpful and which were least helpful when completing the tests. Many of the responses for the most helpful features noted the colour of lines as being most useful. Other responses highlighted straight lines, good spacing between stations and the labels and the angle between lines. The least helpful features included the use of curved lines, squashed up text and lines and overlapping labels. Other comments mentioned that some text was too small to read (although it wasn't actually necessary to read the text of the labels for our tests) and that yellow lines were difficult to see on a white background. These last two comments come about from the restrictions on the way that the test was conducted using a computer screen.

The final question asked whether the subjects found any questions ambiguous. Some responses mentioned that there were several potential routes to choose from. While this was possibly the case, we tried to be careful in selecting questions and the set of possible

answers based on what appeared to be the best route and trying to make the answers as unambiguous as possible. Other responses highlighted the difficulty in reading some of the yellow parts of maps and others appeared to be unsure as to what constituted a change.

### 7.5 Conclusion

Our experiment set out to confirm four hypotheses related to comparing automaticallydrawn, geographic and the official published maps of various real-world metro systems in terms of their use for finding optimal routes and the preferences of these maps.

We were able to confirm Hypothesis 1 that compared our automatically-drawn maps with geographic maps in finding the optimal route. Hypothesis 2 remained inconclusive as the performance using the automatically-drawn maps and the official published maps was too similar. However, this is also promising as it shows that maps produced using our method stand up well compared to the published maps.

The observations made of the preferences of the group of individuals surveyed support both Hypothesis 3 and Hypothesis 4 that the automatically drawn maps are more preferable than the geographic maps or the official published maps.

★ What were the limitations of the experiment? ★

# Chapter 8

# Applications of the Metro Map Metaphor

- ★ What needed to be done to the parameters/weightings of the metro map model to tune it for the new applications? ★
- $\bigstar$  Not clear what needs to be done to make the application to new areas produce useful information  $\bigstar$
- ★ Why does the metro map metaphor lend itself to these application areas over other potential layout methods? What specific features of these domains mean that the metro map criteria are appropriate? ★
- ★ It might be useful to discuss each of the metro map layout criteria and discuss if and how it may be relevant to the other domains—especially as it becomes clear that the abstract nature of these two domains make them very different from the geographic nature of maps ★
- ★ No discussion of criteria weightings is included in either of the applications ★

## 8.1 Flexibility and Extensibility

One of the key advantages of multicriteria optimisation is that it is relatively easy to change the criteria that are used when laying out a map. This allows the flexibility

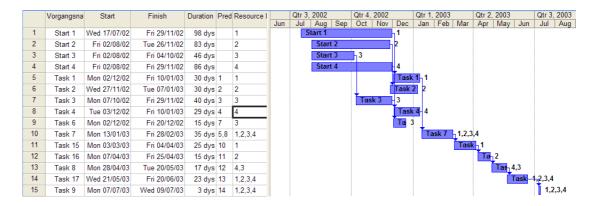


Figure 8.1: A typical Gantt chart.

to draw maps with different characteristics. These different characteristics allow us to consider using our method of drawing metro maps for different application areas beyond drawing public transport maps. Previous work on applying the metaphor to other application areas was described in Section 2.7.

This chapter describes how our metro map layout method can be adapted to two different application areas: the visualisation of project plans (Section 8.2) and website navigation visualisation (Section 8.3).

### 8.2 Visualisation of Project Plans

#### ★ How does this differ from PERT charts? ★

At present, Gantt charts are predominantly used for the mapping of projects in organisations [Gan73]. Gantt charts show each task in a project as a horizontal bar, aligned against a time axis. The time axis allows temporal relationships between the tasks to be visualized. Each task bar can also annotated with the resources (people, teams of people, etc.) that are involved in that task. Dependencies of one task on another are shown by arrows pointing between the two tasks. A typical, simple Gantt chart is shown in Figure 8.1.

While Gantt charts are effective for planning a project they are not effective for communication purposes, especially when different groups are involved (so-called interfunctional communication). This introduces a set of challenges: how to attract the attention of users; how to provide orientation and a shared vision; how to present both an overview of the project as well as certain details; and how to initiate discussions and motivate individuals to participate in the project. Burkhard and Meier [BM04] introduced the Tube Map Visualisation for projects in an organisation and evaluated its strengths and limitations in an comparative study [BMR<sup>+</sup>05].  $\star$  What did they say? How did the metro map visualisation fit into their comparison? Did it show any of the strengths they mention? Did have have any limitations?  $\star$ 

Modifications to our method for laying out metro maps are described in Section 8.2.1. In particular, we introduce a new criterion to take the time axis into account and to change the method for initial positioning of nodes.

### 8.2.1 Timescale Criterion

With the introduction of the time axis, it becomes important that task nodes appear correctly in relation to each other. For example, a task that starts before another should appear to the left of the other task. For a task node n with x-coordinate  $x_n$  and start time  $t_n$  the timescale criterion is found by calculating  $(|t_n - x_n|)^2$ . This has the effect of severely penalizing nodes that have strayed a long way from their start time while having little effect for nodes that are close to their start time. As with other criteria, the timescale criterion is sufficiently weighted so as to be effective when used in combination with the other criteria.

A side effect of the timescale criterion is that the edge length criterion is no longer required for this application, as otherwise the two criteria would conflict with each other.

### 8.2.2 Initial Embedding

When drawing metro maps, the starting position for the multicriteria optimisation method was the geographic position of the stations. However this does not apply in the case of project plan data where the diagram is abstract. Initially, all the nodes are placed along a line such that they all have the same y-coordinate. The initial x-coordinate of the node is determined by the start time of the task.

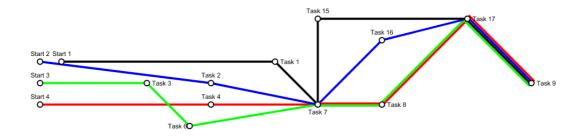


Figure 8.2: The resulting project map drawn using our automated metro map layout method. The chart is aligned from left to right along a time axis. Each line represents a group in the project; each station an individual or collective milestone in the project.

### 8.2.3 Implementation

### ★ What parameters are being used? Same as for the metro maps? ★

Figure 8.2 shows an example of a project map that has been automatically laid out using the metro map metaphor. The project plan that the map represents is the one shown in Figure 8.1. Each line is drawn with a different color to allow them to be easily distinguished. Therefore, each member of the project can trace his or her route through the project, and in particular, which order their tasks take place and how they relate to other tasks. It is also clear as to which tasks are important for more than one member of the project (such as Task 7) where the lines from more than one member intersect. Project members working together are also clearly shown where two lines run in parallel (such as either side of Task 8 and between Task 17 and Task 9).

While the automatically generated project map has obvious advantages, it suffers from a number of disadvantages such as an apparently unnecessary edge crossing, edges drawn very close to nodes (as with the Start 1 node) and edges which are not four-gonal. These problems could obviously be solved with manual editing of the project map, but this is not really desirable as it is meant to automatically generate usable project maps.

Cosmetic problems are also evident. These include the way in which different lines swap places with each other as they pass through a node (e.g. the lines passing through the Task~8 node) and the way that many parallel lines (as between the Task~17 and Task~9 nodes) are wider than the nodes.  $\bigstar$  How could these problems be fixed?  $\bigstar$ 

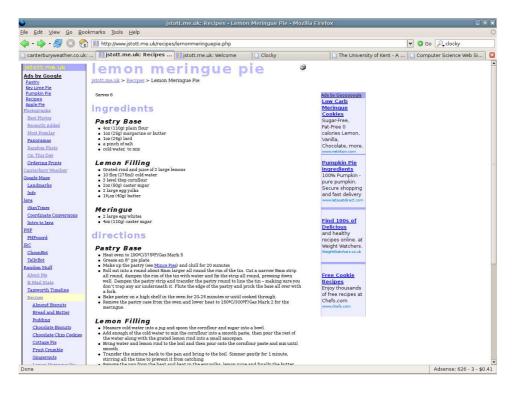


Figure 8.3: Example of a website that uses both breadcrumb trails (underneath the page heading) and a hierarchical list of contents (down the left-hand side). From jstott.me.uk.

 $\bigstar$  Should mention that the example does not have a geographic constraint  $\bigstar$ 

### 8.3 Website Navigation Visualisation

Making websites easy to navigate for the user is a constant challenge facing website designers. Several different approaches are commonly used, including simple tabulated menus, breadcrumb trails (which show the trail of pages from the home page to the current page), as shown in Figure 8.3, and site maps (which show all the pages on the website, normally grouped into categories or listed alphabetically), as shown in Figure 8.4. All of these methods are created using simple text—very few actually use a graphical visualisation of the website structure. There could be several reasons for this, including a reluctance to use a navigation technique which might be unfamiliar to the user, lack of time to develop new techniques, or lack of expertise with creating graphical navigation aids.

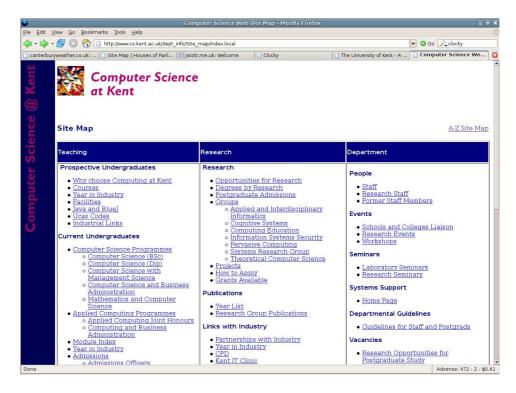


Figure 8.4: Example of a website site map. From the University of Kent Computer Science website.

We believe that the use of the metro map as a metaphor for navigating websites could prove to be a more effective solution than a simple site map of the website. Indeed, one might be of the opinion that the phrase "site map" would already imply some kind of graphical visualisation of the site. However, nearly all site maps are simple lists of pages, perhaps organised into groups or a hierarchy. With a metro map of a website, each station (node) represents a single page (or possibly a group of pages) with logical connections between pages represented by edges between nodes. Concepts that connect several related pages can be shown by distinguishing that concept as a line in the map with a separate colour.

The metro map metaphor can be used when designing websites, not just as a visualisation tool, but also to influence the fundamental design of the structure of the website. By making sure that the metaphor influences the design of the website, the strength of the metro map as a navigation visualisation is greatly enhanced.

In this case study, we examine how a metro map can be used as the navigational basis for a website with the aim of providing an online tutoring website for a programming language. This application is particularly appropriate as the mappings between individual language concepts and stations on a metro map are likely to be fairly intuitive to the website user. Also, metro lines can be used to highlight particular key concepts that join together several smaller language components (such as graphical user interfaces or file input/output).

One of the key differences in the approach required to draw a metro map style diagram for this case is that none of the nodes in the graph have any association with a geographic location, be it an absolute location (such as a grid reference or latitude and longitude) or a relative location (such as one node must be north of another node). This has one clear benefit in that we no longer need be concerned with the relative positions of nodes, allowing for greater flexibility when drawing the graph. However, deciding on an initial embedding for the graph is problematic. This is due to our method being based on optimizing an existing layout: if the initial embedding is not adequate, then our method may struggle to produce an acceptable optimisation.

Another difference the way in which the user is likely to interact with the diagram. In this case, the user must be able to accept the visualisation and be able to use it effectively in order to justify our claim that using a metro map metaphor enhances peoples ability to navigate a website. In order to achieve this, we need to include cues to the user for such things as where to start and where to go next. Such cues could be implicit (for example, the map is oriented with the start at the top with navigation flowing from top to bottom) or explicit (such as directed edges indicating direction of navigation or emphasis of the starting point in some way).

In the following two sections, we present two different results using a different initial embedding for each one.

### 8.3.1 Initial Embedding with a Spring Embedder

The first of our website navigation results uses an initial embedding created using a spring embedder [Ead84]. The spring embedder operates on a randomly produced embedding with the intention of untangling the edges and separating the various edges.

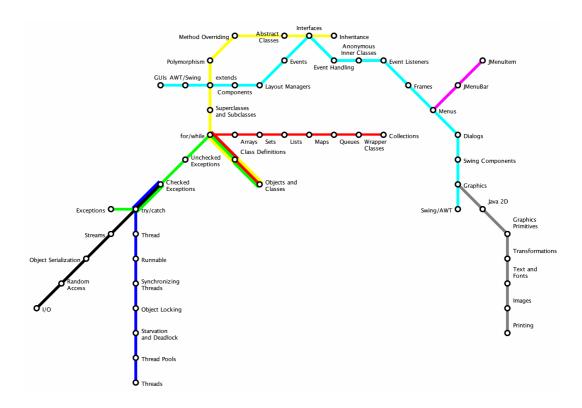


Figure 8.5: A website navigation map using the metro map metaphor using a spring embedder to produce an initial embedding. ★ Where to start? Point it out! ★

The repulsive force between unconnected nodes ensures that nodes that are not immediately adjacent are placed further apart allowing more space for labelling. Enough iterations of the spring embedder are executed such that as many edge crossings as possible are removed from the initial random embedding.

After the spring embedder has finished running, the nodes are moved to the nearest point where grid lines intersect and the metro map method is applied. Figure 8.5 shows the result of drawing the website navigation map using the metro map metaphor when using a spring embedder to produce the initial embedding.

# ★ How are the labels generated? Automatically from the web pages or user supplied? Is this a problem? ★

The map clearly exhibits many of the characteristics of metro maps. As metro maps are a familiar visualisation to many people, this immediately grabs the attention of the user and causes them to become interested in the map. The metro map metaphor gives the implicit impression that the map is designed for the purpose of navigation. Each line

is emphasised in a different colour, allowing a clear distinction to be made between each of the other line. Nearly all of the nodes are unambiguously labelled (with exceptions on the line between the for/while and Collections nodes).

In this case, the labelling is quite good in that only one label (Abstract Classes at the top of the map) occludes an edge and no labels occlude any other labels or nodes. The occlusion with the Abstract Classes label occurs because the edge passes between each line of the label and does not intersect the bounding boxes for each line of the label. A number of examples of local minima in the labelling process are evident on the line between the for/while and Collections nodes and on the line between the Graphics and Printing nodes. In these cases, all the labels are in positions where the label position criterion has not been minimised because the label position consistency criterion has taken precedence. It could be possible to move the position of a group of labels like this if more than one label was being moved at the same time (similar to how node clustering works).

As it stands, one of the main problems with the map is that the starting point for navigation is not obvious. The intention is for the *Objects and Classes* node to be the point where navigation should start, but the lack of any visual cues to support this doesnt make the start point very clear. Also, the direction of navigation from one node to the next is not particularly intuitive. These drawbacks could be solved to come extent by the introduction of extra visual cues such as directed edges and by highlighting the start node by making the label larger or increasing the size of the node symbol. However, adding these visual cues would make the map look less like a metro map therefore negating some of the effect of the metaphor.  $\star$  *DOES THIS MATTER?! The metro map metaphor fails here anyway as there is no start station. Just have a station called "start here"!* 

### 8.3.2 Initial Embedding with a Hierarchical Layout

Our second result uses a hierarchical layout algorithm [STT81] to find an initial embedding. Using a hierarchical layout algorithm makes sense in the case of website navigation as websites generally have some kind of hierarchy. The root of the hierarchy is selected

as the start point in the navigation—for a website, this would be the home page. In our example, the intention is to have the most fundamental concepts at the top of the map with a downwards movement implying more complex concepts. A modification to the hierarchical layout algorithm is therefore required such that this conceptual ordering is maintained. Figure 8.6 shows the result of drawing a website navigation map using the metro map metaphor when using a hierarchical layout algorithm to produce the initial embedding. To provide an idea of how the visualisation might be incorporated into a website, a mock-up is available online at http://www.jstott.me.uk/javatube/.

As with the first result, which used a spring embedder for the initial layout, this result clearly shows many of the characteristics of metro maps. One of the advantages of this result over the first is that the hierarchical embedding forces the starting point for navigation to be at the top of the map with a top-to-bottom ordering of concepts. From there, each subsequent node is either below or at the same level as the previous node. The short branch from extends to GUIs is a result of our modification to the hierarchical layout algorithm to enforce the conceptual ordering. A top-to-bottom navigation makes it more intuitive for a user to find their way around the map. The direction of navigation could again be reinforced using directed edges as extra visual cues.  $\bigstar$  Directed edges are not part of the metro map metaphor?  $\bigstar$ 

One of the features of the hierarchical layout of the second result is that there are a number of parallel lines, making it quite difficult to find enough space for labels. As such, this result contains two examples of labels that occlude edges—the *Unchecked Exceptions* label and the *Method Overriding* label. There are also more examples of ambiguously positioned labels than in the first result. These are labels that are not clearly associated with any particular node because they are too close to another unrelated node. Examples of ambiguously positioned labels in this result include the *Object Locking* and *Synchronizing Threads* nodes.

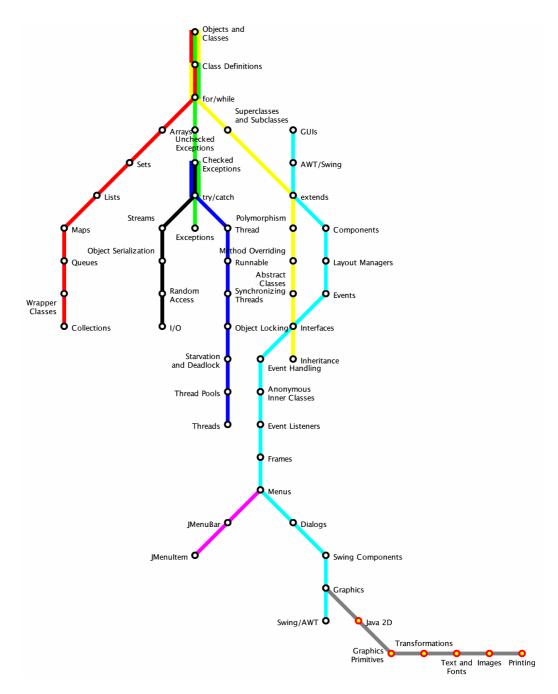


Figure 8.6: A website navigation map using the metro map metaphor using a hierarchical layout algorithm to produe an initial embedding.

# Chapter 9

## Conclusion

- $\bigstar$  Fails to say why the contributions are significant and novel  $\bigstar$ 
  - $\bigstar$  No discussion of the possible limitations of this research  $\bigstar$

This chapter provides a summary of the contributions that this research makes and also describes enhancements that are likely to improve our method further given time for further work.

### 9.1 Summary of Contributions

This thesis introduced a novel method for the layout of metro maps using a hill climbing multicriteria optimiser. The method incorporates a number of node movement and label positioning criteria which, when combined, allow nodes and labels to be moved to more optimal locations in the search space (Chapter 3). Clusters of nodes were identified and moved similarly using a number of clustering and partitioning methods (Chapter 4). This method addresses our motivation for the automatic layout of metro maps and has been demonstrated with a number of real-world examples (Chapter 6). The effectiveness of this method was assessed in an empirical experiment (Chapter 7).

Using empirical experiments, we showed that the automatically-drawn metro maps can be more effective for route planning than geographic maps, and as effective as official published maps. In addition, in many cases, the automatically-drawn metro map was preferred over both the geographic and published maps.

The use of an automatic layout method for drawing metro maps allows this metaphor to be used much more readily for the visualisation of concepts away from the traditional use for public transport networks. We demonstrated how our method can be modified to allow the production of maps of project plans and for the visualisation of website navigation hierarchies (Chapter 8).

### 9.2 Further Work

This section discusses some potential ideas that can be applied to our method in the future.

 $\star$  supplement these ideas with figures  $\star$ 

### 9.2.1 Other Metro Map Characteristics

A number of other characteristics of metro maps can be incorporated into our method. These include drawing lines using several segments (polylines), using rounded corners rather than angular corners for lines, and better handling of multiple parallel lines.

In our method we consider each edge between two nodes as a single line. Allowing edges between nodes to be formed of several segments will allow a much greater freedom for the positioning of nodes. This is likely to help a great deal in the situations where lines can not quite be drawn perfectly octilinearly. Many real world maps also use rounded corners at the point where lines change direction—these corners have a finite size that would certainly need to be taken into consideration. It is worth noting that some maps—notably that of the London Underground—have barely any situations where a line changes direction under a node. If a line does need to change direction it is usually just before or just after a node, thereby improving the appearance of the continuity of a line through the node.

We did not explicitly handle multiple parallel lines in our method and we also modelled stations as point features. Many maps use finite-sized devices to represent interchange stations between several lines, as shown in Figure 2.11 on page 23. When considering drawing parallel lines like this, the order that these lines pass through stations becomes important. Benkert *et al.* discuss how the ordering of parallel edges can be optimised [BNUW07].

Finally, we did not consider incorporating topographic metadata in our method, yet many real-world maps feature some form of topographic metadata. This is a particularly challenging enhancement when one considers the variety of different types of metadata that could be included on a map. Metadata might be in the form of boundary line (such as a coastline), a line feature (such as a road or river) or even an area feature (such as parkland).

### 9.2.2 Node Movement Criteria and Weightings

Several areas of future work involve the node movement criteria and their weightings. There is the possibility of introducing further criteria, perhaps that consider aesthetic features other than nodes or edges. For example, it may be desirable to maximise the area of faces thereby increasing the space available for labels. Also, a more rigorous and methodical approach to determining the weightings to use for each of the criteria is required. There is scope for this to be done empirically.

The results presented in Chapter 6 highlighted a number of examples of local minima in the search space. It would be useful to characterise these local minima in order to investigate how the method could be adapted or extended in order to reduce the possibility of the minima occurring.

#### 9.2.3 Labelling

A number of features of labelling on existing metro maps have been omitted from our handling of labelling. The most prominent of these is to include diagonal labels, and extra criteria can also be added to enhance the labelling results.

Diagonal labels are a fairly common feature of metro maps. While our results using only horizontal labels are generally good, certain circumstances can produce occasions where labelling with diagonal labels would be more aesthetically pleasing. Particular cases include long, horizontal lines where there is very little space for length labels above

or below the line, or diagonal lines where it is desirable to keep the labels perpendicular to the line. Diagonal labels impose an extra level of complexity when testing for occlusions or proximity as the rectangular bounding box of the label can no longer be used for these purposes.

Further criteria can be added: the proximity of labels to other labels and edges would be useful in some circumstances (the proximity to nodes was necessary to avoid ambiguities and was discussed in Section 3.9.5), and the cases where horizontal labels are allowed to alternate either side of a horizontal line could be explicitly handled with a new criterion.

### 9.2.4 Computational Efficiency

### ★ What was the efficiency performance of our implementation like? ★

Several enhancements can be made to our method to improve its computational efficiency, for example, only calculating local criteria or making judgements about whether it is worth considering a node or label for movement. In the implementation of our method we incorporated a number of efficiency improvements, but there is still room for more.

Frequently, nodes are positioned once in the first iteration of our method and they never move again. It should be possible to identify at least some nodes that have already found an optimum position and exclude those from the set of nodes to process for each iteration. This could be done by assessing whether anything in the vicinity of a node has changed such that a more optimal position for that node may have become available. Clearly, if the node was not moved in previous iterations and nothing has changed in the locality of that node, it is unlikely that it can be moved.

Local criteria can be used to significantly speed up our method. For example, it would be pointless to test whether a label on one side of the map occluded labels on the other side of the map. Equally, when moving a node, the only edges that need be considered when calculating the edge length criterion are those that are adjacent to that node.

#### 9.2.5 Other Enhancements

One enhancement that could make a significant effect of the quality of our maps is to apply some pre-processing steps on the initial layout. For example, the centrality-based scaling methods of Merrick and Gudmundsson [MG06] could help make space for labels in dense or complex maps. Centrality-based scaling allows a graph to be scaled based on some measure of the density or centrality of areas of the graph (see Section 2.8.4). The measures of centrality could be extended to incorporate not only the density of nodes but also the density of labels and lines (or edges). For metro maps, this would allow a greater scale not just for areas where there are lots of stations but also for areas where many lines converge or where there are a large number of labels.

### 9.3 In Conclusion

★ Final paragraph to round off whole thesis. THE BIG SELL for people who want to find out what's so great about my stuff! ★

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## Appendix A

# **Empirical Evaluation**

### A.1 Questions and Maps Used

#### A.1.1 Atlanta

- Geographic map (Figure A.1).
- Normalised published map (Figure A.2).
- Automatically-drawn map (Figure A.3).

Table A.1: Atlanta questions.

Number	Question	Answer Options	Answer
1.a	How many stations do you pass	$\{8, 9, 10, 11, 12\}$	10
	through to get from 'Bankhead' to		
	'Lenox'		
1.b	How many stations do you pass	$\{4, 5, 6, 7, 8\}$	5
	through to get from 'Vine City' to		
	'Midtown'		
1.c	What is the minimum number of	$\{0, 1, 2, 3, 4\}$	0
	changes to get from 'Indian Creek'		
	to 'West Lake'		
1.d	What is the minimum number of	$\{0, 1, 2, 3, 4\}$	0
	changes to get from 'College Park'		
	to 'Five Points'		

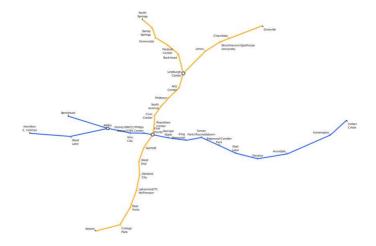


Figure A.1: Atlanta MARTA geographic map.



Figure A.2: Atlanta MARTA normalised published map.

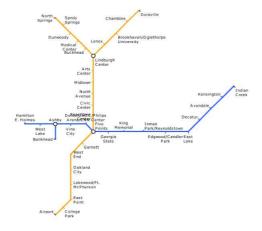


Figure A.3: Atlanta MARTA automatically-drawn map.

#### A.1.2 Bucharest

- Geographic map (Figure A.4).
- Normalised published map (Figure A.5).
- Automatically-drawn map (Figure A.6).

Table A.2: Bucharest questions.

Number	Question	Answer Options	Answer
2.a	How many stations do you pass	${9,10,11,12,13}$	10
	through to get from 'Pacii' to 'Ti-		
	tan'		
2.b	How many stations do you pass	$\{1, 2, 3, 4, 5\}$	5
	through to get from 'Romana' to		
	'Piata Sudului'		
2.c	What is the minimum number of	$\{0, 1, 2, 3, 4\}$	0
	changes to get from 'Eroilor' to		
	'Iancului'		
2.d	What is the minimum number of	$\{0, 1, 2, 3, 4\}$	2
	changes to get from 'Eroii Revolu-		
	tiei' to '1 Mai'		

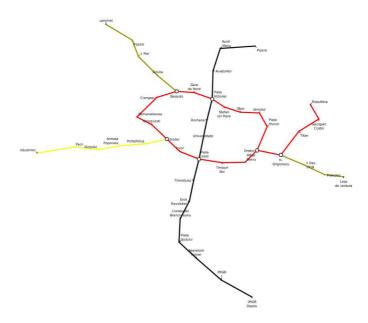


Figure A.4: Bucharest geographic map.

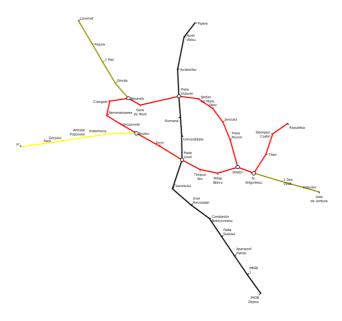


Figure A.5: Bucharest normalised published map.

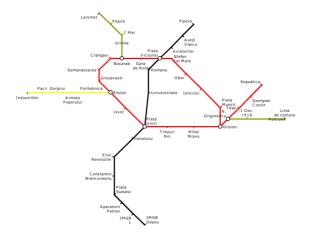


Figure A.6: Bucharest automatically-drawn map.

#### A.1.3 Mexico City

- Geographic map (Figure A.7).
- Normalised published map (Figure A.8).
- Automatically-drawn map (Figure A.9).

Table A.3: Mexico City questions.

Number	Question	Answer Options	Answer
3.a	How many stations do you pass	$\{4, 5, 6, 7, 8\}$	7
	through to get from 'Balderas' to		
	'Consulado'		
3.b	How many stations do you pass	$\{4, 5, 6, 7, 8\}$	6
	through to get from 'Refineria' to		
	'Patriotismo'		
3.c	What is the minimum number of	$\{0, 1, 2, 3, 4\}$	2
	changes to get from 'Sevilla' to		
	'Aragon'		
3.d	What is the minimum number of	$\{0, 1, 2, 3, 4\}$	2
	changes to get from 'Martin Car-		
	rera' to ''		

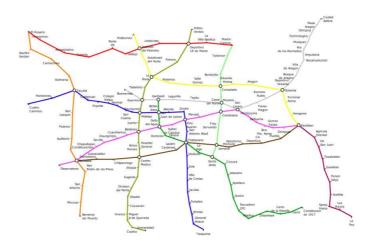


Figure A.7: Mexico City geographic map.



Figure A.8: Mexico City normalised published map.

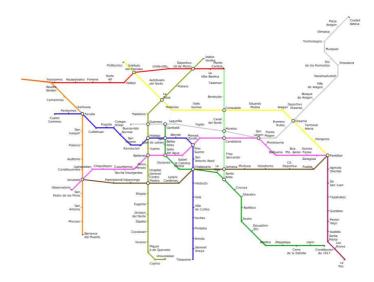


Figure A.9: Mexico City automatically-drawn map.

#### A.1.4 Stockholm

- Geographic map (Figure A.10).
- $\bullet\,$  Normalised published map (Figure A.11).
- Automatically-drawn map (Figure A.12).

Table A.4: Stockholm questions.

Number	Question	Answer Options	Answer
4.a	How many stations do you pass	$\{6, 7, 8, 9, 10\}$	10
	through to get from 'Stora Mossen'		
	to 'Karlaplan'		
4.b	How many stations do you pass	$\{2, 3, 4, 5, 6\}$	6
	through to get from 'Liljeholmen'		
	to 'Kungstrådgarden'		
4.c	What is the minimum number of	$\{0, 1, 2, 3, 4\}$	0
	changes to get from 'Kista' to 'T-		
	Centralen'		
4.d	What is the minimum number of	$\{0, 1, 2, 3, 4\}$	1
	changes to get from 'Bergamossen'		
	to 'Axelsburg'		

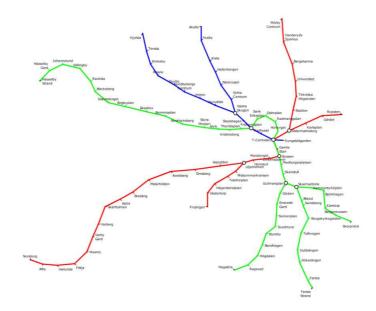


Figure A.10: Stockholm geographic map.

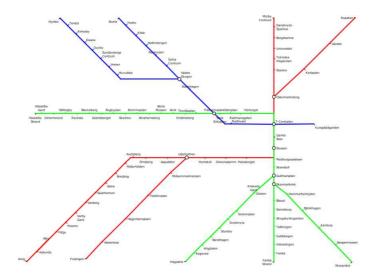


Figure A.11: Stockholm normalised published map.



Figure A.12: Stockholm automatically-drawn map.

#### A.1.5 Toronto

- $\bullet$  Geographic map (Figure A.13).
- $\bullet\,$  Normalised published map (Figure A.14).
- Automatically-drawn map (Figure A.15).

Table A.5: Toronto questions.

Number	Question	Answer Options	Answer
5.a	How many stations do you pass	$\{6, 7, 8, 9, 10\}$	7
	through to get from 'Dupont' to		
	'Chester'		
5.b	How many stations do you pass	$\{10, 11, 12, 13, 14\}$	14
	through to get from 'Lansdowne' to		
	'York Mills'		
5.c	What is the minimum number of	$\{0, 1, 2, 3, 4\}$	1
	changes to get from 'Bayview' to		
	'Union'		
5.d	What is the minimum number of	$\{0, 1, 2, 3, 4\}$	2
	changes to get from 'Midland' to		
	'Dundas'		



Figure A.13: Toronto geographic map.



Figure A.14: Toronto normalised published map.



Figure A.15: Toronto automatically-drawn map.

#### A.1.6 Washington D.C.

- Geographic map (Figure A.16).
- $\bullet\,$  Normalised published map (Figure A.17).
- Automatically-drawn map (Figure A.18).

Table A.6: Washington questions.

Number	Question	Answer Options	Answer	
6.a	How many stations do you pass	$\{1, 2, 3, 4, 5\}$	2	
	through to get from 'Pentagon' to			
	'Court House'			
6.b	How many stations do you pass	$\{4, 5, 6, 7, 8\}$	4	
	through to get from 'Cleveland			
	Park' to 'Federal Triangle'			
6.c	What is the minimum number of	$\{0, 1, 2, 3, 4\}$	0	
	changes to get from 'Metro Center'			
	to 'Takoma'			
6.d	What is the minimum number of	$\{0, 1, 2, 3, 4\}$	2	
	changes to get from 'Largo Town			
	Center' to 'Eisenhower Ave.'			

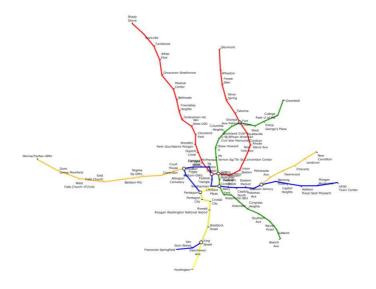


Figure A.16: Washington D.C. geographic map.

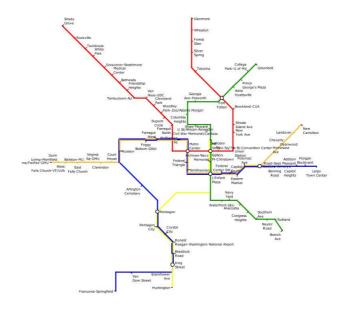


Figure A.17: Washington D.C. normalised published map.

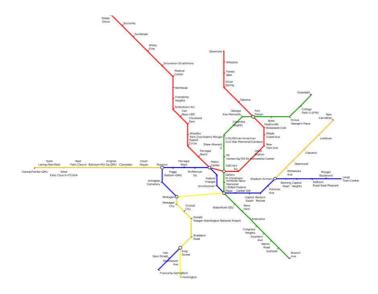


Figure A.18: Washington D.C. automatically-drawn map.

#### A.2 Associated Material

As a number of sessions of the empirical experiment were run at different times, it was essential that each session was as similar as possible. The following scripts and handouts were standard across all sessions.

#### A.2.1 Preliminary Script

This script was read aloud to candidates before they started the experiment.

[The test software should be shown on the projector]

Please do not start using a computer until told to do so. During this test, do not talk, or attempt to see what other participants are answering. If you have a query, please raise your hand.

Although we ask for your login so that we can collate the data, the results of this test and questionnaire will be anonymized.

You will be presented with a sequence of metro map diagrams. The test will pose a question for each map which requires an answer to be selected. The question will require you to plan a route between two stations on the map.

You will first need to enter your login, level of study, year of study, age and gender and click OK. Do not do this yet, you will be told when to start the test.

[Enter login test level of study Undergraduate, year of study 2, Age 25, Gender Male, then press OK. Press Start]

A metro map is used to depict the interconnections on a public transport system so that the user is able to plan and undertake a specific journey. Stations are represented by circles which are labelled with the name of the station. A line in a single colour indicates which stations are connected by direct services. Where two or more lines pass through a single station you are able to change from one line to the other. See an example of a metro map on the projector.

When you start the test, you will be shown a metro map in the main part of the screen [Point to map]. The question will be shown in the top-right corner [Point to question] with a selection of answers below [Point to answers].

The questions that will be asked will involve planning a journey between two stations. The stations will be highlighted on the map in order that you can identify them more easily [Point to highlighted stations].

Once you have worked out the route for the question, click the button next to the answer in the list shown before clicking the Go button.

In this case the question is How many changes are required to get from 'Shopping' to 'Barro'. I can see that you need to change once from one coloured line to another [point on screen]. So I click 1 [Click option 1]. Then I click GO [Click GO].

After each question you need to give an indication of the difficulty of the question. To do this, select the appropriate option from the list from very easy to very hard. Then click the OK button.

The period between clicking Go and answering the difficulty question are an opportunity to rest, if you need to do so, as timing does not start again until the OK button is clicked.

#### [Click Average then OK]

This next question asks me How many stations do I go through to get from Aeroporto to Santa Luzia. With this sort of question, you do not count the end stations, only the stations in-between. Counting the stations, including the station that requires me to change I get an answer of 9 [Point at each intermediate station, counting]. [Click option 9, then Go]

Please do not rush the questions, and take some effort to get the questions correct. Whilst we are measuring the time it takes to complete each answer, we do not mind if you do not complete all the questions.

You will be presented with questions for 20 minutes. After this time is up you will be shown how many questions you got correct as well the answers to any questions that you got incorrect.

At the end of the test, do not log off.

After the test, you need to complete a short questionnaire

Enter your details and press OK then the start button to begin the test now.

#### A.2.2 Postliminary Script

This script was read out after the interactive part of the experiment had concluded.

[The first slide should be showing on the projector]

[Hand out 1 questionnaire and 1 pen to each participant]

Please first fill in your login on the sheet in front of you, and then look up at the projector screen. You will be shown three metro maps at a time. Please decide which of these maps would be best for navigating a metro map system. As each slide is shown, write down 1, 2 or 3 in the spaces below, where 1 is the most preferable map and 3 is the least preferable map

I will count down from 5 before showing the next set of metro maps

[Count down 5, 4, 3, 2, 1, then show the next slide]

[Wait for a minute and then count down 5, 4, 3, 2, 1, for each slide, until the end of the presentation appears]

Please now take 5 minutes to fill in questions 1, 2, 3 and 4 by hand.

[Wait for 5 minutes and then start handing out five pound notes, getting signatures and handing out debriefing scripts. The experiment is now over, and you can answer questions about the tasks].

#### A.2.3 Questionnaire

This is the questionnaire that each candidate was asked to fill in in relation to the postliminary script (Section A.2.2).

#### Your Login:

Please first fill in your login above, and then look up at the projector screen. You will be shown three metro maps at a time. Please decide which of these maps would be best for navigating a metro map system. First write down your login, then, as each slide is shown, write down 1, 2 or 3 in the spaces below, where 1 is the most preferable map and 3 is the least preferable map.

Slide Preference for Map A for Map B for Map C

Slide 1

Slide 2

Slide 3

Slide 4

Slide 5

Slide 6

Table A.7: Preference grid.

Please now take five minutes to answer the questions below

- 1. Have you seen any of the metro maps shown here before these tests? If so, which ones?
- 2. Which features of the metro map layout did you find most helpful when completing the tests?
- 3. Which features of the metro map layout did you find least helpful when completing the tests?
- 4. Did you find any of the questions ambiguous?

#### A.2.4 Concluding Handout

This text was given to candidates after they had completed the questionnaire and before they left the experiment.

Thank you for participating in this research

You were presented with a number of maps which were drawn using three different techniques. One version was drawn using the geographic layout of stations; the second version was drawn from a published map of the network; the final version was drawn using an automatic method which balances aesthetic criteria to try and find an optimal layout.

The purpose of this research is to qualify some design aesthetics for automatically laying out metro maps and to see if our automated method is good at producing comprehensible diagrams.

The idea is that being able to automatically produce metro maps might improve their use for navigating metro networks. In addition, being able to automatically generate the such maps could lead to them being more widely used for many other application areas.

We would appreciate it if you did not discuss this experiment with other students in the university. These experiments will be continuing through the last two weeks of term, and having subjects who have prior knowledge of what the tests are about makes the data less useful.

Thank you again for your contribution.

#### A.3 Results

#### A.3.1 Route Planning Tasks

Table A.8: Route planning task results.

Subj.	Grp	$\mathrm{Map}^a$	Qu.	Map	Answer	Given	Correct	$\mathrm{Ease}^b$	Time
				$\mathrm{Type}^c$		Answer			(s)
1	С	1	a	F	10	11	FALSE	3	38.58
2	С	1	a	F	10	10	TRUE	2	24.06
3	$^{\mathrm{C}}$	1	a	F	10	10	TRUE	0	26.58
4	A	1	a	Р	10	10	TRUE	1	17.92
5	В	1	a	G	10	10	TRUE	2	16.81

<sup>&</sup>lt;sup>a</sup>1 = Atlanta; 2 = Bucharest; 3 = Mexico City; 4 = Stockholm; 5 = Toronto; 6 = Washington D.C.

 $<sup>^{</sup>b}1 = \text{very easy } \dots 5 = \text{very hard}$ 

 $<sup>{}^</sup>c{\bf G}=$  geographic layout;  ${\bf P}=$  normalised published layout;  ${\bf F}=$  automatically-drawn layout

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		m prev						-	m.
Subj.	Grp	Map	Qu.	Map	Answer	Given	Correct	Ease	Time
				Type	1.0	Answer	747.07		(s)
6	В	1	a	G	10	9	FALSE	2	17.7
7	В	1	a	G	10	10	TRUE	2	27.74
8	A	1	a	P	10	10	TRUE	1	17.06
9	A	1	a	P	10	10	TRUE	3	24.3
10	A	1	a	P	10	10	TRUE	2	27.86
11	C	1	a	F	10	10	TRUE	3	32.45
12	В	1	a	G	10	10	TRUE	2	26.49
13	С	1	a	$\mathbf{F}$	10	10	TRUE	2	25.66
14	A	1	a	Р	10	10	TRUE	2	16.92
15	A	1	a	P	10	10	TRUE	1	17.8
16	С	1	a	$\mathbf{F}$	10	10	TRUE	2	18.69
17	В	1	a	$\mathbf{G}$	10	11	FALSE	2	28.81
18	В	1	a	G	10	10	TRUE	0	38.89
19	С	1	a	$\mathbf{F}$	10	10	TRUE	1	16.56
20	С	1	a	$\mathbf{F}$	10	10	TRUE	2	19.75
21	A	1	a	P	10	10	TRUE	1	21.17
22	В	1	a	G	10	10	TRUE	2	15.31
23	A	1	a	P	10	10	TRUE	2	21.58
24	В	1	a	G	10	9	FALSE	1	26.28
25	$^{\mathrm{C}}$	1	a	F	10	10	TRUE	1	14.83
26	В	1	a	G	10	9	FALSE	1	20
27	A	1	a	P	10	10	TRUE	0	34.31
28	В	1	a	G	10	9	FALSE	2	18.05
29	A	1	a	P	10	10	TRUE	2	14.94
30	С	1	a	$\mathbf{F}$	10	10	TRUE	3	19.25
31	A	1	a	Р	10	10	TRUE	2	11.2
32	С	1	a	F	10	10	TRUE	2	13.49
33	В	1	a	G	10	11	FALSE	2	17.39
34	В	1	a	G	10	10	TRUE	0	27.56
35	С	1	a	F	10	10	TRUE	3	13.39
36	С	1	a	F	10	11	FALSE	3	18.11
37	В	1	a	G	10	9	FALSE	2	32.55
38	Α	1	a	Р	10	10	TRUE	2	15.2
39	A	1	a	Р	10	12	FALSE	4	15.83
40	В	1	a	G	10	10	TRUE	0	18.97
41	В	1	a	G	10	11	FALSE	$\overset{\circ}{2}$	18.28
42	$^{-}$ C	1	a	F	10	10	TRUE	3	20.02
43	Č	1	a	F	10	10	TRUE	1	19.9
1	Č	1	b	G	5	5	TRUE	2	30.66
2	Č	1	b	G	5	5	TRUE	2	18.19
3	$\stackrel{\circ}{\mathrm{C}}$	1	b	Ğ	5	5	TRUE	3	13.16
4	A	1	b	F	5	4	FALSE	4	11.05
5	В	1	b	P	5	5	TRUE	2	16.36
6	В	1	b	P	5	5	TRUE	0	10.89
7	В	1	b	P	5	5	TRUE	1	13.72
8	A	1	b	F	5	5	TRUE	1	9
9	A	1	b	F	5	5	TRUE	2	20.58
10	A	1	b	F	5	5	TRUE	2	19.45
10	11	1	IJ	T,	J	J	11001		19.40

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		m prev.				~.			
Subj.	Grp	Map	Qu.	Map	Answer	Given	Correct	Ease	Time
				Type		Answer			(s)
11	С	1	b	G	5	5	TRUE	2	17.2
12	В	1	b	P	5	5	TRUE	2	23.63
13	С	1	b	G	5	5	TRUE	2	10.81
14	A	1	b	F	5	5	TRUE	2	11.59
15	A	1	b	F	5	5	TRUE	2	12.2
16	С	1	b	$\mathbf{G}$	5	5	TRUE	1	17.98
17	В	1	b	P	5	5	TRUE	1	17.17
18	В	1	b	P	5	5	TRUE	1	12.1
19	С	1	b	G	5	5	TRUE	1	19.91
20	$^{\rm C}$	1	b	G	5	5	TRUE	2	15.89
21	A	1	b	F	5	5	TRUE	1	12.12
22	В	1	b	Р	5	5	TRUE	1	9.23
23	A	1	b	$\mathbf{F}$	5	8	FALSE	2	19.34
24	В	1	b	Р	5	5	TRUE	1	13.24
25	С	1	b	G	5	5	TRUE	2	9.8
26	В	1	b	Р	5	5	TRUE	0	15.5
27	Α	1	b	F	5	5	TRUE	1	19.21
28	В	1	b	Р	5	5	TRUE	2	10.57
29	Α	1	b	F	5	5	TRUE	3	15.56
30	С	1	b	G	5	5	TRUE	3	18.3
31	A	1	b	F	5	5	TRUE	2	10.03
32	C	1	b	G	5	5	TRUE	$\overline{2}$	10.37
33	В	1	b	P	5	5	TRUE	1	10.56
34	В	1	b	Р	5	5	TRUE	0	14.41
35	С	1	b	G	5	5	TRUE	2	13.25
36	Č	1	b	G	5	5	TRUE	1	9.58
37	В	1	b	P	5	5	TRUE	3	12.69
38	A	1	b	F	5	5	TRUE	4	12.98
39	A	1	b	F	5	8	FALSE	4	10.41
40	В	1	b	P	5	5	TRUE	0	8.29
41	В	1	b	P	5	5	TRUE	1	6.79
42	C	1	b	G	5	5	TRUE	1	12.99
43	$\stackrel{\circ}{\mathrm{C}}$	1	b	G	5	5	TRUE	1	15.99
1	$\stackrel{\circ}{\mathrm{C}}$	1	c	F	0	0	TRUE	2	13.69
2	$^{\rm C}$	1	c	F	0	1	FALSE	2	25.27
3	$^{\rm C}$	1	c	F	0	0	TRUE	$\frac{2}{2}$	26.76
4	A	1	c	P	0	0	TRUE	1	13.69
5	В	1		G	0	0	TRUE	0	24.88
6	В	1	С	G	0	0	TRUE	0	25.63
7	В	1	c	G		0	TRUE		16.64
8	A	1	c	P	0	0	TRUE	1 1	10.04 $12.08$
9		1	С	P P	0	0			
	A	1	c	P P		1	TRUE FALSE	1	11.97
10	A		c		0		TRUE	0	13.61
11	С	1	c	F	0	0		3	35.59
12	В	1	c	G	0	1	FALSE	3	86.96
13	$\mathbf{C}$	1	c	F	0	0	TRUE	2	18.69
14	A	1	С	Р	0	0	TRUE	1	9.45
15	A	1	c	P	0	0	TRUE	1	8.78

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		m prev							
Subj.	Grp	Map	Qu.	Map	Answer	Given	Correct	Ease	Time
				Type		Answer			(s)
16	С	1	c	F	0	0	TRUE	1	22.61
17	В	1	c	G	0	1	FALSE	3	44.19
18	В	1	c	G	0	1	FALSE	1	29.71
19	С	1	c	$\mathbf{F}$	0	0	TRUE	2	44.86
20	С	1	c	F	0	1	FALSE	1	25.58
21	A	1	c	Р	0	0	TRUE	0	32.6
22	В	1	c	G	0	0	TRUE	1	15.13
23	Α	1	c	P	0	0	TRUE	2	30.73
24	В	1	c	G	0	0	TRUE	2	51.58
25	С	1	c	F	0	0	TRUE	0	15.25
26	В	1	c	G	0	2	FALSE	0	12.22
27	A	1	c	Р	0	0	TRUE	0	11.53
28	В	1	c	G	0	0	TRUE	2	23.15
29	Α	1	c	Р	0	0	TRUE	1	18
30	С	1	c	F	0	0	TRUE	2	28.92
31	Α	1	c	Р	0	0	TRUE	1	14.54
32	С	1	c	F	0	0	TRUE	1	14.08
33	В	1	c	G	0	0	TRUE	0	13.3
34	В	1	c	G	0	1	FALSE	1	35.03
35	С	1	c	F	0	0	TRUE	1	18.58
36	C	1	c	F	0	0	TRUE	2	21.22
37	В	1	c	G	0	1	FALSE	$\overline{2}$	33.63
38	A	1	c	P	0	0	TRUE	$\overline{2}$	14.06
39	A	1	c	Р	0	0	TRUE	1	9.84
40	В	1	c	G	0	0	TRUE	0	17.45
41	В	1	c	G	0	0	TRUE	2	15.74
42	С	1	c	F	0	0	TRUE	1	19.63
43	Č	1	c	$\overline{\mathrm{F}}$	0	0	TRUE	1	11.98
1	Č	1	d	G	0	0	TRUE	0	9.53
2	Č	1	d	G	0	0	TRUE	$\overset{\circ}{2}$	14.95
3	$\tilde{\mathrm{C}}$	1	d	Ğ	0	0	TRUE	3	31.44
4	Ā	1	d	F	0	0	TRUE	0	13.81
5	В	1	d	P	0	0	TRUE	0	10.03
6	В	1	d	Р	0	0	TRUE	0	11.39
7	В	1	d	Р	0	0	TRUE	0	18.94
8	A	1	d	F	0	0	TRUE	1	10.94
9	A	1	d	F	0	0	TRUE	1	10.31
10	A	1	d	F	0	0	TRUE	0	11
11	C	1	d	G	0	0	TRUE	1	19.54
12	В	1	d	P	0	0	TRUE	0	29.94
13	C	1	d	G	0	0	TRUE	0	10.58
14	A	1	d	F	0	0	TRUE	0	7.64
15	A	1	d	F	0	0	TRUE	$\frac{0}{2}$	11.28
16	C	1	d	G	0	0	TRUE	0	11.23
17	В	1	d	P	0	0	TRUE	0	15.05
18	В	1	d	P	0	1	FALSE	1	17.28
19	С	1	d	G	0	0	TRUE	1	19.55
20	C	1	d	G	0	0	TRUE	$\frac{1}{2}$	19.53 $14.53$
20		1	u	G	U	U	continued		

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		m prev.				-	- C		
Subj.	Grp	Map	Qu.	Map	Answer	Given	Correct	Ease	Time
				Type		Answer			(s)
21	A	1	d	F	0	0	TRUE	0	13.58
22	В	1	d	P	0	0	TRUE	1	11.55
23	A	1	d	F	0	0	TRUE	1	17.26
24	В	1	d	P	0	0	TRUE	0	14.77
25	С	1	d	G	0	0	TRUE	0	8.74
26	В	1	d	P	0	0	TRUE	0	16.98
27	A	1	d	F	0	0	TRUE	0	14.01
28	В	1	d	P	0	0	TRUE	2	13.16
29	A	1	d	F	0	0	TRUE	1	16.08
30	$^{\rm C}$	1	d	G	0	0	TRUE	1	15.2
31	A	1	d	F	0	0	TRUE	1	10.47
32	С	1	d	G	0	0	TRUE	1	14.4
33	В	1	d	Р	0	0	TRUE	0	6
34	В	1	d	P	0	0	TRUE	0	14.46
35	С	1	d	G	0	0	TRUE	0	13
36	С	1	d	G	0	0	TRUE	0	10.84
37	В	1	d	Р	0	0	TRUE	3	19.81
38	A	1	d	F	0	0	TRUE	2	13.12
39	Α	1	d	F	0	0	TRUE	1	20.56
40	В	1	d	Р	0	0	TRUE	0	18.54
41	В	1	d	Р	0	0	TRUE	0	5.73
42	С	1	d	G	0	0	TRUE	0	13.73
43	C	1	d	G	0	0	TRUE	0	12.76
1	С	2	a	G	10	10	TRUE	2	32.38
2	С	2	a	G	10	9	FALSE	2	37.41
3	С	2	a	G	10	9	FALSE	3	24.21
4	A	2	a	F	10	10	TRUE	2	41.6
5	В	2	a	Р	10	10	TRUE	2	15.03
6	В	2	a	Р	10	10	TRUE	1	13.81
7	В	2	a	Р	10	10	TRUE	2	29.5
8	Α	2	a	F	10	10	TRUE	2	34.1
9	Α	2	a	F	10	10	TRUE	3	66.83
10	Α	2	a	F	10	10	TRUE	2	34.36
11	C	$\overline{2}$	a	G	10	9	FALSE	3	37.26
12	В	$\overline{2}$	a	P	10	10	TRUE	$\overset{\circ}{2}$	31.55
13	C	2	a	G	10	10	TRUE	3	21.75
14	A	2	a	F	10	10	TRUE	3	39.02
15	A	2	a	F	10	10	TRUE	1	13.33
16	C	2	a	G	10	9	FALSE	1	15.56
17	В	2	a	P	10	10	TRUE	2	16.61
18	В	2	a	P	10	10	TRUE	0	57.44
19	C	2	a	G	10	10	TRUE	$\frac{0}{2}$	43.83
20	$^{\rm C}$	2	a	G	10	9	FALSE	$\frac{2}{2}$	13.53
21	A	2	a	F	10	10	TRUE	3	28.84
22	В	$\frac{2}{2}$	a	P	10	10	TRUE	2	13.2
23	A	$\frac{2}{2}$	a	F	10	10	TRUE	3	50.81
24	В	$\frac{2}{2}$		P	10	10	TRUE	2	13.72
25	С	2	a	G	10	9	FALSE	1	23.66
∠0	$\cup$		a	G	10	9	FALSE	1	∠5.00

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		m prev				-	- C		
Subj.	Grp	Map	Qu.	Map	Answer	Given	Correct	Ease	Time
				Type		Answer			(s)
26	В	2	a	P	10	10	TRUE	2	23.53
27	A	2	a	F	10	10	TRUE	0	45.08
28	В	2	a	P	10	10	TRUE	2	16.73
29	A	2	a	$\mathbf{F}$	10	11	FALSE	3	25.43
30	С	2	a	$\mathbf{G}$	10	9	FALSE	3	18.77
31	A	2	a	$\mathbf{F}$	10	10	TRUE	3	35.23
32	С	2	a	G	10	10	TRUE	2	18.1
33	В	2	a	P	10	10	TRUE	2	11.44
34	В	2	a	P	10	11	FALSE	0	51.63
35	С	2	a	G	10	9	FALSE	2	37.02
36	С	2	a	G	10	10	TRUE	3	23.77
37	В	2	a	Р	10	10	TRUE	2	12.97
38	A	2	a	$\mathbf{F}$	10	10	TRUE	2	10.45
39	A	2	a	$\mathbf{F}$	10	10	TRUE	2	12.19
40	В	2	a	Р	10	10	TRUE	0	21.25
41	В	2	a	Р	10	11	FALSE	2	18.17
42	С	2	a	G	10	10	TRUE	3	16.73
43	С	2	a	G	10	9	FALSE	1	22.77
1	С	2	b	Р	5	5	TRUE	1	13.38
2	C	2	b	Р	5	5	TRUE	2	17.33
3	C	2	b	Р	5	5	TRUE	3	18.78
4	Ā	$\overline{2}$	b	G	5	5	TRUE	0	9.81
5	В	$\overline{2}$	b	F	5	5	TRUE	$\overset{\circ}{2}$	10.02
6	В	2	b	F	5	5	TRUE	2	10.63
7	В	2	b	F	5	5	TRUE	2	27.35
8	Α	2	b	G	5	5	TRUE	1	15.11
9	A	$\overline{2}$	b	G	5	5	TRUE	$\overline{2}$	22.17
10	A	$\overline{2}$	b	G	5	5	TRUE	1	9.27
11	C	2	b	P	5	5	TRUE	1	20.68
12	В	2	b	F	5	5	TRUE	$\frac{1}{2}$	32.69
13	C	2	b	P	5	5	TRUE	1	20.23
14	A	2	b	G	5	5	TRUE	$\frac{1}{2}$	9.64
15	A	2	b	G	5	5	TRUE	2	14.25
16	C	2	b	P	5	5	TRUE	0	9.83
17	В	2	b	F	5	5	TRUE	1	21.17
18	В	2	b	F	5	5	TRUE	2	16.97
19	C	2	b	P	5	5	TRUE	1	26.04
20	C	2	b	P	5	5	TRUE	2	13.6
21	A	2	b	G	5	3	FALSE	3	25.31
22	В	2	b	F	5	5	TRUE	1	8.3
23	A	$\frac{2}{2}$	b	G	5	5	TRUE	1	34.8
24	В	$\frac{2}{2}$	b	F	5	5	TRUE	1	10.86
25	C	$\frac{2}{2}$	b	P	5	5	TRUE	1	10.05
26	В	2	b	F	5 5	5	TRUE	0	10.03 $14.2$
20	A	$\frac{2}{2}$	b	G G	5 5	5 5	TRUE	2	18.4
28	В	$\frac{2}{2}$	b	F	5 5	3 4	FALSE	$\frac{2}{2}$	8.95
29	A	$\frac{2}{2}$	b	G G	5 5	5 5	TRUE	$\frac{2}{2}$	$\frac{8.95}{21.57}$
30	C	$\frac{2}{2}$		P	5 5	5 5			
ა∪			b	۲	Э	Э	TRUE	1	10.67

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Subj.	$\operatorname{Grp}$	Map	Qu.	Map	Answer	Given	Correct	Ease	Time
				Type		Answer			(s)
31	A	2	b	G	5	5	TRUE	3	32.85
32	$^{\mathrm{C}}$	2	b	P	5	5	TRUE	1	14.7
33	В	2	b	$\mathbf{F}$	5	5	TRUE	1	12.67
34	В	2	b	$\mathbf{F}$	5	5	TRUE	0	30.66
35	$^{\mathrm{C}}$	2	b	Р	5	5	TRUE	1	16.52
36	С	2	b	Р	5	5	TRUE	1	7.38
37	В	2	b	F	5	5	TRUE	2	9.59
38	A	2	b	G	5	5	TRUE	3	10
39	A	2	b	G	5	5	TRUE	4	20.25
40	В	2	b	F	5	5	TRUE	0	10.27
41	В	2	b	$^{-}$	5	5	TRUE	1	7.79
42	C	2	b	P	5	5	TRUE	1	19.57
43	C	2	b	P	5	5	TRUE	1	11.88
1	C	2	c	G	0	0	TRUE	1	12.75
$\frac{1}{2}$	C	2	c	G	0	0	TRUE	2	26.02
3	C	2	c	G	0	0	TRUE	2	22.15
$\frac{3}{4}$	A	$\frac{2}{2}$	c	F	0	0	TRUE	0	14.13
5	В	$\frac{2}{2}$	c	P	0	0	TRUE	1	19.09
6	В	$\frac{2}{2}$		P	0	0	TRUE	0	$\frac{13.03}{22.81}$
7	В	$\frac{2}{2}$	c	P	0	0	TRUE	1	34.25
8	A	$\frac{2}{2}$	c	F	0	0			12.95
			c				TRUE	1	
9	A	$\frac{2}{2}$	c	F	0	0	TRUE	1	29.66
10	A		С	F	0	0	TRUE	1	12.83
11	С	2	С	G	0	0	TRUE	3	48.67
12	В	2	c	Р	0	0	TRUE	2	52.5
13	$\mathbf{C}$	2	c	G	0	2	FALSE	3	23.83
14	A	2	c	F	0	0	TRUE	1	8.03
15	A	2	c	F	0	0	TRUE	1	18.67
16	С	2	c	G	0	0	TRUE	0	9.64
17	В	2	c	P	0	0	TRUE	2	35.5
18	В	2	c	P	0	3	FALSE	2	45.7
19	С	2	c	G	0	0	TRUE	2	27.77
20	$^{\mathrm{C}}$	2	c	G	0	0	TRUE	1	19.21
21	A	2	c	F	0	0	TRUE	1	17.86
22	В	2	$^{\mathrm{c}}$	P	0	0	TRUE	1	22.52
23	A	2	$^{\mathrm{c}}$	F	0	0	TRUE	2	44.15
24	В	2	c	P	0	1	FALSE	3	42.82
25	$^{\rm C}$	2	c	G	0	0	TRUE	1	10.88
26	В	2	$^{\mathrm{c}}$	P	0	2	FALSE	1	12.77
27	A	2	c	F	0	0	TRUE	0	10.42
28	В	2	С	P	0	0	TRUE	2	34.99
29	A	2	c	$\mathbf{F}$	0	0	TRUE	1	12.67
30	$^{\mathrm{C}}$	2	c	G	0	0	TRUE	2	19.41
31	A	2	С	F	0	0	TRUE	1	16.62
32	$^{\mathrm{C}}$	2	c	G	0	0	TRUE	1	9.41
33	В	$\overline{2}$	c	P	0	0	TRUE	0	11.03
34	В	2	c	P	0	0	TRUE	0	38.6
35	C	2	c	G	0	0	TRUE	1	18.13
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Subj.	$\operatorname{Grp}$	Map	Qu.	Map	Answer	Given	Correct	Ease	Time
				Type		Answer			(s)
36	С	2	$\mathbf{c}$	G	0	0	TRUE	2	23.59
37	В	2	$\mathbf{c}$	P	0	0	TRUE	1	12.63
38	A	2	$^{\mathrm{c}}$	F	0	0	TRUE	3	24.25
39	A	2	$^{\mathrm{c}}$	$\mathbf{F}$	0	0	TRUE	1	9.23
40	В	2	$^{\mathrm{c}}$	P	0	1	FALSE	0	14.5
41	В	2	$^{\mathrm{c}}$	Р	0	0	TRUE	2	14.72
42	$^{\mathrm{C}}$	2	$\mathbf{c}$	G	0	0	TRUE	1	29.35
43	$^{\mathrm{C}}$	2	$\mathbf{c}$	G	0	0	TRUE	0	7.28
1	$^{\mathrm{C}}$	2	d	Р	2	3	FALSE	1	15.89
2	$^{\mathrm{C}}$	2	d	Р	2	2	TRUE	2	23.31
3	$^{\mathrm{C}}$	2	d	Р	2	2	TRUE	2	20.6
4	A	2	d	G	2	2	TRUE	0	15.09
5	В	2	d	F	2	2	TRUE	2	15.81
6	В	2	d	F	2	2	TRUE	1	10.14
7	В	2	d	F	2	2	TRUE	2	24.58
8	A	2	d	G	2	2	TRUE	1	13.8
9	A	2	d	G	2	2	TRUE	2	31.27
10	A	2	d	G	2	2	TRUE	3	37.78
11	C	$\overline{2}$	d	P	$\overline{2}$	$\overline{2}$	TRUE	$\overset{\circ}{2}$	26.7
$\overline{12}$	В	$\overline{2}$	d	$^{-}$	$\overline{2}$	3	FALSE	$\overline{2}$	42.46
13	C	2	d	P	2	$\overset{\circ}{2}$	TRUE	2	27.95
14	A	2	d	G	2	2	TRUE	2	17.31
15	A	2	d	Ğ	$\frac{2}{2}$	2	TRUE	2	9.55
16	C	2	d	P	$\frac{z}{2}$	2	TRUE	1	15.19
17	В	2	d	F	2	2	TRUE	1	19.3
18	В	2	d	F	$\frac{2}{2}$	3	FALSE	1	15.24
19	С	2	d	P	$\frac{2}{2}$	2	TRUE	2	44.14
20	C	2	d	P	$\frac{2}{2}$	2	TRUE	2	23.82
21	A	2	d	G	$\frac{2}{2}$	2	TRUE	2	25.59
22	В	$\frac{2}{2}$	d	F	$\frac{2}{2}$	2	TRUE	1	14.05
23	A	$\frac{2}{2}$	d	G	$\frac{2}{2}$	$\frac{2}{2}$	TRUE	3	56.6
$\frac{23}{24}$	В	$\frac{2}{2}$	d	F	$\frac{2}{2}$	$\frac{2}{2}$	TRUE	1	14.55
$\frac{24}{25}$	С	$\frac{2}{2}$	d	P	$\frac{2}{2}$	$\frac{2}{2}$	TRUE	0	11.38
26	В	$\frac{2}{2}$	d	F	$\frac{2}{2}$	$\frac{2}{2}$	TRUE	1	21.19
27	A	2	d	G	$\frac{2}{2}$	$\frac{2}{2}$	TRUE	0	$\frac{21.13}{22.13}$
28	В	$\frac{2}{2}$	d	F	$\frac{2}{2}$	$\frac{2}{2}$	TRUE	$\frac{0}{2}$	19.93
29	A	$\frac{2}{2}$	d	G	$\frac{2}{2}$	$\frac{2}{2}$	TRUE	$\frac{2}{2}$	19.33
$\frac{29}{30}$	$\stackrel{\mathbf{A}}{\mathrm{C}}$	$\frac{2}{2}$	d	P	$\frac{2}{2}$	$\frac{2}{2}$	TRUE	2	22.34
31		$\frac{2}{2}$	d	G	$\frac{2}{2}$	$\frac{2}{2}$	TRUE	2	19.56
$\frac{31}{32}$	${ m A} \over { m C}$	$\frac{2}{2}$		P	$\frac{2}{2}$	$\frac{2}{2}$	TRUE	2	
	В	$\frac{2}{2}$	d	F	$\frac{2}{2}$	$\frac{2}{2}$		$\frac{2}{2}$	14.51
33		$\frac{2}{2}$	d		$\frac{2}{2}$		TRUE		20.05
34	В С		d	F		2	TRUE	2	35.46
35 26		2	d	P	2	2	TRUE	1	16.72
36	С	2	d	Р	2	3	FALSE	2	21.63
37	В	2	d	F	2	2	TRUE	3	21.41
38	A	2	d	G	2	2	TRUE	2	14.23
39	A	2	d	G	2	2	TRUE	2	16.5
40	В	2	d	F	2	2	TRUE	0	16.23

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Subj.	Grp	Map	Qu.	Map	Answer	Given	Correct	Ease	Time
				Type		Answer			(s)
41	В	2	d	F	2	2	TRUE	1	11.75
42	С	2	d	P	2	2	TRUE	2	13.54
43	С	2	d	P	2	2	TRUE	1	19.52
1	С	3	a	P	7	8	FALSE	2	25.19
2	С	3	$\mathbf{a}$	P	7	7	TRUE	2	76.92
3	С	3	a	P	7	6	FALSE	3	22.82
4	A	3	a	G	7	7	TRUE	0	22.19
5	В	3	a	$\mathbf{F}$	7	7	TRUE	2	14.41
6	В	3	a	F	7	6	FALSE	2	20.64
7	В	3	a	F	7	7	TRUE	2	23.11
8	A	3	a	G	7	7	TRUE	2	17.74
9	A	3	a	G	7	7	TRUE	3	28.7
10	A	3	a	G	7	7	TRUE	2	29.11
11	С	3	a	Р	7	7	TRUE	4	23.75
12	В	3	a	F	7	6	FALSE	2	26.05
13	С	3	a	Р	7	6	FALSE	3	17.27
14	Α	3	a	G	7	7	TRUE	3	25.67
15	Α	3	a	G	7	7	TRUE	1	16.33
16	С	3	a	P	7	5	FALSE	1	26.06
17	В	3	a	F	7	7	TRUE	$\overset{-}{2}$	37.99
18	В	3	a	F	7	6	FALSE	2	59.47
19	C	3	a	P	7	6	FALSE	2	50.61
20	Č	3	a	P	7	7	TRUE	2	18.55
21	A	3	a	G	7	7	TRUE	3	34.97
22	В	3	a	F	7	7	TRUE	2	22
23	A	3	a	G	7	7	TRUE	2	33.78
24	В	3	a	F	7	7	TRUE	2	54.94
25	C	3	a	P	7	7	TRUE	1	34.97
26	В	3	a	F	7	7	TRUE	2	42.17
27	A	3	a	G	7	8	FALSE	1	37.19
28	В	3		F	7	7	TRUE	2	15.66
29	A	3	a	G	7	7	TRUE	3	16.89
30	C	3	a	P	7	7	TRUE	3	18.61
31	A	3	a	G	7	7	TRUE	3	29.13
32	C	3	a	P	7	8	FALSE	2	$\frac{29.13}{16.2}$
			a				TRUE		
33	В	3	a	F	7 7	7 6		3	39.03 $25.92$
34	В	3	a	F	7		FALSE	0	
35	С	3	a	P	7	7	TRUE	2	19.27
36	С	3	a	Р	7	6	FALSE	3	24.99
37	В	3	a	F	7	7	TRUE	3	13.85
38	A	3	a	G	7	5	FALSE	2	13.03
39	A	3	a	G	7	7	TRUE	2	9.31
40	В	3	a	F	7	6	FALSE	0	21.19
41	В	3	a	F	7	6	FALSE	1	15.74
42	C	3	a	P	7	7	TRUE	4	18.03
43	C	3	a	P	7	7	TRUE	1	16.84
1	C	3	b	F	6	6	TRUE	2	16.89
2	С	3	b	F	6	6	TRUE	2	30.98

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Subj.	$\operatorname{Grp}$	Map	Qu.	Map	Answer	Given	Correct	Ease	Time
	~		,	Type		Answer	TD IID	2	(s)
3	C	3	b	F	6	6	TRUE	3	19.6
4	A	3	b	P	6	5	FALSE	0	11.84
5	В	3	b	G	6	6	TRUE	1	15.59
6	В	3	b	G	6	6	TRUE	2	12.05
7	В	3	b	G	6	6	TRUE	1	16.3
8	A	3	b	P	6	6	TRUE	1	11.86
9	A	3	b	P	6	6	TRUE	2	23.05
10	A	3	b	P	6	6	TRUE	1	12.42
11	$^{\mathrm{C}}$	3	b	F	6	6	TRUE	2	12.25
12	В	3	b	G	6	6	TRUE	2	34.39
13	$^{\mathrm{C}}$	3	b	F	6	6	TRUE	3	18.25
14	A	3	b	P	6	6	TRUE	1	13.7
15	A	3	b	P	6	6	TRUE	2	13.52
16	$^{\mathrm{C}}$	3	b	F	6	8	FALSE	2	26.55
17	В	3	b	G	6	6	TRUE	2	20.41
18	В	3	b	G	6	6	TRUE	2	12.42
19	$^{\rm C}$	3	b	$\mathbf{F}$	6	5	FALSE	2	41.38
20	$^{\rm C}$	3	b	$\mathbf{F}$	6	6	TRUE	2	12.69
21	A	3	b	Р	6	6	TRUE	1	17.81
22	В	3	b	G	6	6	TRUE	2	9.47
23	A	3	b	Р	6	7	FALSE	1	16.95
24	В	3	b	G	6	6	TRUE	1	15.94
25	$^{\mathrm{C}}$	3	b	$\mathbf{F}$	6	6	TRUE	1	17.23
26	В	3	b	G	6	6	TRUE	1	12.91
27	A	3	b	Р	6	6	TRUE	1	16.07
28	В	3	b	G	6	6	TRUE	2	11.5
29	A	3	b	Р	6	6	TRUE	2	11.26
30	$^{\mathrm{C}}$	3	b	F	6	6	TRUE	1	15.73
31	A	3	b	Р	6	6	TRUE	2	18.48
32	$^{\mathrm{C}}$	3	b	F	6	6	TRUE	2	12.8
33	В	3	b	G	6	6	TRUE	1	7.44
34	В	3	b	G	6	6	TRUE	0	51.39
35	$^{\mathrm{C}}$	3	b	F	6	6	TRUE	1	11.81
36	$^{\mathrm{C}}$	3	b	F	6	6	TRUE	1	10.56
37	В	3	b	G	6	6	TRUE	3	11.71
38	A	3	b	P	6	6	TRUE	3	11.7
39	A	3	b	P	6	6	TRUE	3	11.98
40	В	3	b	G	6	6	TRUE	0	15.47
41	В	3	b	G	6	6	TRUE	2	12.21
42	C	3	b	F	6	6	TRUE	3	17.12
43	C	3	b	F	6	6	TRUE	1	13.93
1	$^{\rm C}$	3	c	P	2	2	TRUE	3	27.19
2	C	3	c	P	2	2	TRUE	3	129.64
3	C	3	c	P	$\frac{2}{2}$	1	FALSE	2	20.68
4	A	3	c	G	$\frac{2}{2}$	1	FALSE	1	14.05
5	В	3		F	$\frac{2}{2}$	1	FALSE	1	14.05 $17.56$
6	В	3	c	F	$\frac{2}{2}$	2	TRUE	2	18.23
0 7	В	3	c	F F	$\frac{2}{2}$	1		$\frac{2}{2}$	
(	D	3	c	Г		1	FALSE		23.77

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Subj.	$\operatorname{Grp}$	Map	Qu.	Map	Answer	Given	Correct	Ease	Time
				Type		Answer	D4 T 0 D		(s)
8	A	3	c	G	2	1	FALSE	1	14.17
9	A	3	c	G	2	2	TRUE	2	21.62
10	A	3	c	G	2	1	FALSE	1	11.92
11	С	3	c	P	2	2	TRUE	2	51.46
12	В	3	c	F	2	2	TRUE	3	45.05
13	C	3	$^{\mathrm{c}}$	P	2	2	TRUE	1	24.84
14	A	3	$^{\mathrm{c}}$	G	2	1	FALSE	2	23.5
15	A	3	$^{\mathrm{c}}$	G	2	3	FALSE	2	13.08
16	$^{\mathrm{C}}$	3	$^{\mathrm{c}}$	P	2	2	TRUE	1	22.39
17	В	3	$^{\mathrm{c}}$	F	2	1	FALSE	1	36.31
18	В	3	$^{\mathrm{c}}$	F	2	1	FALSE	0	28.61
19	$^{\mathrm{C}}$	3	$^{\mathrm{c}}$	Р	2	1	FALSE	2	26.91
20	$^{\mathrm{C}}$	3	c	Р	2	2	TRUE	2	17.6
21	A	3	c	G	2	1	FALSE	2	26.41
22	В	3	$\mathbf{c}$	$\mathbf{F}$	2	1	FALSE	1	18.94
23	A	3	c	G	2	2	TRUE	2	37.69
24	В	3	$\mathbf{c}$	$\mathbf{F}$	2	2	TRUE	2	42.57
25	$^{\rm C}$	3	$^{\mathrm{c}}$	Р	2	1	FALSE	1	15.48
26	В	3	$^{\mathrm{c}}$	F	2	2	TRUE	2	170.64
27	A	3	$^{\mathrm{c}}$	G	2	1	FALSE	2	38.49
28	В	3	c	$\mathbf{F}$	2	2	TRUE	2	40.19
29	A	3	c	G	2	1	FALSE	2	21.89
30	$^{\mathrm{C}}$	3	c	Р	2	1	FALSE	2	24
31	A	3	c	G	2	1	FALSE	2	28.07
32	$^{\mathrm{C}}$	3	c	Р	2	1	FALSE	1	13.14
33	В	3	c	$\mathbf{F}$	2	2	TRUE	2	41.77
34	В	3	c	$\mathbf{F}$	2	1	FALSE	0	30.15
35	$^{\mathrm{C}}$	3	c	Р	2	1	FALSE	2	22.74
36	$^{\mathrm{C}}$	3	c	Р	2	1	FALSE	2	33.17
37	В	3	c	F	2	1	FALSE	1	19.94
38	A	3	c	G	2	1	FALSE	2	13.9
39	A	3	c	G	2	2	TRUE	2	14.66
40	В	3	c	F	2	2	TRUE	0	12.78
41	В	3	c	F	2	2	TRUE	2	22.74
42	$^{\mathrm{C}}$	3	c	Р	2	1	FALSE	2	15.12
43	$^{\mathrm{C}}$	3	c	Р	2	1	FALSE	2	17.93
1	$^{\mathrm{C}}$	3	d	F	2	3	FALSE	2	53.73
2	$^{\mathrm{C}}$	3	d	F	2	3	FALSE	2	30.96
3	$^{\mathrm{C}}$	3	d	F	2	2	TRUE	3	30.39
4	Ā	3	d	P	2	2	TRUE	0	15.42
5	В	3	d	G	2	3	FALSE	1	14.74
6	В	3	d	G	2	2	TRUE	2	18.25
7	В	3	d	G	2	2	TRUE	2	19.63
8	A	3	d	P	2	2	TRUE	$\frac{2}{2}$	23.25
9	A	3	d	P	$\frac{2}{2}$	2	TRUE	$\frac{2}{2}$	39.76
10	A	3	d	P	$\frac{2}{2}$	1	FALSE	4	42.13
11	C	3	d	F	$\frac{2}{2}$	2	TRUE	2	18.37
12	В	3	d	G	$\frac{2}{2}$	3	FALSE	$\frac{2}{2}$	41.8
12	D	ა	u	G		3	FALSE		41.0

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		m previ							
Subj.	Grp	Map	Qu.	Map	Answer	Given	Correct	Ease	Time
				Type		Answer			(s)
13	С	3	d	F	2	2	TRUE	2	29.72
14	A	3	d	P	2	3	FALSE	2	28.74
15	A	3	d	P	2	2	TRUE	2	12.92
16	$^{\rm C}$	3	d	F	2	2	TRUE	0	23.95
17	В	3	d	G	2	2	TRUE	4	66.03
18	В	3	d	G	2	4	FALSE	2	16
19	$^{\mathrm{C}}$	3	d	F	2	2	TRUE	1	46.8
20	$^{\rm C}$	3	d	F	2	3	FALSE	2	36.52
21	A	3	d	P	2	2	TRUE	2	33.89
22	В	3	d	G	2	2	TRUE	1	14.47
23	A	3	d	Р	2	2	TRUE	2	28.4
24	В	3	d	G	2	3	FALSE	1	18.91
25	$^{\mathrm{C}}$	3	d	F	2	2	TRUE	0	24.3
26	В	3	d	G	2	2	TRUE	1	26.87
27	A	3	d	P	2	2	TRUE	0	40.95
28	В	3	d	G	$\frac{1}{2}$	1	FALSE	$\overset{\circ}{2}$	27.44
29	A	3	d	P	$\frac{1}{2}$	2	TRUE	3	34.38
30	C	3	d	F	2	2	TRUE	1	26.81
31	A	3	d	P	2	2	TRUE	2	25.67
32	C	3	d	F	$\frac{2}{2}$	2	TRUE	3	19.59
33	В	3	d	G	$\frac{2}{2}$	$\frac{2}{2}$	TRUE	3	26.67
34	В	3	d	G	$\frac{2}{2}$	2	TRUE	$\frac{3}{2}$	67.62
35	C	3	d	F	$\frac{2}{2}$	2	TRUE	1	21.69
36	C	3	d	F	$\frac{2}{2}$	2	TRUE	2	12.52
37	В	3	d	G	$\frac{2}{2}$	2	TRUE	3	21.6
38	A	3	d	P	$\frac{2}{2}$	3	FALSE	2	19.08
39	A	3	d	P	$\frac{2}{2}$	2	TRUE	$\frac{2}{2}$	21.61
40	В	3	d	G	$\frac{2}{2}$	$\frac{2}{2}$	TRUE	1	21.01 $24.93$
40	В	3	d	G	$\frac{2}{2}$	$\frac{2}{2}$	TRUE	3	19.94
41	С	3	d	F	$\frac{2}{2}$	$\frac{2}{2}$	TRUE	2	
43	C	3	d	F	$\overset{2}{2}$	$\overset{2}{2}$	TRUE	$\frac{2}{2}$	18.92 20.33
43 1	C	3 4		F	10	10	TRUE	$\frac{2}{2}$	
$\frac{1}{2}$	C		a				FALSE		31.13
		4	a	F	10	7		2	101.92 $21.1$
3	$\mathbf{C}$	4	a	F P	10	10	TRUE	4	
4	A	4	a	_	10	10	TRUE	0	15.39
5	В	4	a	G	10	7	FALSE	2	16.92
6	В	4	a	G	10	7	FALSE	3	14.89
7	В	4	a	G	10	6	FALSE	2	23.42
8	A	4	a	Р	10	9	FALSE	3	29.19
9	A	4	a	P	10	7	FALSE	3	56.17
10	A	4	a	Р	10	9	FALSE	3	31.8
11	С	4	a	F	10	7	FALSE	2	30.9
12	В	4	a	G	10	7	FALSE	2	26.72
13	С	4	a	F	10	7	FALSE	2	20.36
14	A	4	a	P	10	9	FALSE	2	25.69
15	A	4	a	P	10	7	FALSE	2	24.44
16	С	4	a	F	10	10	TRUE	1	17.83
17	В	4	a	G	10	7	FALSE	3	28.36

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		m prev							·
Subj.	Grp	Map	Qu.	Map	Answer	Given	Correct	Ease	Time
				Type		Answer			(s)
18	В	4	a	G	10	7	FALSE	2	18.36
19	С	4	a	F	10	7	FALSE	1	41.08
20	С	4	a	F	10	7	FALSE	2	19.27
21	A	4	a	P	10	9	FALSE	2	16.89
22	В	4	a	G	10	7	FALSE	2	23.22
23	A	4	a	P	10	8	FALSE	3	54.4
24	В	4	a	G	10	7	FALSE	1	25.6
25	$^{\rm C}$	4	a	$\mathbf{F}$	10	6	FALSE	1	13.77
26	В	4	a	G	10	7	FALSE	2	31.36
27	A	4	a	Р	10	10	TRUE	2	34.11
28	В	4	a	G	10	7	FALSE	2	26.03
29	A	4	a	P	10	10	TRUE	3	15.13
30	С	4	a	F	10	7	FALSE	3	21.25
31	A	4	a	Р	10	10	TRUE	4	25.1
32	С	4	a	$\mathbf{F}$	10	7	FALSE	3	22.91
33	В	4	a	G	10	7	FALSE	2	18.14
34	В	4	a	G	10	7	FALSE	1	34.72
35	С	4	a	F	10	7	FALSE	2	21.42
36	С	4	a	F	10	7	FALSE	3	35.72
37	В	4	a	G	10	8	FALSE	3	13.94
38	A	4	a	P	10	9	FALSE	3	18.43
39	A	4	a	P	10	7	FALSE	$\frac{3}{2}$	23.38
40	В	4	a	G	10	7	FALSE	0	15.67
41	В	4	a	Ğ	10	7	FALSE	$\overset{\circ}{2}$	10.7
42	C	4	a	F	10	7	FALSE	3	18.72
43	Č	4	a	F	10	7	FALSE	$\overset{\circ}{2}$	27.61
1	Č	4	b	G	6	5	FALSE	2	17.58
2	Č	4	b	Ğ	6	5	FALSE	2	16.33
3	C	4	b	G	6	4	FALSE	2	10.89
4	A	4	b	F	6	6	TRUE	0	7.44
5	В	4	b	P	6	6	TRUE	3	20.31
6	В	4	b	P	6	6	TRUE	1	11
7	В	4	b	P	6	6	TRUE	$\frac{1}{2}$	33.21
8	A	4	b	F	6	6	TRUE	2	19.81
9	A	4	b	F	6	6	TRUE	2	12.33
10	A	4	b	F	6	6	TRUE	2	15.95
11	C	4	b	G	6	5	FALSE	3	25.85
12	В	4	b	P	6	6	TRUE	4	82.65
13	C	4	b	G	6	4	FALSE	2	10.64
14	A	4	b	F	6	6	TRUE	2	10.04
15	A	4	b	F	6	6	TRUE	1	9.38
16	C	4	b	G	6	4	FALSE	0	16.53
17	В	4	b	P	6	6	TRUE	3	23.33
18	В	4	b	P	6	5	FALSE	3 1	10.37
19	С	4	b b	G	6	5 5	FALSE	1	19.47
20	C	4	b b	G	6	5 5	FALSE	1	19.47 $12.41$
20	A	4	b b	F	6	6	TRUE	1	12.41 $12.22$
21 22	B B			r P		6		$\frac{1}{3}$	
22	D	4	b	۲	6	Ü	TRUE	ა	22.5

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Subj.	$\operatorname{Grp}$	Map	Qu.	Map	Answer	Given	Correct	Ease	Time
				Type		Answer			(s)
23	A	4	b	F	6	6	TRUE	2	33.55
24	В	4	b	P	6	2	FALSE	2	26.33
25	$^{\mathrm{C}}$	4	b	G	6	5	FALSE	1	8.84
26	В	4	b	P	6	6	TRUE	2	32.08
27	A	4	b	F	6	6	TRUE	1	16.23
28	В	4	b	Р	6	6	TRUE	2	9.37
29	A	4	b	F	6	6	TRUE	3	7.48
30	$^{\mathrm{C}}$	4	b	$\mathbf{G}$	6	5	FALSE	3	11.22
31	A	4	b	$\mathbf{F}$	6	6	TRUE	3	10.64
32	$^{\mathrm{C}}$	4	b	G	6	5	FALSE	2	11.61
33	В	4	b	Р	6	6	TRUE	2	14.08
34	В	4	b	Р	6	6	TRUE	4	65.72
35	$^{\mathrm{C}}$	4	b	G	6	5	FALSE	1	15.91
36	$^{\mathrm{C}}$	4	b	G	6	5	FALSE	1	9.16
37	В	4	b	Р	6	6	TRUE	2	25.03
38	A	4	b	F	6	6	TRUE	2	11.31
39	A	4	b	F	6	6	TRUE	2	8.44
40	В	4	b	Р	6	6	TRUE	0	25.91
41	В	4	b	P	6	6	TRUE	3	28.76
42	$^{-}$ C	4	b	G	6	5	FALSE	4	14.12
43	$\overset{\circ}{\mathrm{C}}$	4	b	Ğ	6	4	FALSE	1	8.64
1	$\overset{\circ}{\mathrm{C}}$	4	c	F	0	0	TRUE	$\frac{1}{2}$	21.7
$\stackrel{1}{2}$	$\overset{\circ}{\mathrm{C}}$	4	c	F	0	1	FALSE	2	28.2
3	$\overset{\circ}{\mathrm{C}}$	4	c	F	0	0	TRUE	3	20.11
4	A	4	c	P	0	0	TRUE	0	10.31
5	В	4	c	G	0	0	TRUE	1	9.95
6	В	4	c	G	0	0	TRUE	2	10.47
7	В	4	c	G	0	0	TRUE	1	16.21
8	A	4	c	P	0	0	TRUE	1	9.44
9	A	4	c	P	0	0	TRUE	2	15.33
10	A	4	c	P	0	0	TRUE	0	8.31
11	C	4	c	F	0	1	FALSE	3	37.82
12	В	4	c	G	0	0	TRUE	2	$\frac{37.02}{26.1}$
13	С	4	c	F	0	0	TRUE	1	23.31
14	A	4	c	P	0	0	TRUE	0	6.97
15	A	4		P	0	0	TRUE	1	12.06
$\frac{15}{16}$	C	4	С	F	0	0	TRUE		39.44
17	В	$\frac{4}{4}$	c	G	0	1	FALSE	$\frac{1}{3}$	
			c	G		$\frac{1}{2}$	FALSE	2	35.94 22.39
18	В С	4	c	F	0			$\frac{2}{2}$	22.39
19	C	4	c		0	0	TRUE		
20		4	c	F P	0	0	TRUE	2	22.28
21	A	4	c		0	0	TRUE	1	11.88
22	В	4	c	G	0	0	TRUE	1	13.19
23	A	4	c	Р	0	0	TRUE	2	18.07
24	В	4	С	G	0	0	TRUE	2	16.86
25 26	С	4	С	F	0	1	FALSE	2	46.33
26	В	4	С	G	0	0	TRUE	1	17.44
27	A	4	С	Р	0	0	TRUE	0	14.56

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Subj.	Grp	Map	Qu.	Map	Answer	Given	Correct	Ease	Time
				Type		Answer			(s)
28	В	4	c	G	0	0	TRUE	2	16.94
29	A	4	c	P	0	0	TRUE	1	6.58
30	C	4	c	F	0	0	TRUE	2	17.03
31	A	4	c	P	0	0	TRUE	1	9.43
32	C	4	c	F	0	0	TRUE	1	6.08
33	В	4	c	G	0	0	TRUE	0	9.92
34	В	4	c	G	0	0	TRUE	1	31.99
35	С	4	c	F	0	0	TRUE	1	10.5
36	С	4	c	F	0	0	TRUE	2	26.84
37	В	4	c	G	0	0	TRUE	1	11.7
38	A	4	c	P	0	0	TRUE	2	10.83
39	A	4	c	P	0	0	TRUE	2	10.66
40	В	4	c	G	0	0	TRUE	0	11.38
41	В	4	c	G	0	0	TRUE	2	16.2
42	С	4	c	F	0	0	TRUE	1	9.86
43	С	4	c	F	0	0	TRUE	0	6.9
1	С	4	d	G	1	3	FALSE	3	24.8
2	С	4	d	G	1	1	TRUE	2	28.32
3	С	4	d	G	1	1	TRUE	3	21.28
4	A	4	d	F	1	3	FALSE	2	13.92
5	В	4	d	Р	1	1	TRUE	2	37.47
6	В	4	d	Р	1	1	TRUE	2	16.13
7	В	4	d	P	1	1	TRUE	2	57.62
8	A	4	d	$\mathbf{F}$	1	1	TRUE	2	12.58
9	A	4	d	$\mathbf{F}$	1	1	TRUE	2	35.23
10	A	4	d	$\mathbf{F}$	1	1	TRUE	2	16.11
11	С	4	d	G	1	2	FALSE	3	38.27
12	В	4	d	Р	1	2	FALSE	3	95.47
13	С	4	d	G	1	1	TRUE	2	17.25
14	A	4	d	F	1	1	TRUE	2	35.21
15	A	4	d	$\mathbf{F}$	1	1	TRUE	1	13.83
16	С	4	d	G	1	1	TRUE	0	20.05
17	В	4	d	Р	1	1	TRUE	4	39.94
18	В	4	d	Р	1	2	FALSE	1	12.13
19	С	4	d	G	1	1	TRUE	1	17.56
20	С	4	d	G	1	1	TRUE	2	14.45
21	A	4	d	F	1	1	TRUE	2	15.66
22	В	4	d	Р	1	1	TRUE	3	39.63
23	A	4	d	F	1	1	TRUE	3	59.89
24	В	4	d	Р	1	1	TRUE	0	21.07
25	С	4	d	G	1	1	TRUE	0	8.67
26	В	4	d	P	1	1	TRUE	1	48.66
27	A	4	d	F	1	1	TRUE	0	12.21
28	В	4	d	P	1	1	TRUE	2	46.44
29	A	4	d	F	1	1	TRUE	3	31.8
30	C	4	d	G	1	4	FALSE	4	10.38
31	A	4	d	F	1	1	TRUE	2	17.98
32	C	4	d	G	1	1	TRUE	2	12.13
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Subj.	$\operatorname{Grp}$	Map	Qu.	Map	Answer	Given	Correct	Ease	Time
				Type		Answer			(s)
33	В	4	d	P	1	1	TRUE	2	17.67
34	В	4	d	P	1	0	FALSE	3	52.16
35	$^{\mathrm{C}}$	4	d	G	1	1	TRUE	1	6.59
36	$^{\mathrm{C}}$	4	d	G	1	1	TRUE	2	11.88
37	В	4	d	P	1	2	FALSE	4	37.66
38	A	4	d	$\mathbf{F}$	1	1	TRUE	2	12.43
39	A	4	d	F	1	4	FALSE	4	28.44
40	В	4	d	Р	1	2	FALSE	2	20.48
41	В	4	d	Р	1	2	FALSE	2	17.31
42	С	4	d	G	1	1	TRUE	2	10.77
43	Č	4	d	G	1	1	TRUE	1	16.15
1	$\overset{\circ}{\mathrm{C}}$	5	a	Ğ	7	6	FALSE	2	17.95
2	C	5	a	G	7	6	FALSE	2	20.47
3	C	5	a	G	7	7	TRUE	2	21.62
$\frac{3}{4}$	A	5	a	F	7	6	FALSE	1	14.11
5	В	5		P	7	6	FALSE	3	27.14
6	В	5	a	P	7	6	FALSE	4	39.75
7	В	5	a	P	7	6	FALSE	2	55.24
			a						
8	A	5	a	F	7	6	FALSE	3	23.45
9	A	5	a	F	7	6	FALSE	2	24.72
10	A	5	a	F	7	6	FALSE	2	26.5
11	С	5	a	G	7	6	FALSE	4	45.51
12	В	5	a	P	7	6	FALSE	2	70.93
13	С	5	a	G	7	6	FALSE	3	25.48
14	A	5	a	F	7	6	FALSE	1	11.89
15	A	5	a	F	7	6	FALSE	1	11.94
16	С	5	a	G	7	6	FALSE	4	54.92
17	В	5	a	P	7	6	FALSE	3	27.56
18	В	5	a	P	7	6	FALSE	3	96.4
19	$^{\mathrm{C}}$	5	a	G	7	6	FALSE	2	49.14
20	$^{\mathrm{C}}$	5	a	G	7	6	FALSE	2	16.63
21	A	5	a	F	7	6	FALSE	1	11.95
22	В	5	a	P	7	6	FALSE	2	27.42
23	A	5	a	F	7	7	TRUE	2	27.93
24	В	5	a	P	7	6	FALSE	2	36.5
25	$\mathbf{C}$	5	a	G	7	6	FALSE	3	85.13
26	В	5	a	Р	7	10	FALSE	3	103.14
27	A	5	a	F	7	6	FALSE	0	16.34
28	В	5	a	Р	7	6	FALSE	2	25.97
29	A	5	a	F	7	6	FALSE	2	9.31
30	C	5	a	G	7	6	FALSE	3	22.34
31	A	5	a	F	7	6	FALSE	$\frac{3}{2}$	24.06
32	$\stackrel{\Lambda}{\mathrm{C}}$	5	a	G	7	6	FALSE	$\frac{2}{2}$	9.05
33	В	5		P	7	6	FALSE	$\frac{2}{4}$	68
34	В	5	a	P	7	6	FALSE	2	33.59
$\frac{34}{35}$	С		a						
		5	a	G	7	6	FALSE	4	47.42
36	С	5	a	G	7	6	FALSE	4	71.85
37	В	5	a	Р	7	6	FALSE	4	20.63

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		m prev							
Subj.	Grp	Map	Qu.	Map	Answer	Given	Correct	Ease	Time
				Type		Answer			(s)
38	A	5	a	F	7	6	FALSE	3	15.17
39	A	5	a	$\mathbf{F}$	7	6	FALSE	2	11.42
40	В	5	a	P	7	6	FALSE	0	63.36
41	В	5	a	P	7	6	FALSE	3	32.08
42	С	5	a	G	7	6	FALSE	4	32.1
43	С	5	a	G	7	6	FALSE	3	55.08
1	С	5	b	Р	14	12	FALSE	2	29.75
2	С	5	b	Р	14	11	FALSE	2	21.39
3	С	5	b	Р	14	11	FALSE	4	21.72
4	A	5	b	$\mathbf{G}$	14	13	FALSE	2	9.77
5	В	5	b	$\mathbf{F}$	14	14	TRUE	2	15.99
6	В	5	b	$\mathbf{F}$	14	14	TRUE	3	18.2
7	В	5	b	$\mathbf{F}$	14	14	TRUE	2	13.67
8	A	5	b	G	14	13	FALSE	3	32.63
9	A	5	b	G	14	13	FALSE	2	26.23
10	A	5	b	G	14	13	FALSE	3	33.52
11	С	5	b	Р	14	14	TRUE	4	30.74
12	В	5	b	F	14	13	FALSE	3	53.22
13	С	5	b	Р	14	13	FALSE	3	19.12
14	Α	5	b	G	14	13	FALSE	2	13.25
15	Α	5	b	G	14	13	FALSE	2	12.56
16	С	5	b	P	14	13	FALSE	2	19.5
17	В	5	b	F	14	14	TRUE	3	35.75
18	В	5	b	F	14	13	FALSE	1	14.44
19	С	5	b	Р	14	12	FALSE	3	27.06
20	C	5	b	Р	14	13	FALSE	3	20.92
21	Ā	5	b	G	14	10	FALSE	2	16.39
22	В	5	b	F	14	14	TRUE	3	15.14
23	A	5	b	G	14	13	FALSE	2	35.42
24	В	5	b	$\overset{\smile}{\mathrm{F}}$	14	14	TRUE	2	19.66
25	C	5	b	P	14	13	FALSE	1	14.91
26	В	5	b	F	14	14	TRUE	$\frac{1}{2}$	30.08
27	A	5	b	G	14	13	FALSE	1	12.37
28	В	5	b	F	14	14	TRUE	$\frac{1}{2}$	14.61
29	A	5	b	G	14	13	FALSE	3	13.93
30	C	5	b	P	14	13	FALSE	3	20.64
31	A	5	b	G	14	13	FALSE	2	13.9
32	$^{\Lambda}$ C	5	b	P	14	14	TRUE	2	15.57
33	В	5	b	F	14	14	TRUE	2	14
34	В	5	b	F	14	14	TRUE	0	29.21
35	C	5	b	P	14	13	FALSE	1	14.95
36	C	5	b	P	14	13	FALSE	2	17.05
37	В	5	b	F	14	14	TRUE	3	12.07
38	A	5	b	G	14	13	FALSE	3	25.29
39	A	5	b	G	14	13	FALSE	2	12.28
40	В	5	b	F	14	14	TRUE	0	12.26
40	В	5	b	F	$\frac{14}{14}$	14	TRUE	3	15.15
41	С	5	b	r P	$\frac{14}{14}$	13	FALSE	3 4	16.78
42		J	D	1	1.4	10	continue		

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		m prev						-	TD1
Subj.	Grp	Map	Qu.	Map	Answer	Given	Correct	Ease	Time
				Type		Answer			(s)
43	С	5	b	P	14	13	FALSE	1	13.95
1	C	5	c	G	1	1	TRUE	1	13.42
2	C	5	c	G	1	1	TRUE	2	26.44
3	C	5	c	G	1	1	TRUE	2	13.68
4	A	5	c	F	1	1	TRUE	1	9.36
5	В	5	c	Р	1	1	TRUE	1	7.83
6	В	5	$^{\mathrm{c}}$	P	1	1	TRUE	2	10.63
7	В	5	c	Р	1	1	TRUE	2	13.85
8	A	5	c	$\mathbf{F}$	1	1	TRUE	2	13.16
9	A	5	$^{\mathrm{c}}$	$\mathbf{F}$	1	1	TRUE	2	13.41
10	A	5	$^{\mathrm{c}}$	F	1	1	TRUE	1	15.11
11	С	5	$^{\mathrm{c}}$	G	1	1	TRUE	2	26.81
12	В	5	c	Р	1	2	FALSE	1	20.77
13	С	5	c	G	1	1	TRUE	2	16.52
14	A	5	c	$\mathbf{F}$	1	1	TRUE	0	8.38
15	A	5	c	$\mathbf{F}$	1	1	TRUE	2	13.78
16	С	5	c	G	1	1	TRUE	1	15
17	В	5	c	Р	1	1	TRUE	0	28.99
18	В	5	c	Р	1	2	FALSE	0	23.31
19	С	5	c	G	1	1	TRUE	1	21.45
20	C	5	c	G	1	1	TRUE	1	13.27
21	Ā	5	c	F	1	1	TRUE	1	8.52
22	В	5	c	P	1	1	TRUE	1	10.28
23	A	5	c	F	1	1	TRUE	$\frac{1}{2}$	11.01
$\frac{23}{24}$	В	5	c	P	1	1	TRUE	1	12.92
25	C	5	c	G	1	1	TRUE	1	9.8
26	В	5	c	P	1	1	TRUE	1	10.13
$\frac{20}{27}$	A	5	c	F	1	1	TRUE	0	11.38
28	В	5	c	P	1	1	TRUE	$\frac{0}{2}$	$\frac{11.36}{22.77}$
29	A	5	c	F	1	1	TRUE	$\frac{2}{2}$	4.44
$\frac{29}{30}$	C	5	c	G	1	1	TRUE	$\frac{2}{2}$	11.12
31	A	5	c	F	1	1	TRUE	$\frac{2}{2}$	8.67
$\frac{31}{32}$	C	5		G	1	1	TRUE	1	9.57
33	В	5	c	P	1	1	TRUE	1	13.59
34	В	5	c	P	1	1			
			c				TRUE	0	21.42
$\frac{35}{26}$	С	5	c	G	1	1	TRUE	2	7.77
36	С	5	С	G	1	1	TRUE	2	11.27
37	В	5	С	Р	1	1	TRUE	2	12
38	A	5	c	F	1	1	TRUE	2	10.83
39	A	5	c	F	1	1	TRUE	2	9.11
40	В	5	С	Р	1	1	TRUE	0	9.28
41	В	5	c	Р	1	1	TRUE	1	8.19
42	С	5	С	G	1	1	TRUE	1	20.35
43	C	5	c	G	1	1	TRUE	1	10.24
1	C	5	d	P	2	3	FALSE	2	10.64
2	C	5	d	P	2	1	FALSE	2	11.87
3	C	5	d	P	2	2	TRUE	3	12.69
4	A	5	d	G	2	1	FALSE	2	7.05

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Subj.	Grp	Map	Qu.	Map	Answer	Given	Correct	Ease	Time
				Type		Answer			(s)
5	В	5	d	F	2	2	TRUE	2	11.2
6	В	5	d	$\mathbf{F}$	2	1	FALSE	2	21.13
7	В	5	d	$\mathbf{F}$	2	1	FALSE	1	15.97
8	A	5	d	G	2	1	FALSE	2	19.22
9	A	5	d	G	2	2	TRUE	2	20.33
10	A	5	d	G	2	1	FALSE	4	40.17
11	$^{\mathrm{C}}$	5	d	Р	2	2	TRUE	3	21.32
12	В	5	d	$\mathbf{F}$	2	1	FALSE	1	19.61
13	С	5	d	P	2	2	TRUE	1	15.5
14	A	5	d	G	2	2	TRUE	2	34.28
15	Α	5	d	G	2	2	TRUE	2	15.22
16	С	5	d	Р	2	1	FALSE	4	28.64
17	В	5	d	F	2	2	TRUE	4	20.05
18	В	5	d	F	2	1	FALSE	0	15.96
19	С	5	d	Р	2	2	TRUE	2	38.86
20	Č	5	d	P	$\overline{2}$	1	FALSE	1	13.33
21	Ā	5	d	G	$\overline{2}$	1	FALSE	1	19.06
22	В	5	d	$\overset{\smile}{\mathrm{F}}$	$\frac{z}{2}$	1	FALSE	1	16.5
23	A	5	d	G	$\frac{2}{2}$	1	FALSE	3	26.69
24	В	5	d	F	$\frac{2}{2}$	$\overset{1}{2}$	TRUE	2	29.5
25	C	5	d	P	$\frac{2}{2}$	$\frac{2}{2}$	TRUE	4	19.44
26	В	5	d	F	$\frac{2}{2}$	1	FALSE	1	14.67
27	A	5	d	G	$\frac{2}{2}$	$\frac{1}{2}$	TRUE	3	47.1
28	В	5	d	F	$\frac{2}{2}$	1	FALSE	2	8.82
29	A	5	d	G	$\frac{2}{2}$	1	FALSE	3	25.26
30	C	5	d	P	$\frac{2}{2}$	1	FALSE	1	17.34
31	A	5	d	G	$\frac{2}{2}$	$\frac{1}{2}$	TRUE	3	23.08
32	C	5	d	P	$\frac{2}{2}$	$\frac{2}{2}$	TRUE	2	10.37
33	В	5	d	F	$\frac{2}{2}$	$\frac{2}{2}$	TRUE	3	19.43
34	В	5 5	d	F	$\frac{2}{2}$	1	FALSE	0	
	С	5 5	d	г Р	$\frac{2}{2}$	$\frac{1}{2}$	TRUE	2	17.05
35	C	5 5	d d	P P	$\frac{2}{2}$	$\frac{2}{2}$	TRUE	$\frac{2}{3}$	15.59
36	В					$\frac{2}{2}$	TRUE		25.88
37		5	d	F	2			3	10.42
38	A	5 5	d	G	$\frac{2}{2}$	1	FALSE	$\frac{1}{3}$	9.04 $22.2$
39	A	_	d	G	_	_	FALSE	_	
40	В	5	d	F	$\frac{2}{2}$	2	TRUE	2	25.58
41	В	5	d	F	2	1	FALSE	2	10.34
42	С	5	d	P	2	1	FALSE	1	10.8
43	C	5	d	Р	2	2	TRUE	2	23.67
1	С	6	a	Р	2	1	FALSE	1	12.34
2	C	6	a	Р	2	2	TRUE	2	55.77
3	C	6	a	P	2	3	FALSE	0	28.71
4	A	6	a	G	2	1	FALSE	2	10.88
5	В	6	a	F	2	2	TRUE	1	15.61
6	В	6	a	F	2	1	FALSE	1	16.03
7	В	6	a	F	2	2	TRUE	0	20.75
8	A	6	a	G	2	2	TRUE	3	14.97
9	A	6	a	G	2	1	FALSE	1	22.05

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		m previ							
Subj.	Grp	Map	Qu.	Map	Answer	Given	Correct	Ease	Time
				Type		Answer			(s)
10	A	6	a	G	2	1	FALSE	1	12.27
11	С	6	a	P	2	5	FALSE	4	61.13
12	В	6	a	F	2	2	TRUE	1	24.77
13	$^{\rm C}$	6	a	P	2	2	TRUE	3	12.23
14	A	6	a	G	2	2	TRUE	1	8.97
15	A	6	a	G	2	2	TRUE	2	13.92
16	$^{\mathrm{C}}$	6	a	Р	2	5	FALSE	4	72.85
17	В	6	a	F	2	2	TRUE	0	16.13
18	В	6	a	F	2	2	TRUE	0	21.71
19	$^{\mathrm{C}}$	6	a	Р	2	2	TRUE	2	48.1
20	$^{\mathrm{C}}$	6	a	Р	2	1	FALSE	1	11.41
21	A	6	a	G	2	1	FALSE	1	10.8
22	В	6	a	F	2	2	TRUE	1	7.13
23	A	6	a	G	2	2	TRUE	1	16.64
$\frac{1}{24}$	В	6	a	F	$\frac{1}{2}$	2	TRUE	1	33.33
25	$^{-}$ C	6	a	P	$\frac{1}{2}$	5	FALSE	$\overline{4}$	102.06
26	В	6	a	F	$\frac{1}{2}$	2	TRUE	1	16.97
27	A	6	a	G	2	1	FALSE	0	15.16
28	В	6	a	F	2	1	FALSE	$\overset{\circ}{2}$	18.72
29	A	6	a	G	$\frac{2}{2}$	5	FALSE	2	12.94
30	C	6	a	P	$\frac{2}{2}$	5	FALSE	4	42.42
31	A	6	a	G	$\frac{2}{2}$	1	FALSE	1	20.56
$\frac{31}{32}$	$\stackrel{\Lambda}{\mathrm{C}}$	6	a	P	$\frac{2}{2}$	5	FALSE	4	76.19
33	В	6	a	F	$\frac{2}{2}$	$\frac{3}{2}$	TRUE	1	9.11
34	В	6	a	F	$\frac{2}{2}$	2	TRUE	1	14.52
$\frac{34}{35}$	С	6		P	$\frac{2}{2}$	$\frac{2}{2}$	TRUE	2	28.03
$\frac{35}{36}$	C	6	a	P	$\frac{2}{2}$	3	FALSE	4	82.63
$\frac{30}{37}$	В	6	a	F	$\frac{2}{2}$	2	TRUE	1	27.3
38	A	6	a	G	$\frac{2}{2}$	1	FALSE	4	9.94
39	A	6	a	G	$\frac{2}{2}$	1	FALSE	2	8.2
39 40	В	6	a	F	$\frac{2}{2}$	$\frac{1}{2}$	TRUE	0	10.18
			a	F	$\frac{2}{2}$				
41	В	6	a	r P		2	TRUE	1	6.31
42	С	6	a		2	2	TRUE	3	28.64
43	${ m C}$	6	a	Р	2	1	FALSE	0	10.9
1	~	6	b	F	4	4	TRUE	2	11.92
2	С	6	b	F	4	4	TRUE	2	14.94
3	C	6	b	F	4	4	TRUE	3	20.48
4	A	6	b	P	4	4	TRUE	2	9.75
5	В	6	b	G	4	4	TRUE	3	20.05
6	В	6	b	G	4	4	TRUE	3	15.33
7	В	6	b	G	4	4	TRUE	3	26.39
8	A	6	b	P	4	4	TRUE	2	19.49
9	A	6	b	P	4	4	TRUE	2	27.39
10	A	6	b	P	4	5	FALSE	4	28.44
11	С	6	b	F	4	4	TRUE	1	23.36
12	В	6	b	G	4	4	TRUE	3	54.49
13	$^{\mathrm{C}}$	6	b	F	4	4	TRUE	1	13.94
14	A	6	b	P	4	4	TRUE	3	12.69

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Type			m prev							
15	Subj.	Grp	Map	Qu.	Map	Answer	Given	Correct	Ease	Time
16         C         6         b         F         4         4         TRUE         0         9.4           17         B         6         b         G         4         4         TRUE         3         25.            18         B         6         b         G         4         4         TRUE         2         31.           19         C         6         b         F         4         4         TRUE         1         24.           20         C         6         b         F         4         4         TRUE         1         24.           21         A         6         b         F         4         4         TRUE         2         12.           21         A         6         b         F         4         4         TRUE         2         28.           22         B         6         b         G         4         4         TRUE         1         21.           25         C         6         b         F         4         4         TRUE         1         21.           25         C         6         b         F							Answer			(s)
17         B         6         b         G         4         4         TRUE         3         25.           18         B         6         b         G         4         4         TRUE         2         31.            19         C         6         b         F         4         4         TRUE         1         24.           20         C         6         b         F         4         4         TRUE         1         24.           21         A         6         b         P         4         4         TRUE         3         22.           22         B         6         b         G         4         4         TRUE         3         22.           23         A         6         b         F         4         4         TRUE         1         21.           25         C         6         b         F         4         4         TRUE         1         21.           25         C         6         b         F         4         4         TRUE         1         21.           25         C         6         b         F						4			2	14.02
18         B         6         b         G         4         4         TRUE         2         31.           19         C         6         b         F         4         4         TRUE         1         24.            20         C         6         b         F         4         4         TRUE         2         12.           21         A         6         b         P         4         4         TRUE         3         22.           22         B         6         b         G         4         4         TRUE         3         22.           23         A         6         b         P         4         4         TRUE         3         8.4           24         B         6         b         G         4         4         TRUE         1         6.7           26         B         6         b         G         4         5         FALSE         4         21.           27         A         6         b         P         4         4         TRUE         1         16.           29         A         6         b         F				b		4	4	TRUE		9.42
19         C         6         b         F         4         4         TRUE         1         24.           20         C         6         b         F         4         4         TRUE         2         12.            21         A         6         b         P         4         4         TRUE         3         22.           22         B         6         b         G         4         4         TRUE         3         8.4           23         A         6         b         P         4         4         TRUE         2         28.           24         B         6         b         G         4         4         TRUE         1         21.           25         C         6         b         F         4         4         TRUE         1         21.           26         B         6         b         G         4         5         FALSE         4         21.           27         A         6         b         P         4         4         TRUE         2         27.           28         B         6         b         G	17	В	6	b		4	4	TRUE	3	25.52
20         C         6         b         F         4         4         TRUE         2         12.2           21         A         6         b         P         4         4         TRUE         3         22.3           22         B         6         b         G         4         4         TRUE         3         8.4           23         A         6         b         P         4         4         TRUE         2         28.           24         B         6         b         G         4         4         TRUE         1         21.4           25         C         6         b         F         4         4         TRUE         1         6.7           26         B         6         b         G         4         5         FALSE         4         21.6           27         A         6         b         P         4         4         TRUE         2         27.3           28         B         6         b         G         4         4         TRUE         2         21.3           30         C         6         b         F <td>18</td> <td></td> <td>6</td> <td>b</td> <td></td> <td>4</td> <td>4</td> <td>TRUE</td> <td>2</td> <td>31.15</td>	18		6	b		4	4	TRUE	2	31.15
21         A         6         b         P         4         4         TRUE         3         22.           22         B         6         b         G         4         4         TRUE         3         8.4           23         A         6         b         P         4         4         TRUE         2         28.           24         B         6         b         G         4         4         TRUE         1         21.           25         C         6         b         F         4         4         TRUE         1         6.7           26         B         6         b         G         4         5         FALSE         4         21.         27.           28         B         6         b         G         4         6         FALSE         2         20.           29         A         6         b         F         4         4         TRUE         3         14.           30         C         6         b         F         4         4         TRUE         3         14.           31         A         6         b	19	С	6	b	F	4	4	TRUE	1	24.39
22         B         6         b         G         4         4         TRUE         3         8.4           23         A         6         b         P         4         4         TRUE         2         28.            24         B         6         b         G         4         4         TRUE         1         21.           25         C         6         b         F         4         4         TRUE         1         6.7           26         B         6         b         G         4         5         FALSE         4         21.           27         A         6         b         P         4         4         TRUE         2         27.           28         B         6         b         G         4         6         FALSE         2         20.           29         A         6         b         F         4         4         TRUE         2         21.           30         C         6         b         F         4         4         TRUE         2         10.           31         A         6         b         F	20	С	6	b	F	4	4	TRUE	2	12.97
23         A         6         b         P         4         4         TRUE         2         28.           24         B         6         b         G         4         4         TRUE         1         21.            25         C         6         b         F         4         4         TRUE         1         6.           26         B         6         b         G         4         5         FALSE         4         21.           27         A         6         b         P         4         4         TRUE         2         27.           28         B         6         b         G         4         6         FALSE         2         20.           29         A         6         b         F         4         4         TRUE         3         14.           30         C         6         b         F         4         4         TRUE         2         10.           31         A         6         b         F         4         4         TRUE         2         11.           32         C         6         b         F	21	A	6	b	Р	4	4	TRUE	3	22.53
24         B         6         b         G         4         4         TRUE         1         21.0           25         C         6         b         F         4         4         TRUE         1         6.7           26         B         6         b         G         4         5         FALSE         4         21.           27         A         6         b         P         4         4         TRUE         2         27.           28         B         6         b         G         4         6         FALSE         2         20.           29         A         6         b         P         4         4         TRUE         3         14.           30         C         6         b         F         4         4         TRUE         2         10.           31         A         6         b         F         4         4         TRUE         2         11.           32         C         6         b         F         4         4         TRUE         1         10.           33         B         6         b         G	22	В	6	b	G	4	4	TRUE	3	8.48
25         C         6         b         F         4         4         TRUE         1         6.7           26         B         6         b         G         4         5         FALSE         4         21.           27         A         6         b         P         4         4         TRUE         2         27.           28         B         6         b         G         4         6         FALSE         2         20.           29         A         6         b         P         4         4         TRUE         3         14.           30         C         6         b         F         4         4         TRUE         2         10.           31         A         6         b         F         4         4         TRUE         2         11.           32         C         6         b         F         4         4         TRUE         1         10.           33         B         6         b         G         4         4         TRUE         1         11.           34         B         6         b         F	23	A	6	b	Р	4	4	TRUE	2	28.18
25         C         6         b         F         4         4         TRUE         1         6.7           26         B         6         b         G         4         5         FALSE         4         21.           27         A         6         b         P         4         4         TRUE         2         27.           28         B         6         b         G         4         6         FALSE         2         20.           29         A         6         b         P         4         4         TRUE         3         14.           30         C         6         b         F         4         4         TRUE         2         10.           31         A         6         b         F         4         4         TRUE         2         11.           32         C         6         b         F         4         4         TRUE         1         10.           33         B         6         b         G         4         4         TRUE         1         11.           34         B         6         b         F	24	В	6	b	G	4	4	TRUE	1	21.69
26         B         6         b         G         4         5         FALSE         4         21           27         A         6         b         P         4         4         TRUE         2         27           28         B         6         b         G         4         6         FALSE         2         20           29         A         6         b         P         4         4         TRUE         3         14           30         C         6         b         F         4         4         TRUE         2         10           31         A         6         b         F         4         4         TRUE         2         11           32         C         6         b         F         4         4         TRUE         1         10           33         B         6         b         G         4         4         TRUE         1         11           34         B         6         b         G         4         4         TRUE         1         11           36         C         6         b         F <td< td=""><td></td><td>С</td><td>6</td><td>b</td><td></td><td>4</td><td>4</td><td></td><td>1</td><td>6.78</td></td<>		С	6	b		4	4		1	6.78
27         A         6         b         P         4         4         TRUE         2         27.3           28         B         6         b         G         4         6         FALSE         2         20.           29         A         6         b         P         4         4         TRUE         3         14.0           30         C         6         b         F         4         4         TRUE         2         10.3           31         A         6         b         F         4         4         TRUE         2         11.3           32         C         6         b         F         4         4         TRUE         2         11.3           33         B         6         b         G         4         4         TRUE         1         10.3           34         B         6         b         G         4         4         TRUE         1         11.3           35         C         6         b         F         4         4         TRUE         1         11.3           36         C         6         b         F<				b						21.3
28         B         6         b         G         4         6         FALSE         2         20.           29         A         6         b         P         4         4         TRUE         3         14.0           30         C         6         b         F         4         4         TRUE         2         10.3           31         A         6         b         F         4         4         TRUE         2         11.3           32         C         6         b         F         4         4         TRUE         1         10.0           33         B         6         b         G         4         4         TRUE         1         11.3           34         B         6         b         G         4         4         TRUE         1         11.3           35         C         6         b         F         4         4         TRUE         1         11.3           36         C         6         b         F         4         4         TRUE         1         11.3           37         B         6         b         G<		Α		b						27.21
29         A         6         b         P         4         4         TRUE         3         14.4           30         C         6         b         F         4         4         TRUE         2         10.3           31         A         6         b         P         4         4         TRUE         2         11.3           32         C         6         b         F         4         4         TRUE         1         10.3           33         B         6         b         G         4         4         TRUE         1         11.3           34         B         6         b         G         4         5         FALSE         1         34.3           35         C         6         b         F         4         4         TRUE         1         11.3           36         C         6         b         F         4         4         TRUE         1         11.3           37         B         6         b         G         4         4         TRUE         2         11.3           38         A         6         b         P										20.12
30         C         6         b         F         4         4         TRUE         2         10.3           31         A         6         b         P         4         4         TRUE         2         11.3           32         C         6         b         F         4         4         TRUE         1         10.3           33         B         6         b         G         4         4         TRUE         1         11.3           34         B         6         b         G         4         5         FALSE         1         34.3           35         C         6         b         F         4         4         TRUE         1         11.3           36         C         6         b         F         4         4         TRUE         1         11.3           37         B         6         b         G         4         4         TRUE         2         11.3           38         A         6         b         P         4         4         TRUE         2         11.3           39         A         6         b         G										14.01
31         A         6         b         P         4         4         TRUE         2         11.3           32         C         6         b         F         4         4         TRUE         1         10.4           33         B         6         b         G         4         4         TRUE         1         11.4           34         B         6         b         G         4         5         FALSE         1         34.3           35         C         6         b         F         4         4         TRUE         1         11.3           36         C         6         b         F         4         4         TRUE         1         11.3           37         B         6         b         G         4         4         TRUE         2         11.3           38         A         6         b         P         4         4         TRUE         2         11.3           39         A         6         b         P         4         4         TRUE         3         12.4           40         B         6         b         G										10.34
32         C         6         b         F         4         4         TRUE         1         10.2           33         B         6         b         G         4         4         TRUE         4         34.3           34         B         6         b         G         4         5         FALSE         1         34.3           35         C         6         b         F         4         4         TRUE         1         11.3           36         C         6         b         F         4         4         TRUE         1         11.3           37         B         6         b         G         4         4         TRUE         2         11.3           38         A         6         b         P         4         4         TRUE         2         11.3           39         A         6         b         P         4         4         TRUE         3         12.4           40         B         6         b         G         4         4         TRUE         0         21.3           41         B         6         b         F										11.88
33       B       6       b       G       4       4       TRUE       4       34.3         34       B       6       b       G       4       5       FALSE       1       34.3         35       C       6       b       F       4       4       TRUE       1       11.3         36       C       6       b       F       4       4       TRUE       1       11.3         37       B       6       b       G       4       4       TRUE       2       11.3         38       A       6       b       P       4       4       TRUE       2       11.3         39       A       6       b       P       4       4       TRUE       3       12.4         40       B       6       b       G       4       4       TRUE       0       21.3         41       B       6       b       G       4       4       TRUE       0       21.3         42       C       6       b       F       4       4       TRUE       0       11.3         4       A       TRUE										10.49
34       B       6       b       G       4       5       FALSE       1       34.3         35       C       6       b       F       4       4       TRUE       1       11.3         36       C       6       b       F       4       4       TRUE       1       11.3         37       B       6       b       G       4       4       TRUE       2       11.3         38       A       6       b       P       4       4       TRUE       2       11.3         39       A       6       b       P       4       4       TRUE       3       12.4         40       B       6       b       G       4       4       TRUE       0       21.3         41       B       6       b       G       4       4       TRUE       0       21.3         42       C       6       b       F       4       4       TRUE       3       18.0         43       C       6       b       F       4       4       TRUE       0       11.5         1       C       6       c </td <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td>34.82</td>										34.82
35         C         6         b         F         4         4         TRUE         1         11.3           36         C         6         b         F         4         4         TRUE         1         11.3           37         B         6         b         G         4         4         TRUE         2         11.3           38         A         6         b         P         4         4         TRUE         2         11.3           39         A         6         b         P         4         4         TRUE         3         12.4           40         B         6         b         G         4         4         TRUE         0         21.3           41         B         6         b         G         4         4         TRUE         0         21.3           42         C         6         b         F         4         4         TRUE         2         12.3           43         C         6         b         F         4         4         TRUE         0         11.3           1         C         6         c         P </td <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td>34.39</td>										34.39
36         C         6         b         F         4         4         TRUE         1         11.3           37         B         6         b         G         4         4         TRUE         2         11.4           38         A         6         b         P         4         4         TRUE         3         12.5           39         A         6         b         P         4         5         FALSE         4         16.4           40         B         6         b         G         4         4         TRUE         0         21.5           41         B         6         b         G         4         4         TRUE         0         21.5           42         C         6         b         F         4         4         TRUE         2         12.5           43         C         6         b         F         4         4         TRUE         0         11.5           1         C         6         c         P         0         0         TRUE         2         20.           2         C         6         c         P <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td>11.7</td>										11.7
37       B       6       b       G       4       4       TRUE       2       11.3         38       A       6       b       P       4       4       TRUE       3       12.3         39       A       6       b       P       4       5       FALSE       4       16.4         40       B       6       b       G       4       4       TRUE       0       21.3         41       B       6       b       G       4       4       TRUE       0       21.3         42       C       6       b       F       4       4       TRUE       2       12.3         43       C       6       b       F       4       4       TRUE       0       11.3         1       C       6       c       P       0       0       TRUE       2       20.4         2       C       6       c       P       0       0       TRUE       2       32.4         3       C       6       c       P       0       0       TRUE       0       17.0         4       A       6       c										11.88
38       A       6       b       P       4       4       TRUE       3       12.         39       A       6       b       P       4       5       FALSE       4       16.         40       B       6       b       G       4       4       TRUE       0       21.         41       B       6       b       G       4       4       TRUE       2       12.         42       C       6       b       F       4       4       TRUE       3       18.         43       C       6       b       F       4       4       TRUE       0       11.         1       C       6       c       P       0       0       TRUE       2       20.         2       C       6       c       P       0       0       TRUE       2       32.         3       C       6       c       P       0       0       TRUE       0       17.         4       A       6       c       G       0       0       TRUE       0       7.0         4       A       6       c										11.51
39     A     6     b     P     4     5     FALSE     4     16.       40     B     6     b     G     4     4     TRUE     0     21.       41     B     6     b     G     4     4     TRUE     2     12.       42     C     6     b     F     4     4     TRUE     3     18.       43     C     6     b     F     4     4     TRUE     0     11.       1     C     6     c     P     0     0     TRUE     2     20.       2     C     6     c     P     0     0     TRUE     2     32.       3     C     6     c     P     0     0     TRUE     0     17.       4     A     6     c     G     0     0     TRUE     1     11.       5     B     6     c     F     0     0     TRUE     0     7.0       6     B     6     c     F     0     2     FALSE     3     14.										12.9
40       B       6       b       G       4       4       TRUE       0       21.         41       B       6       b       G       4       4       TRUE       2       12.         42       C       6       b       F       4       4       TRUE       3       18.         43       C       6       b       F       4       4       TRUE       0       11.         1       C       6       c       P       0       0       TRUE       2       20.         2       C       6       c       P       0       0       TRUE       2       32.         3       C       6       c       P       0       0       TRUE       0       17.         4       A       6       c       G       0       0       TRUE       11.       11.         5       B       6       c       F       0       0       TRUE       0       7.0         6       B       6       c       F       0       2       FALSE       3       14.										16.47
41       B       6       b       G       4       4       TRUE       2       12.3         42       C       6       b       F       4       4       TRUE       3       18.0         43       C       6       b       F       4       4       TRUE       0       11.3         1       C       6       c       P       0       0       TRUE       2       20.0         2       C       6       c       P       0       0       TRUE       2       32.0         3       C       6       c       P       0       0       TRUE       0       17.3         4       A       6       c       G       0       0       TRUE       1       11.3         5       B       6       c       F       0       0       TRUE       0       7.0         6       B       6       c       F       0       2       FALSE       3       14.3										21.12
42       C       6       b       F       4       4       TRUE       3       18.0         43       C       6       b       F       4       4       TRUE       0       11.1         1       C       6       c       P       0       0       TRUE       2       20.2         2       C       6       c       P       0       0       TRUE       2       32.3         3       C       6       c       P       0       0       TRUE       0       17.3         4       A       6       c       G       0       0       TRUE       1       11.5         5       B       6       c       F       0       0       TRUE       0       7.0         6       B       6       c       F       0       2       FALSE       3       14.5										12.21
43     C     6     b     F     4     4     TRUE     0     11.6       1     C     6     c     P     0     0     TRUE     2     20.4       2     C     6     c     P     0     0     TRUE     2     32.       3     C     6     c     P     0     0     TRUE     0     17.5       4     A     6     c     G     0     0     TRUE     1     11.5       5     B     6     c     F     0     0     TRUE     0     7.0       6     B     6     c     F     0     2     FALSE     3     14.5										
1     C     6     c     P     0     0     TRUE     2     20.       2     C     6     c     P     0     0     TRUE     2     32.       3     C     6     c     P     0     0     TRUE     0     17.       4     A     6     c     G     0     0     TRUE     1     11.       5     B     6     c     F     0     0     TRUE     0     7.0       6     B     6     c     F     0     2     FALSE     3     14.										
2         C         6         c         P         0         0         TRUE         2         32.           3         C         6         c         P         0         0         TRUE         0         17.5           4         A         6         c         G         0         0         TRUE         1         11.5           5         B         6         c         F         0         0         TRUE         0         7.0           6         B         6         c         F         0         2         FALSE         3         14.5										
3         C         6         c         P         0         0         TRUE         0         17.3           4         A         6         c         G         0         0         TRUE         1         11.3           5         B         6         c         F         0         0         TRUE         0         7.0           6         B         6         c         F         0         2         FALSE         3         14.3										
4     A     6     c     G     0     0     TRUE     1     11.3       5     B     6     c     F     0     0     TRUE     0     7.0       6     B     6     c     F     0     2     FALSE     3     14.3										
5         B         6         c         F         0         0         TRUE         0         7.0           6         B         6         c         F         0         2         FALSE         3         14.3										
6 B 6 c F 0 2 FALSE 3 14.										
							_			
	7	В	6	С	F	0	0	TRUE	1	13.6
										22.41
										36.36
										12.97
										29.91
										36
										22.66
				С						12.09
				С						14.58
				С						15.24
				c					1	31.28
				c						32.63
19 C 6 C P 0 TRUE 1 21.	19	С	6	c	Р	0	0	TRUE	1	21.48

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		m prev							
Subj.	Grp	Map	Qu.	Map	Answer	Given	Correct	Ease	Time
				Type		Answer			(s)
20	С	6	c	P	0	0	TRUE	2	19.55
21	A	6	c	G	0	0	TRUE	1	12.33
22	В	6	c	F	0	0	TRUE	1	12.73
23	A	6	c	G	0	2	FALSE	2	16.92
24	В	6	c	F	0	4	FALSE	2	28.91
25	С	6	c	P	0	0	TRUE	1	8.05
26	В	6	c	F	0	0	TRUE	1	16.66
27	A	6	c	G	0	0	TRUE	0	11.59
28	В	6	c	F	0	0	TRUE	2	9.48
29	A	6	c	G	0	0	TRUE	1	18.36
30	С	6	c	P	0	0	TRUE	2	11.89
31	A	6	c	G	0	0	TRUE	1	13.35
32	С	6	c	P	0	0	TRUE	2	12.97
33	В	6	c	F	0	0	TRUE	1	10.27
34	В	6	c	$\mathbf{F}$	0	0	TRUE	0	16.63
35	С	6	c	Р	0	0	TRUE	1	13.41
36	С	6	c	Р	0	0	TRUE	1	14.64
37	В	6	c	F	0	0	TRUE	1	13.73
38	A	6	c	G	0	0	TRUE	4	14.32
39	A	6	c	G	0	0	TRUE	2	11.78
40	В	6	c	F	0	0	TRUE	4	7.77
41	В	6	c	F	0	0	TRUE	1	12.75
42	С	6	c	Р	0	0	TRUE	1	13.58
43	С	6	c	Р	0	0	TRUE	1	22.1
1	С	6	d	F	1	2	FALSE	2	25.09
2	С	6	d	F	1	2	FALSE	2	28.09
3	С	6	d	F	1	1	TRUE	2	24.04
4	A	6	d	Р	1	1	TRUE	0	15.72
5	В	6	d	G	1	1	TRUE	2	11.94
6	В	6	d	G	1	2	FALSE	2	23.72
7	В	6	d	G	1	1	TRUE	2	14.03
8	A	6	d	Р	1	1	TRUE	4	27.49
9	A	6	d	Р	1	1	TRUE	4	79.26
10	A	6	d	Р	1	0	FALSE	1	17.42
11	С	6	d	F	1	1	TRUE	2	31.54
12	В	6	d	G	1	1	TRUE	2	46.27
13	С	6	d	F	1	1	TRUE	3	26.45
14	A	6	d	Р	1	1	TRUE	2	31.13
15	A	6	d	Р	1	1	TRUE	2	23.36
16	C	6	d	F	1	2	FALSE	$\overline{2}$	22.22
17	В	6	d	G	1	1	TRUE	$\frac{2}{4}$	41.63
18	В	6	d	G	1	2	FALSE	2	20.52
19	C	6	d	F	1	1	TRUE	2	32.35
20	C	6	d	F	1	3	FALSE	2	20.44
21	A	6	d	P	1	1	TRUE	2	34.13
22	В	6	d	G	1	1	TRUE	1	17.73
23	A	6	d	P	1	1	TRUE	2	31.69
$\frac{23}{24}$	В	6	d	G	1	1	TRUE	1	20.52
44		U	u	u	1	1	1100		20.02

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Subj.	Grp	Map	Qu.	Map	Answer	Given	Correct	Ease	Time
				Type		Answer			(s)
25	С	6	d	F	1	1	TRUE	1	13.14
26	В	6	d	G	1	1	TRUE	1	14.28
27	A	6	d	Р	1	1	TRUE	2	20.96
28	В	6	d	G	1	1	TRUE	2	22.85
29	A	6	d	Р	1	1	TRUE	4	57.18
30	$^{\mathrm{C}}$	6	d	F	1	1	TRUE	2	15.94
31	A	6	d	Р	1	1	TRUE	3	48.23
32	С	6	d	F	1	1	TRUE	3	13.07
33	В	6	d	G	1	1	TRUE	2	14.51
34	В	6	d	G	1	1	TRUE	2	35.46
35	$^{\mathrm{C}}$	6	d	F	1	1	TRUE	1	23.59
36	$^{\mathrm{C}}$	6	d	$\mathbf{F}$	1	1	TRUE	2	15.08
37	В	6	d	G	1	1	TRUE	3	11.23
38	A	6	d	Р	1	2	FALSE	4	18.66
39	A	6	d	Р	1	1	TRUE	4	24.05
40	В	6	d	G	1	1	TRUE	1	19.17
41	В	6	d	G	1	1	TRUE	2	11.45
42	$^{\mathrm{C}}$	6	d	F	1	2	FALSE	2	21.08
43	C	6	d	F	1	1	TRUE	2	22.69

# A.3.2 Design Preferences

Table A.9: Design Preferences: Geographic Map. "1" represents the design that the candidate felt was most preferable and "3" represents the least preferable map.

Candidate	Atlanta	Bucharest	Mexico	Stockholm	Toronto	Washington
1	3	2	2	3	3	3
2	3	2	3	3	3	3
3	2	1	2	3	2	2
4	3	3	2	3	3	3
5	3	2	3	3	3	3
6	2	1	3	3	3	3
7	3	2	2	3	3	3
8	3	2	2	3	3	3
9	2	2	2	3	2	3
10	3	2	2	2	2	3
11	3	2	2	3	2	3
12	3	3	2	3	3	3
13	1	3	3	1	2	3
14	3	2	2	3	2	3
15	1	3	2	3	1	3
16	2	3	2	3	2	3
17	3	2	2	3	3	3
18	3	3	3	3	3	3
19	2	1	1	2	3	3

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Candidate	Atlanta	Bucharest	Mexico	Stockholm	Toronto	Washington
20	2	3	2	3	1	3
21	1	2	1	2	1	3
22	1	1	2	3	2	3
23	2	1	2	3	1	3
24	1	2	2	2	3	3
25	1	1	2	3	1	3
26	3	2	2	3	3	3
27	1	1	2	3	2	3
28	2	1	2	3	2	2
29	3	3	2	3	3	3
30	3	3	2	3	3	3
31	3	2	$\frac{2}{2}$	2	2	3
32	2	2		3	2	3
33	2	1	2	2	2	$\frac{2}{3}$
34	3	3	2	3	3	
35	3	2	3	2	2	3
36	3	2	2	2	2	3
37	1	2	2	1	2	3
38	3	2	2	3	2	3
39	3	2	2	3	2	3
40	3	3	2	3	2	3
41	1	2	2	2	2	3
42	1	3	$\frac{2}{2}$	3	3	3
43	3	2	2	3	3	3

Table A.10: Design Preferences: Normalised Published Map. "1" represents the design that the candidate felt was most preferable and "3" represents the least preferable map.

Candidate	Atlanta	Bucharest	Mexico	Stockholm	Toronto	Washington
1	2	3	3	1	1	1
2	1	3	2	1	1	2
3	3	3	3	1	1	1
4	2	2	3	1	1	2
5	2	3	2	1	1	1
6	3	2	2	1	1	2
7	2	3	3	1	2	1
8	1	3	3	2	2	2
9	3	3	3	2	3	2
10	1	3	3	1	3	2
11	2	3	3	1	3	2
12	2	2	3	1	1	2
13	3	2	2	3	3	2
14	2	3	3	1	3	2

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Candidate	Atlanta	Bucharest	Mexico	Stockholm	Toronto	Washington
15	2	1	3	1	3	1
16	1	2	3	1	3	2
17	2	3	3	2	1	2
18	2	2	2	1	1	1
19	1	3	2	1	1	2
20	1	1	3	1	3	2
21	3	1	3	1	3	2
22	2	2	3	1	3	2
23	3	2	3	1	3	2
24	3	3	3	1	2	2
25	3	3	3	1	3	1
26	2	3	3	1	2	2
27	2	2	3	1	3	2
28	1	3	3	2	1	3
29	2	2	3	1	2	2
30	1	2	3	1	1	1
31	2	3	3	1	3	2
32	1	1	3	1	3	2
33	1	2	3	1	3	3
34	1	2	3	1	1	1
35	1	3	2	1	3	2
36	2	3	3	1	3	2
37	3	3	3	2	3	2
38	2	3	3	1	3	2
39	2	3	3	2	3	2
40	2	2	3	1	3	$\frac{2}{2}$
41	3	1	3	1	3	2
42	2	2	3	1	2	1
43	2	3	3	1	1	2

Table A.11: Design Preferences: Automatically-Drawn Map. "1" represents the design that the candidate felt was most preferable and "3" represents the least preferable map.

Candidate	Atlanta	Bucharest	Mexico	Stockholm	Toronto	Washington
1	1	1	1	2	2	2
2	2	1	1	2	2	1
3	1	2	1	2	3	3
4	1	1	1	2	2	1
5	1	1	1	2	2	2
6	1	3	1	2	2	1
7	1	1	1	2	1	2
8	2	1	1	1	1	1
9	1	1	1	1	1	1
10	2	1	1	3	1	1

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Candidate	Atlanta	Bucharest	Mexico	Stockholm	Toronto	Washington
11	1	1	1	2	1	1
12	1	1	1	2	2	1
13	2	1	1	2	1	1
14	1	1	1	2	1	1
15	3	2	1	2	2	2
16	3	1	1	2	1	1
17	1	1	1	1	2	1
18	1	1	1	2	2	2
19	3	2	3	3	2	1
20	3	2	1	2	2	1
21	2	3	2	3	2	1
22	3	3	1	2	1	1
23	1	3	1	2 3	2	1
24	2	1	1		1	1
25	2	2	1	2	2	2
26	1	1	1	2	1	1
27	3	3	1	2	1	1
28	3	2	1	1	3	1
29	1	1	1	2	1	1
30	2	1	1	2	2	2
31	1	1	1	3	1	1
32	3	3	1	2	1	1
33	3	3	1	3	1	1
34	2	1	1	2	2	2
35	2	1	1	3	1	1
36	1	1	1	3	1	1
37	2	1	1	3	1	1
38	1	1	1	2	1	1
39	1	1	1	1	1	1
40	1	1	1	2	1	1
41	2	3	1	3	1	1
42	3	1	1	2	1	2
43	1	1	1	2	2	1

# A.3.3 Questionnaire Feedback

Table A.12: Have you seen any of the metro maps shown here before these tests? If so, which ones?

Candidate	Answer
1	Yes, I have seen some metro maps before the test. I have seen and
	used the New York Metro
2	None, but some layouts seemed familiar, like the london under-
	ground

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	om previous page
Candidate	Answer
3	Yes Slide 6
4	No
5	No
6	No
7	N/A
8	No
9	None
10	Yes, the one of the Mexico City subway system
11	Map C from slide 1, most familiar metro maps are straight lined,
	definite in shape
12	No
13	No
14	No
15	No
16	No
17	None
18	No, I dont use metro
19	No
20	No
21	No
22	No
23	No, never
24	No
25	No
26	London Underground
27	Think so, London ones including DLR?
28	No
29	No
30	No -Have seen the London Underground ones but do not remem-
	ber them in the test
31	No
32	None
33	Briefly seen book of Metro maps, but not used/studied
34	None
35	Yes, alas I cannot remember which ones!
36	Can't remember!
37	No
38	No. Very familiar with London Underground which is similar
39	None
40	I dont know
41	No
42	No

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Candidate	Answer
43	Washington DC

Table A.13: Which features of the metro map layout did you find most helpful when completing the tests?

Candidate	Answer
1	The maps indicated the relevant stations which we needed to find
2	One that was not too scattered, but had a defined straight and
	clear structure as it was easier to read
3	s+8 line maps
4	Straight lines, lines spread out for ease on eye
5	Clear naming and placement
6	Straight lines and colours
7	Simple non overlaying colours. Basic shapes with straight edges.
	Basic symettry/even distribution of station/rail. Well spaced out
	and easy to read train station names
8	Interchanges clearly marked
9	Geometrical symetry (straight lines); spacing between labels;
	highlighting end points
10	Uniformity, straight lines, labels not on top of each other
11	Definate and spaced out layouts with clear indication of colour
	and station points such as [inverted T stubby diagram]
12	Interchange Signs
13	Different colours, changes clearly pointed out
14	Having enough space between stations to make them distinct -
	some maps had overlapping text and lines which confused things.
15	Colour
16	Layout and colour
17	Circular nodes on interconnecting stations, sensible layout, sensi-
	ble angles on lines, straight lines, sensible colours, lines not ran-
	domly joining
18	Straight lines and the fact that some metro maps had clear sta-
	tions, not confusing
19	The ones with biggest text and biggest ticks
20	Indentations representing stations
21	Different Colours
22	Colour coded tracks, clear circles on join stations
23	Different colours for different lines is helpful; When the map was
	large and different stations were well spaced out and not placed
	close together, it was easier to answer the questions

continued from previous page

Candidate	Answer
24	The blue or red coloured lines (made it easier to see the stations)
25	Clear images, the on which you could see the station stops
26	The small lines indicating a station was their. Also the bigger
	maps which didnt have station names overlapping were much bet-
	ter
27	. for stations, colours of different lines; circle for crossover stations
28	Well spread with appropriate use of colour
29	Large gaps between stations (so names dont overlap)
30	Clear linear route layout, uncluttered
31	Metro maps with straight lines; metro maps with easily distingish-
	able colours; maps where overlapping lines had half one colour,
	half the other, such as the blue/orange line to suggest the line
	occupied the same tracks
32	Straight lines
33	Names of stations next to the markers on the lines
34	The colours indicating the lines
35	Circles for track interchange, some had a more 'sensible' layout
36	distance between station lables; straight routes not curly; colour
	coding of routes
37	clearly marked stations that were spaced out
38	Stations spread out; Stations especially where lines cross not too
	close together; Maps with fewest line crossings/colours
39	When the lines could only be from lines at other angles made it
	harder. When station names were not near each other. If stations
	near one above one below [DIAGRAMS]
40	Maps normalized to straight lines (where possible); where the sta-
	tion and names were spaced out were most helpful but if there was
	a choice between a crowded straight line map and a cursive spaced
4.4	out map, I favoured the spaced out map
41	Enough spacing between station names; Clearly marked stations.
40	Spacing between tracks
42	Colour coding of line, compact text close to stations
43	Different symbols where lines meet

Table A.14: Which features of the metro map layout did you find least helpful when completing the tests?

Candidate	Answer
1	The maps were hard to read and would be better enlarged
2	When there was more than three change connections and the
	routes did not have a clear straight grid map which made it diffi-
	cult to read

continued from previous page

	om previous page
Candidate	Answer
3	ones with all the curves
4	All bundelled together, not clear, station name writing overlaps
5	Messy naming and the misplaced change station
6	When names were too close together
7	Curved lines representing routes. Squashed text and squashed
	track/lines into small areas. Overlayed colours for multiple tracks.
8	Yellow 'tick' marks against white background; Station names over-
	alapping onto other station names; lines overlapping each other
	when going in the same direction
9	Vertical length
10	Labels that cant be read cuz they're on top of each other; lines
	changing colour without a hollow circle where they meet
11	When routes were the same colour and followed curved or jagged
	lines, and when route changes were not clearly indicated with a
	circle
12	untidy lines (not straight)
13	Unexplained changes, from one colour to the next
14	NULL
15	Size of map (when stations were too close to each other to distin-
	guish between them)
16	Positioning of station names and station indicators
17	Poor space management, differently shaped station nodes, changes
10	with no icon indication, poor choice of colours
18	The confusing lines
19	Couldn't read some of them as text label obscured ticks or obscured
20	other labels
20	stations with long names usually overwrite something
21	When the font was too small and the names were too close together
22	Cramp spacing between station writing, unclear ticks on tracks
23	When the stations were placed too close together it was hard to
	distinguish between them, especially when they had long names
24	which took up a lot of space on the map
24 25	The yellow coloured lines (made it harder to see the stations)  Station names which ran into each other and everlapped
26	Station names which ran into each other and overlapped Overlapping station names. Very squashed maps were bad.
27	Close together station names, change of line colour with no circle
21	to indicate line crossover
28	The almost handdrawn metro maps without space between the
20	stations. Ever though this may not be precise but provides better
	navigation and understanding
29	When line colours overlapped or "jumped" stations
30	Station names overlapping
<b>3</b> U	station names overlapping

continued from previous page

Candidate	Answer
31	Metro maps where lines fully overlapped one lines. Maps where
	names overlapped the lines. Maps with bright colours, such as
	yellow
32	Yellow lines were hard to see on white background
33	Overlapping text with lines or other text
34	None
35	Colours were often hard to see clearly
36	overlapping lines; non-distinctive "change"-station symbols
37	Cramped maps where it was difficult to see stations or connections
38	Station names not spread out or stations too close together; Some
	names not easy to read
39	When you could not see join for a line. Station names too close
	together
40	Being crowded, swapping line colurs made it difficult and made
	me think changes were needed. Also, where interchange stations
	lacked a white circle made more difficult
41	Places where two tracks follow the same route shown as an alter-
	nating line
42	Overlapping text/stations; yellow-an-white station marks hard to
	see; text close to the stations
43	Station names overlapping

Table A.15: Did you find any of the questions ambiguous?

Candidate	Answer
1	No, the questions were clearly laid out
2	Do you count a change if the route. Has a same colour or do you
	always assume change lead to only one to one mapping
3	Yes
4	There were a few, but I dont remember which ones
5	No
6	No
7	No.
8	Not really
9	No
10	yes
11	None
12	No
13	Whether you have seen some metro maps. Might have seen some
	but not remembering them.
14	Several of the 'how many do you pass though' questions had mul-
	tiple routes

continued from previous page

continued from previous page	
Candidate	Answer
15	No
16	Yes, some questions seemed impossible to answer given the an-
	swers provided. Because the route from A to B is not possible by
	train, e.g. routes dont connect
17	Some questions were potentially ambiguous, due to the variety of
	routes in some circumstances
18	No
19	No
20	no
21	Yes, most of them
22	Yes, some tracks did not join
23	No
24	No
25	Yes, coloured lines which run into other colours without line
	change marks - why were these diff colours? Also lines which
	were the same colour with line change marks. Why change of line
	but no change of colour?
26	No
27	Yes
28	Yes
29	Some did not have the correct list of questions
30	It was unclear if you had to change when two lines intersect but
	the route continues
31	No
32	There were often many routes with different numbers of stations
	to go by
33	Stora Massen to Karlaplan; Dupont to Chester; Pentagon to
	Court House
34	Yes, some of the questions were not clear
35	Some, as stations were not always clear
36	Yes, Q1, as we were not told to remember map layouts beforehand
37	When it asked for a number of changes on lines with multiple
	endings, what counts as a change?
38	No
39	No
40	Many, especially the ones that were incorrect. The abiguous ones
	were the ones with inconsistent station and line labeling
41	No
42	No -Although some of the metro maps had yellow-an-white
	lines/stations - hard to read
43	No

# Appendix B

# Map Input Data

This Appendix contains the input data for all of the maps presented in Chapter 6. The data for each map consists of a set of node definitions and a set of edge definitions. Each node definition contains the label for that node together with the starting x and y coordinates for that node. The starting co-ordinates are usually the latitude or longitude in degrees of that station, but sometimes other reference systems are used. The edge definitions consist of a label (the name of the line that that edge is part of), a colour (red, green, blue), followed by a list of nodes. The list of nodes can either be just two nodes for a single edge or it can be a longer list. Longer lists are just shorthand for writing lots of individual edge definitions.

The format is also explained in Section 5.2.

#### B.1 Atlanta

```
NODE: label=Five Points: x=-84.3916: y=33.7539

NODE: label=Bankhead: x=-84.4288: y=33.4288

NODE: label=Indian Creek: x=-84.2295: y=33.7697

NODE: label=North Avenue: x=-84.3863: y=33.7719

NODE: label=Buckhead: x=-84.3673: y=33.8479

NODE: label=Medical Center: x=-84.3527: y=33.9108

NODE: label=Dunwoody: x=-84.3444: y=33.9213

NODE: label=Sandy Springs: x=-84.3531: y=33.9326

NODE: label=North Springs: x=-84.3574: y=33.9439

NODE: label=Brookhaven/Oglethorpe University: x=-84.3389: y=33.8605
```

```
NODE: label=Chamblee: x = -84.3075: y = 33.8869
NODE: label=Doraville: x = -84.2805: y = 33.9028
NODE: label=West End: x = -84.4135: y = 33.7361
NODE: label=Oakland City: x = -84.4256: y = 33.7153
NODE: label=Lakewood/Ft. McPherson: x = -84.4287: y = 33.7003
NODE: label=East Point: x = -84.4409: y = 33.6773
NODE: label=College Park: x = -84.4487: y = 33.6516
NODE: label=Airport: x = -84.4462: y = 33.6395
NODE: label=Georgia State: x = -84.3863: y = 33.7507
NODE: label=King Memorial: x = -84.3746: y = 33.7501
NODE: label=Inman Park/Reynoldstown: x = -84.3529: y = 33.7574
NODE: label=Edgwood/Candler Park: x = -84.3390: y = 33.7621
NODE: label=East Lake: x = -84.3130: y = 33.7653
NODE: label=Decatur: x = -84.2954: y = 33.7752
NODE: label=Avondale: x = -84.2828: y = 33.7749
NODE: label=Kensington: x = -84.2502: y = 33.7723
NODE: label=Dome/GWCC/Philips Arena/CNN Center:x=-84.3988:y
    =33.7565
NODE: label=Vine City: x = -84.4061: y = 33.7565
NODE: label=Ashby: x = -84.4183: y = 33.7566
NODE: label=West Lake: x = -84.4449: y = 33.7537
NODE: label=Hamilton E. Holmes: x = -84.4698: y = 33.755
NODE: label=Peachtree Center: x = -84.3876: y = 33.7598
NODE: label=Civic Center: x = -84.3873: y = 33.7674
NODE: label=Midtown: x = -84.3864: y = 33.7809
NODE: label=Arts Center: x = -84.3867: y = 33.7893
NODE: label=Garnett: x = -84.3963: y = 33.7481
NODE: label=Lenox : x = -84.3583 : y = 33.8454
NODE: label=Lindburgh Center: x = -84.3697: y = 33.8212
EDGE: label =: color = 255,171,25: adjlist = Five Points, Peachtree
    Center
EDGE: label =: color = 255,171,25: adjlist = Peachtree Center, Civic
EDGE: label =: color = 255,171,25: adjlist=Civic Center, North Avenue
EDGE: label =: color = 255,171,25: adjlist = North Avenue, Midtown
EDGE: label =: color = 255,171,25: adjlist=Midtown, Arts Center
EDGE: label =: color = 255,171,25: adjlist = Arts Center, Lindburgh
    Center
EDGE: label =: color = 255,171,25: adjlist=Lindburgh Center, Buckhead
EDGE: label =: color = 255,171,25: adjlist = Buckhead, Medical Center
EDGE: label =: color = 255,171,25: adjlist = Medical Center, Dunwoody
EDGE: label =: color = 255,171,25: adjlist = Dunwoody, Sandy Springs
EDGE: label =: color = 255,171,25: adjlist=Sandy Springs, North
    Springs
EDGE: label =: color = 255,171,25: adjlist=Lindburgh Center, Lenox
EDGE: label =: color = 255,171,25: adjlist = Lenox, Brookhaven/
    Oglethorpe University
```

```
EDGE: label =: color = 255,171,25: adjlist = Brookhaven/Oglethorpe
    University, Chamblee
EDGE: label =: color = 255,171,25: adjlist = Chamblee, Doraville
EDGE: label =: color = 255,171,25: adjlist=Five Points, Garnett
EDGE: label=:color=255,171,25: adjlist=Garnett, West End
EDGE: label =: color = 255,171,25: adjlist=West End, Oakland City
EDGE: label =: color = 255,171,25: adjlist = Oakland City, Lakewood/Ft.
    McPherson
EDGE: label =: color = 255,171,25: adjlist = Lakewood/Ft. McPherson,
   East Point
EDGE: label =: color = 255,171,25: adjlist = East Point, College Park
EDGE: label =: color = 255,171,25: adjlist = College Park, Airport
EDGE: label =: color = 50,103,254: adjlist=Five Points, Dome/GWCC/
    Philips Arena/CNN Center
EDGE: label =: color = 50,103,254: adjlist = Dome/GWCC/Philips Arena/
   CNN Center, Vine City
EDGE: label =: color = 50,103,254: adjlist=Vine City, Ashby
EDGE: label=:color=50,103,254: adjlist=Ashby, West Lake
EDGE: label =: color = 50,103,254: adjlist=West Lake, Hamilton E.
EDGE: label =: color = 50,103,254: adjlist = Ashby, Bankhead
EDGE: label =: color = 50,103,254: adjlist=Five Points, Georgia State
EDGE: label =: color = 50,103,254: adjlist=Georgia State, King
   Memorial
EDGE: label =: color = 50,103,254: adjlist=King Memorial, Inman Park/
   Reynoldstown
EDGE: label =: color = 50,103,254: adjlist = Inman Park/Reynoldstown,
   Edgwood/Candler Park
EDGE: label =: color = 50,103,254: adjlist = Edgwood/Candler Park, East
EDGE: label =: color = 50,103,254: adjlist = East Lake, Decatur
EDGE: label =: color = 50,103,254: adjlist = Decatur, Avondale
EDGE: label =: color = 50,103,254: adjlist=Avondale, Kensington
EDGE: label =: color = 50,103,254: adjlist = Kensington, Indian Creek
```

#### B.2 Auckland

```
\begin{aligned} & \text{NODE: label=Boston} & \text{Road: } x\!=\!174.7688\!:\!y\!=\!-36.8661 \\ & \text{NODE: label=Newmarket: } x\!=\!174.7791\!:\!y\!=\!-36.8696 \\ & \text{NODE: label=Remuera: } x\!=\!174.7851\!:\!y\!=\!-36.8812 \\ & \text{NODE: label=Panmure: } x\!=\!174.8459\!:\!y\!=\!-36.9012 \\ & \text{NODE: label=Glen Innes: } x\!=\!174.8540\!:\!y\!=\!-36.8791 \\ & \text{NODE: label=Greenlane: } x\!=\!174.8540\!:\!y\!=\!-36.8905 \\ & \text{NODE: label=Ellerslie: } x\!=\!174.8075\!:\!y\!=\!-36.8978 \\ & \text{NODE: label=Penrose: } x\!=\!174.8152\!:\!y\!=\!-36.9096 \\ & \text{NODE: label=Meadowbank: } x\!=\!174.8208\!:\!y\!=\!-36.8663 \\ & \text{NODE: label=Orakei: } x\!=\!174.8099\!:\!y\!=\!36.8627 \\ & \text{NODE: label=Britomart: } x\!=\!174.7769\!:\!y\!=\!-36.8484 \end{aligned}
```

```
NODE: label=Mt Eden: x = 174.7587: y = -36.8680
NODE: label=Kingsland: x = 174.7452: y = -36.8722
NODE: label=Morningside: x = 174.7353: y = -36.8751
NODE: label=Baldwin Avenue: x = 174.7203: y = -36.8778
NODE: label=Mt Albert: x = 174.7138: y = -36.8850
NODE: label=Avondale: x = 174.7000: y = -36.8950
NODE: label=New Lynn: x = 174.6840: y = -36.9094
NODE: label=Fruitvale Road: x = 174.6698: y = -36.9115
NODE: label=Glen Eden: x = 174.6532: y = -36.9103
NODE: label=Sunnyvale: x = 174.6318: y = -36.8967
NODE: label=Henderson: x = 174.6308: y = -36.8820
NODE: label=Sturges Road: x = 174.6203: y = -36.8734
NODE: label=Ranui: x = 174.6015: y = -36.8677
NODE: label=Swanson: x = 174.5769: y = -36.8664
NODE: label=Waitakere: x = 174.5436: y = -36.8495
NODE: label=Westfield: x = 174.8319: y = -36.9381
NODE: label=Otahuhu: x = 174.8334: y = -36.9475
NODE: label=Mangere: x = 174.8348: y = -36.9558
NODE: label=Middlemore: x = 174.8387: y = -36.9625
NODE: label=Papatoetoe : x = 174.8502 : y = -36.9781
NODE: label=Puhinui: x = 174.8561: y = -36.9897
NODE: label=Wiri: x = 174.8621: y = -37.0014
NODE: label=Homai: x = 174.8748: y = -37.0134
NODE: label=Manurewa: x = 174.8944: y = -37.0219
NODE: label=Te Mahia: x = 174.9062: y = -37.0311
NODE: label=Takanini: x = 174.9194: y = -37.0409
NODE: label=Papakura: x = 174.9469: y = -37.0655
NODE: label=Pukekohe: x = 174.9095: y = -37.2026
EDGE: label=Western Line: color = 0,92,72: adjlist=Waitakere,
   Swanson, Ranui, Sturges Road, Henderson, Sunnyvale, Glen
   Eden, Fruitvale Road, New Lynn, Avondale, Mt Albert, Baldwin
     Avenue, Morningside, Kingsland, Mt Eden, Boston Road,
   Newmarket, Britomart
EDGE: label=Eastern Line: color = 243,171,54: adjlist=Britomart,
   Orakei, Meadowbank, Glen Innes, Panmure, Westfield, Otahuhu,
     Mangere, Middlemore, Papatoetoe, Puhinui, Wiri, Homai,
   Manurewa, Te Mahia, Takanini, Papakura, Pukekohe
EDGE: label=Southern Line: color = 177, 12,62: adjlist=Britomart,
   Newmarket, Remuera, Greenlane, Ellerslie, Penrose, Westfield
     Otahuhu, Mangere, Middlemore, Papatoetoe, Puhinui, Wiri,
   Homai, Manurewa, Te Mahia, Takanini, Papakura, Pukekohe
```

#### B.3 Bucharest

```
NODE: label=Piata Victoriei:x=606.2641208680187: y = 433.4458635842211650.0 NODE: label=Dristor:x=699.4167363134591: y=327.2029155555039650.0
```

```
NODE: label=Piata Unirii:x=578.7945197005204:y
   =308.89474473157026650.0
NODE: label=Romana: x = 595.8992206749997: y = 390.72712717213653650.0
NODE: label=Universtitate: x=587.3001487295211: y
   =349.4681239948213650.0
NODE: label=Tineretului:x=567.9181141917643:y
   =265.5729262051864650.0
NODE: label=1 Dec. 1918: x=795.5529528727775: y
   =297.6031767250176650.0
NODE: label=IMGB Depou: x = 688.6739943368088: y
   =23.548315150782628650.0
NODE: label=IMGB 1:x=625.7856274954758:y=58.491363797161284650.0
NODE: label=Aparatorii Patriei: x=573.0212935317273: y
   =103.50194880468644650.0
NODE: label=Piata Sudului: x=537.7057906017257: y
   =136.73568692331958650.0
NODE: label=Aviatorilor:x=607.8849036661383:y
   =492.0932653311289650.0
NODE: label=Aurel Vlaicu: x=622.9127400559196: y
   =538.5766722489166650.0
NODE: label=Pipera: x = 695.4357268219205: y = 543.4525360325331650.0
NODE: label=Timpuri Noi: x=626.7698426393716: y
   =301.9859921747986650.0
NODE: label=Mihai Bravu: x=674.3500628238654: y
   =302.55148518615425650.0
NODE: label=Piata Muncii:x=720.3201197569026:y
   =376.09459199584273650.0
NODE: label=Iancului: x = 703.9964048318012: y
   =404.4897319649993650.0
NODE: label=Obor: x = 664.8025624545041: y = 406.2798714641121650.0
NODE: label=Stefan cel Mare: x=630.848708702024: y
   =417.0431819804452650.0
NODE: label=Gara de Nord: x=569.0172269077977: y
   =446.9626176284118650.0
NODE: label=Basarab: x=532.7371498641589: y=449.5201817867361650.0
NODE: label=N. Grigorescu: x=748.8211532118221: y
   =317.77808986914954650.0
NODE: label=Titan: x = 785.5155985493016: y = 366.5569523810624650.0
NODE: label=Policolor: x=839.4682956310235: y
   =276.3458991189872650.0
NODE: label=Linia de centura: x=878.9261783614058: y
   =272.75290714201026650.0
NODE: label=Georgian Costin:x=827.7182933412583:y
   =391.90139226871395650.0
NODE: label=Republica: x=809.8523393793713: y
   =422.180049195584650.0
NODE: label=Izvor: x=539.123383562581: y=324.51407802453457650.0
NODE: label=Eroilor: x=512.2918849790794: y=350.787979879442650.0
```

```
NODE: label=Grozavesti:x=480.5348177109894:y
   =380.3371981621366650.0
NODE: label=Semanatoarea: x = 449.90122363171787: y
   =396.66621304905823650.0
NODE: label = Crangasi : x = 491.58463517253836 : y
   =437.0507659028972650.0
NODE: label=Politehnica: x=468.93002929764674: y
   =340.4230528849302650.0
NODE: label=Armata Poporului:x=414.9624677540683:y
   =336.6804151048766650.0
NODE: label=Gorjului:x=370.2854804763591:y
   =328.7175726369838650.0
NODE: label=Pacii: x=319.7311229576297: y=333.88582465139547650.0
NODE: label=Industriilor: x=243.5814299687721: y
   =321.96607733381757650.0
NODE: label=Grivita: x=493.41042747549534: y
   =481.96593777024844650.0
NODE: label=1 Mai: x=453.5388069604622: y=522.0202599289454650.0
NODE: label=Pajura: x = 439.8043258337593: y = 555.9277958381861650.0
NODE: label=Laromet: x = 386.22359689968124: y
   =588.7350791948212650.0
NODE: label=Eroii Revolutiei: x=559.1733019512307: y
   =221.8283558438893650.0
NODE: label=Constantin Brancoveanu: x=540.1549379143057: y
   =185.10983139081907650.0
EDGE: label=M1: color = 255,0,0: adjlist=Stefan cel Mare, Piata
EDGE: label=M1: color = 255,0,0: adjlist=Stefan cel Mare, Obor
EDGE: label=M1: color = 255,0,0: adjlist=Obor, Iancului
EDGE: label=M1: color = 255,0,0: adjlist=Iancului, Piata Muncii
EDGE: label=M1: color = 255,0,0: adjlist=Piata Muncii, Dristor
EDGE: label=M1: color = 255,0,0: adjlist=Mihai Bravu, Dristor
EDGE: label=M1: color = 255,0,0: adjlist=Mihai Bravu, Timpuri Noi
EDGE: label=M1: color = 255,0,0: adjlist=Timpuri Noi, Piata Unirii
EDGE: label=M1: color = 255,0,0: adjlist=Izvor, Piata Unirii
EDGE: label=M1: color = 255,0,0: adjlist=Eroilor, Izvor
EDGE: label=M1: color = 255,0,0: adjlist=Grozavesti, Eroilor
EDGE: label=M1: color = 255,0,0: adjlist=Semanatoarea, Grozavesti
EDGE: label=M1: color = 255,0,0: adjlist=Crangasi, Semanatoarea
EDGE: label=M1: color = 255,0,0: adjlist=Crangasi, Basarab
EDGE: label=M1: color = 255,0,0: adjlist=Basarab, Gara de Nord
EDGE: label=M1: color = 255,0,0: adjlist=Gara de Nord, Piata
   Victoriei
EDGE: label=M1: color = 255,0,0: adjlist=N. Grigorescu, Dristor
EDGE: label=M1: color = 255,0,0: adjlist=Titan, N. Grigorescu
EDGE: label=M1: color = 255,0,0: adjlist=Georgian Costin, Titan
EDGE: label=M1: color = 255,0,0: adjlist=Georgian Costin, Republica
EDGE: label=M2: color = 0,0,0: adjlist=Aurel Vlaicu, Pipera
```

```
EDGE: label=M2: color = 0,0,0: adjlist=Aurel Vlaicu, Aviatorilor
EDGE: label=M2: color = 0,0,0: adjlist=Aviatorilor, Piata Victoriei
EDGE: label=M2: color = 0,0,0: adjlist=Romana, Piata Victoriei
EDGE: label=M2: color = 0,0,0: adjlist=Universitate, Romana
EDGE: label=M2: color = 0,0,0: adjlist=Universtitate, Piata Unirii
EDGE: label=M2: color = 0,0,0: adjlist=Tineretului, Piata Unirii
EDGE: label=M2: color = 0,0,0: adjlist=Eroii Revolutiei, Tineretului
EDGE: label=M2: color = 0,0,0: adjlist=Constantin Brancoveanu, Eroii
     Revolutiei
EDGE: label=M2: color = 0,0,0: adjlist=Constantin Brancoveanu, Piata
     Sudului
EDGE: label=M2: color = 0,0,0: adjlist=Piata Sudului, Aparatorii
    Patriei
EDGE: label=M2: color = 0,0,0: adjlist=Aparatorii Patriei, IMGB 1
EDGE: label=M2: color = 0, 0, 0: adjlist=MGB 1, IMGB Depou
EDGE: label=M3: color = 255,255,0: adjlist=Politehnica, Eroilor
EDGE: label=M3: color = 255,255,0: adjlist=Armata Poporului,
   Politehnica
EDGE: label=M3: color = 255,255,0: adjlist=Gorjului, Armata
   Poporului
EDGE: label=M3: color = 255,255,0: adjlist=Pacii, Gorjului
EDGE: label=M3: color = 255,255,0: adjlist=Pacii, Industriilor
EDGE: label=T1: color = 150,150,0: adjlist=Grivita, Basarab
EDGE: label=T1: color = 150,150,0: adjlist=1 Mai, Grivita
EDGE: label=T1: color = 150,150,0: adjlist=Pajura, 1 Mai
 EDGE: label = T1: color = 150, 150, 0: adjlist = Pajura, Laromet \\
EDGE: label=T2: color = 150,150,0: adjlist=N. Grigorescu, 1 Dec.
   1918
EDGE: label=T2: color = 150,150,0: adjlist=Policolor, 1 Dec. 1918
EDGE: label=T2: color = 150,150,0: adjlist=Policolor, Linia de
   centura
```

#### B.4 London

```
NODE: label=Barbican: x=1320: y=818818
NODE: label=Barking: x=1444: y=843843
NODE: label=Barkingside: x=1447: y=895895
NODE: label=Barons Court: x=1240: y=783783
NODE: label=Bayswater : x=1258 : y=808808
NODE: label=Beckton : x = 1431 : y = 815815
NODE: label=Beckton Park: x=1427: y=808808
NODE: label=Becontree: x=1476: y=845845
NODE: label=Belsize Park: x=1273: y=851851
NODE: label=Bermondsey: x=1345: y=794794
NODE: label=Bethnal Green: x=1349: y=827827
NODE: label=Blackfriars: x=1310: y=813813
NODE: label=Blackhorse Road: x=1358: y=893893
NODE: label=Blackwall: x=1384: y=806806
NODE: label=Bond Street: x=1285: v=811811
NODE: label=Borough: x=1323: y=797797
NODE: label=Boston Manor: x=1163: y=787787
NODE: label=Bounds Green: x=1299: y=914914
NODE: label=Bow Church: x=1373: y=828828
NODE: label=Bow Road: x=1370: y=827827
NODE: label=Brent Cross: x=1238: y=879879
NODE: label=Brixton: x=1312: y=755755
NODE: label=Bromley-by-Bow: x=1379: y=825825
NODE: label=Brondesbury: x=1247: y=844844
NODE: label=Brondesbury Park: x=1242: y=839839
NODE: label=Buckhurst Hill:x=1417:y=939939
NODE: label=Burnt Oak: x=1203: y=907907
NODE: label=Caledonian Road: x=1305: y=849849
NODE: label=Caledonian Road \& Barnsbury:x=1308:y=844844
NODE: label=Camden Road: x=1291: y=842842
NODE: label=Camden Town: x=1289: y=839839
NODE: label=Canada Water: x=1354: y=794794
NODE: label=Canary Wharf: x=1374: y=803803
NODE: label=Canning Town: x=1394: y=813813
NODE: label=Cannon Street:x=1325:y=809809
NODE: label=Canonbury: x=1323: y=850850
NODE: label=Canons Park: x=1181: y=912912
NODE: label=Chalfont \ Latimer: x=996:y=975975
NODE: label=Chalk Farm: x=1281: y=844844
NODE: label=Chancery Lane:x=1311:y=816816
NODE: label=Charing Cross:x=1303:y=803803
NODE: label=Chesham: x=960: y=10161016
NODE: label=Chigwell: x=1437: y=930930
NODE: label=Chiswick Park:x=1203:y=787787
NODE: label=Chorleywood: x=1026: y=961961
NODE: label=Clapham Common: x=1294: y=753753
NODE: label=Clapham North: x=1300: y=756756
NODE: label=Clapham South: x=1287: y=742742
```

```
NODE: label=Cockfosters: x=1281: y=963963
NODE: label = Colindale : x = 1213 : y = 899899
NODE: label=Colliers Wood: x=1268: y=704704
NODE: label=Covent Garden:x=1302:y=810810
NODE: label=Crossharbour \& London Arena:x=1379:y=793793
NODE: label=Croxley: x=1079: y=954954
NODE: label=Custom House: x=1406: y=809809
NODE: label=Cutty Sark: x=1382: y=777777
NODE: label=Cyprus: x=1433: y=808808
NODE: label=Dagenham East: x=1501: y=850850
NODE: label=Dagenham Heathway: x=1489: y=847847
NODE: label=Dalston Kingsland: x=1335:y=850850
NODE: label=Debden: x=1442: y=961961
NODE: label=Deptford Bridge: x=1374:y=769769
NODE: label=Devons Road: x=1376: y=822822
NODE: label=Dollis Hill: x=1221: y=851851
NODE: label=Ealing Broadway: x=1179: y=809809
NODE: label=Ealing Common: x=1188: y=804804
NODE: label=Earl's Court:x=1254:y=784784
NODE: label=East Acton: x=1216: y=812812
NODE: label=East Finchley: x=1272: y=892892
NODE: label=East Ham: x=1424: y=842842
NODE: label=East India: x=1387: y=808808
NODE: label=East Putney: x=1243: y=748748
NODE: label=Eastcote: x=1111:y=876876
NODE: label=Edgware: x=1195: y=919919
NODE: label=Edgware Road: x=1272: y=817817
NODE: label=Elephant \& Castle:x=1319:y=791791
NODE: label=Elm Park: x=1525: y=856856
NODE: label=Elverson Road: x=1378: y=762762
NODE: label=Embankment: x = 1304: y = 803803
NODE: label=Epping: x=1461: y=10151015
NODE: label=Euston: x=1295: y=827827
NODE: label=Euston Square: x=1294: y=823823
NODE: label=Fairlop: x=1449: y=906906
NODE: label=Farringdon: x=1315: y=818818
NODE: label=Finchley Central: x=1253: y=906906
NODE: label=Finchley Road: x=1262: y=847847
NODE: label=Finchley Road \& Frognal:x=1260:y=850850
NODE: label=Finsbury Park:x=1313:y=867867
NODE: label=Fulham Broadway: x=1254: y=772772
NODE: label=Gallions Reach: x=1438: y=809809
NODE: label=Gants Hill: x=1432: y=884884
NODE: label=Gloucester Road: x=1262: y=788788
NODE: label=Golders Green: x=1252: y=874874
NODE: label=Goldhawk Road: x=1231: y=795795
NODE: label=Goodge Street:x=1295:y=818818
NODE: label=Gospel Oak: x=1282: y=856856
```

```
NODE: label=Grange Hill: x=1449: y=925925
NODE: label=Great Portland Street:x=1288:y=821821
NODE: label=Green Park: x=1289: y=802802
NODE: label=Greenford: x=1147: y=839839
NODE: label=Greenwich: x=1379: y=773773
NODE: label=Gunnersbury: x=1197: y=783783
NODE: label=Hackney Central: x=1349: y=849849
NODE: label=Hackney Wick: x=1370: y=845845
NODE: label=Hainault: x=1450: y=914914
NODE: label=Hammersmith: x=1233: y=786786
NODE: label=Hampstead: x=1263: y=857857
NODE: label=Hampstead Heath: x=1272: y=856856
NODE: label=Hanger Lane: x=1184: y=826826
NODE: label=Harlesden: x=1209: y=833833
NODE: label=Harrow \& Wealdstone: x=1154: y=894894
NODE: label=Harrow-on-the-Hill: x=1153: y=880880
NODE: label=Hatton Cross: x=1096: y=753753
NODE: label=Heathrow Terminals 1, 2 \& 3:x=1075:y=758758
NODE: label=Heathrow Terminal 4:x=1080:y=744744
NODE: label=Hendon Central:x=1229:y=886886
NODE: label=Heron Quays: x=1374: y=801801
NODE: label=High Barnet: x=1249: y=962962
NODE: label=High Street Kensington: x=1255: y=794794
NODE: label=Highbury \& Islington:x=1315:y=847847
NODE: label = Highgate : x = 1285 : y = 881881
NODE: label=Hillingdon: x=1074: y=850850
NODE: label=Holborn: x=1305: v=815815
NODE: label=Holland Park: x=1246: y=802802
NODE: label=Holloway Road: x=1309: y=854854
NODE: label=Homerton: x = 1358: y = 849849
NODE: label=Hornchurch: x=1538: y=862862
NODE: label=Hounslow Central:x=1135:y=759759
NODE: label=Hounslow East: x=1142: y=762762
NODE: label=Hounslow West: x=1122: y=761761
NODE: label=Hyde Park Corner: x=1282: y=797797
NODE: label=Ickenham : x = 1081 : y = 859859
NODE: label=Island Gardens: x=1382: y=784784
NODE: label=Kennington: x=1316: y=782782
NODE: label=Kensal Green: x=1232: y=827827
NODE: label=Kensal Rise: x=1228: y=835835
NODE: label=Kensington (Olympia): x=1243: y=792792
NODE: label=Kentish Town: x=1290: y=851851
NODE: label=Kentish Town West: x=1286: y=847847
NODE: label=Kenton: x=1167: y=883883
NODE: label=Kew Gardens: x=1192: y=767767
NODE: label=Kilburn: x=1245: y=846846
NODE: label=Kilburn Park: x=1253: y=833833
NODE: label=King's Cross St. Pancras:x=1302:y=830830
```

```
NODE: label=Kingsbury: x=1193: y=887887
NODE: label=Knightsbridge: x=1276: y=795795
NODE: label=Ladbroke Grove: x=1242: y=813813
NODE: label=Lambeth North: x=1312: y=794794
NODE: label=Lancaster Gate: x=1266: y=807807
NODE: label=Latimer Road: x=1237:y=809809
NODE: label=Leicester Square: x=1299: y=808808
NODE: label=Lewisham : x = 1381 : y = 759759
NODE: label=Leyton: x=1383: y=861861
NODE: label=Leytonstone: x=1392: y=874874
NODE: label=Limehouse: x=1361: y=811811
NODE: label=Liverpool Street:x=1331:y=815815
NODE: label=London Bridge: x=1327: y=802802
NODE: label=Loughton : x = 1423 : y = 956956
NODE: label=Maida Vale: x=1259: y=827827
NODE: label=Manor House: x=1320: y=874874
NODE: label=Mansion House: x=1323: y=809809
NODE: label=Marble Arch: x=1277: y=810810
NODE: label=Marylebone: x=1275: y=819819
NODE: label=Mile End: x=1365: y=825825
NODE: label=Mill Hill East: x=1240: y=914914
NODE: label=Monument: x = 1328: y = 808808
NODE: label=Moor Park: x=1085: y=934934
NODE: label=Moorgate: x=1326: y=816816
NODE: label=Morden: x=1256: y=686686
NODE: label=Mornington Crescent: x=1291: y=833833
NODE: label=Mudchute: x=1379: y=788788
NODE: label=Neasden: x=1213: y=853853
NODE: label=New Cross: x = 1368: y = 770770
NODE: label=New Cross Gate: x=1362: y=769769
NODE: label=Newbury Park: x=1449: y=883883
NODE: label=North Acton: x=1208: y=819819
NODE: label=North Ealing: x=1188: y=804804
NODE: label=North Greenwich: x=1391:y=798798
NODE: label=North Harrow: x=1135: y=886886
NODE: label=North Wembley: x=1176: y=862862
NODE: label=North Woolwich: x=1432: y=798798
NODE: label=Northfields: x=1171: y=792792
NODE: label=Northolt: x=1132: y=845845
NODE: label=Northwick Park:x=1166:y=879879
NODE: label=Northwood: x=1091: y=913913
NODE: label=Northwood Hills:x=1102:y=902902
NODE: label=Notting Hill Gate: x=1252: y=804804
NODE: label=Oakwood: x=1293: y=959959
NODE: label=Old Street: x=1326: y=825825
NODE: label=Osterley: x=1145: y=771771
NODE: label=Oval: x=1310: y=775775
NODE: label=Oxford Circus: x=1290: y=812812
```

```
NODE: label=Paddington: x=1266: y=812812
NODE: label=Park Royal: x=1191: y=822822
NODE: label=Parsons Green: x=1250: y=766766
NODE: label=Perivale: x=1163: y=833833
NODE: label=Piccadilly Circus:x=1294:y=805809
NODE: label=Pimlico: x=1296: y=784784
NODE: label=Pinner: x=1122: y=894894
NODE: label=Plaistow: x=1399: y=833833
NODE: label=Poplar : x=1376 : y=806806
NODE: label=Preston Road: x=1182: y=873873
NODE: label=Prince Regent: x=1412: y=809809
NODE: label=Pudding Mill Lane: x=1378: y=835835
NODE: label=Putney Bridge: x=1244: y=758758
NODE: label=Queen's Park: x=1245: y=832832
NODE: label=Queensbury: x=1188: y=897897
NODE: label=Queensway: x = 1258: y = 806806
NODE: label=Ravenscourt Park: x=1225: y=787787
NODE: label=Rayners Lane: x=1129: y=875875
NODE: label=Redbridge: x=1418: y=883883
NODE: label=Regent's Park: x=1286: y=821821
NODE: label=Richmond: x=1181: y=751751
NODE: label=Rickmansworth: x=1056: y=945945
NODE: label=Roding Valley: x=1415: y=928928
NODE: label=Rotherhithe: x=1352: y=798798
NODE: label=Royal Albert: x=1420: y=808808
NODE: label=Royal Oak: x=1257: y=815815
NODE: label=Royal Victoria:x=1401:y=808808
NODE: label=Ruislip:x=1095:y=870870
NODE: label=Ruislip Gardens: x=1103:y=858858
NODE: label=Ruislip Manor:x=1101:y=872872
NODE: label=Russell Square:x=1301:y=821821
NODE: label=Seven Sisters: x=1335: y=888888
NODE: label=Shadwell: x=1350: y=809809
NODE: label=Shepherd's Bush: x=1231: y=800800
NODE: label=Shepherd's Bush (Central):x=1231:y=801801
NODE: label=Shoreditch: x=1339: y=822822
NODE: label=Silvertown: x=1420: y=801801
NODE: label=Sloane Square:x=1281:y=786786
NODE: label=Snaresbrook: x=1401: y=887887
NODE: label=South Acton: x=1201: y=792792
NODE: label=South Ealing: x=1175: y=793793
NODE: label=South Harrow: x=1143: y=863863
NODE: label=South Kensington: x=1269: y=788788
NODE: label=South Kenton: x=1173: y=870870
NODE: label=South Quay: x=1375: y=798798
NODE: label=South Ruislip:x=1110:y=854854
NODE: label=South Wimbledon: x=1258: y=700700
NODE: label=South Woodford: x=1405: y=900900
```

```
NODE: label=Southfields: x=1247: y=733733
NODE: label=Southgate: x=1296: y=942942
NODE: label=Southwark: x=1315: y=800800
NODE: label=St. James's Park: x=1296: y=794794
NODE: label=St. John's Wood: x=1267: y=833833
NODE: label=St. Paul's: x=1321: y=815815
NODE: label=Stamford Brook: x=1218: y=787787
NODE: label=Stanmore: x=1175: y=925925
NODE: label=Stepney Green: x=1356: y=821821
NODE: label=Stockwell: x=1304: y=765765
NODE: label=Stonebridge Park: x=1196: y=842842
NODE: label=Stratford : x = 1385 : y = 843843
NODE: label=Sudbury Hill: x=1154: y=855855
NODE: label=Sudbury Town: x=1168: y=848848
NODE: label=Surrey Quays: x=1356:y=789789
NODE: label=Swiss Cottage:x=1266:y=843843
NODE: label=Temple: x=1309: y=808808
NODE: label=Theydon Bois: x=1455: y=991991
NODE: label=Tooting Bec: x=1280: y=723723
NODE: label=Tooting Broadway: x=1274: y=713713
NODE: label=Tottenham Court Road: x=1297: y=814809
NODE: label=Tottenham Hale: x=1345: y=895895
NODE: label=Totteridge \ Whetstone: x=1261:y=939939
NODE: label=Tower Gateway: x=1336: y=808808
NODE: label=Tower Hill: x=1335: y=807807
NODE: label=Tufnell Park: x=1291: y=858858
NODE: label=Turnham Green: x=1212: y=787787
NODE: label=Turnpike Lane: x=1315: y=896896
NODE: label=Upminster: x=1560: y=868868
NODE: label=Upminster Bridge: x=1551:y=867867
NODE: label=Upney: x = 1458: y = 843843
NODE: label=Upton Park: x=1412: y=837837
NODE: label=Uxbridge:x=1056:y=842842
NODE: label=Vauxhall: x=1304: y=780780
NODE: label=Victoria: x=1288: y=789789
NODE: label=Walthamstow Central:x=1373:y=889889
NODE: label=Wanstead: x=1406: y=882882
NODE: label=Wapping: x=1350: y=801801
NODE: label=Warren Street: x=1292: y=822822
NODE: label=Warwick Avenue: x=1260: y=821821
NODE: label=Waterloo: x=1310: y=799799
NODE: label=Watford: x=1095: y=965965
NODE: label=Wembley Central:x=1182:y=851851
NODE: label=Wembley Park: x=1192: y=863863
NODE: label=West Acton: x=1193: y=813813
NODE: label=West Brompton: x=1254: y=779779
NODE: label=West Finchley: x=1255: y=916916
NODE: label=West Ham: x=1392: y=829829
```

```
NODE: label=West Hampstead: x=1255: y=846846
NODE: label=West Harrow: x=1141:y=880880
NODE: label=West India Quay: x=1374: y=805805
NODE: label=West Kensington: x=1246: y=783783
NODE: label=West Ruislip:x=1084:y=867867
NODE: label=Westbourne Park: x=1248: y=817817
NODE: label=Westferry: x=1370: y=807807
NODE: label=Westminster: x=1302: y=797797
NODE: label=White City: x=1233: y=807807
NODE: label=Whitechapel: x=1346: y=818818
NODE: label=Willesden Green: x=1233: y=848848
NODE: label=Willesden Junction: x=1218:y=829829
NODE: label=Wimbledon: x=1248: y=706706
NODE: label=Wimbledon Park: x=1252: y=721721
NODE: label=Wood Green: x=1310: y=904904
NODE: label=Woodford: x=1409: y=917917
NODE: label=Woodside Park: x=1257: y=925925
EDGE: label=X: color = 255,255,255; adjlist=Tower Hill, Tower
   Gateway
EDGE: label=X: color = 255,255,255; adjlist=Bank, Monument
EDGE: label=X: color = 255,255,255; adjlist=Bow Road, Bow Church
EDGE: label=Victoria: color = 60,140,255: adjlist=Brixton, Stockwell
    Vauxhall, Pimlico, Victoria, Green Park, Oxford Circus,
   Warren Street, Euston, King's Cross St. Pancras, Highbury \&
    Islington, Finsbury Park, Seven Sisters, Tottenham Hale,
   Blackhorse Road, Walthamstow Central
EDGE: label=District: color = 20,150,0: adjlist=Kensington (Olympia)
   , Earl's Court
EDGE: label=District: color = 20,150,0: adjlist=Ealing Broadway,
   Ealing Common, Acton Town, Chiswick Park, Turnham Green
EDGE: label=District: color = 20,150,0: adjlist=Richmond, Kew
   Gardens, Gunnersbury, Turnham Green, Stamford Brook,
   Ravenscourt Park, Hammersmith, Barons Court, West Kensington
   , Earl's Court, South Kensington, Sloane Square, Victoria,
   St. James's Park, Westminster, Embankment, Temple,
   Blackfriars, Mansion House, Cannon Street, Monument, Tower
   Hill, Aldgate East, Whitechapel, Stepney Green, Mile End,
   Bow Road, Bow Church, Bromley-by-Bow, West Ham, Plaistow,
   Upton Park, East Ham, Barking, Upney, Becontree, Dagenham
   Heathway, Dagenham East, Elm Park, Hornchurch, Upminster
   Bridge, Upminster
EDGE: label=District: color = 20,150,0: adjlist=Wimbledon, Wimbledon
    Park, Southfields, East Putney, Putney Bridge, Parsons
   Green, Fulham Broadway, West Brompton, Earl's Court, High
   Street Kensington, Notting Hill Gate, Bayswater, Paddington,
    Edgware Road
```

- EDGE: label=Bakerloo: color = 100,80,10: adjlist=Harrow \&
   Wealdstone, Kenton, South Kenton, North Wembley, Wembley
   Central, Stonebridge Park, Harlesden, Willesden Junction,
   Kensal Green, Queen's Park, Kilburn Park, Maida Vale,
   Warwick Avenue, Paddington, Edgware Road, Marylebone, Baker
   Street, Regent's Park, Oxford Circus, Piccadilly Circus,
   Charing Cross, Embankment, Waterloo, Lambeth North, Elephant
   \& Castle
- EDGE: label=Metropolitan: color = 140,80,90: adjlist=Uxbridge, Hillingdon, Ickenham, Ruislip, Ruislip Manor, Eastcote, Rayners Lane, West Harrow, Harrow-on-the-Hill, Northwick Park, Preston Road, Wembley Park, Finchley Road, Baker Street, Great Portland Street, Euston Square, King's Cross St. Pancras, Farringdon, Barbican, Moorgate, Liverpool Street, Aldgate
- $$\begin{split} & EDGE: label=Metropolitan: color=140\,,80\,,90: adjlist=Amersham\,,\\ & Chalfont \ \backslash\& \ Latimer\,\,,\ Chorleywood\,\,,\ Rickmansworth\,\,,\ Moor\ Park\,\,,\\ & Northwood\,\,,\ Northwood\,\,Hills\,\,,\ Pinner\,\,,\ North\,\,Harrow\,\,,\ Harrow-on-the-Hill \end{split}$$
- EDGE: label=Metropolitan: color = 140,80,90: adjlist=Chesham, Chalfont & Latimer
- EDGE: label=Metropolitan: color = 140,80,90: adjlist=Watford, Croxley, Moor Park
- EDGE: label=Hammersmith & City: color = 255,100,160: adjlist=
  Hammersmith, Goldhawk Road, Shepherd's Bush, Latimer Road,
  Ladbroke Grove, Westbourne Park, Royal Oak, Paddington,
  Edgware Road, Baker Street, Great Portland Street, Euston
  Square, King's Cross St. Pancras, Farringdon, Barbican,
  Moorgate, Liverpool Street, Aldgate East, Whitechapel,
  Stepney Green, Mile End, Bow Road, Bow Church, Bromley-byBow, West Ham, Plaistow, Upton Park, East Ham, Barking
- EDGE: label=Circle: color = 255,255,0: adjlist=Paddington, Bayswater, Notting Hill Gate, High Street Kensington, Gloucester Road, South Kensington, Sloane Square, Victoria, St. James's Park, Westminster, Embankment, Temple, Blackfriars, Mansion House, Cannon Street, Monument, Tower Hill, Aldgate, Liverpool Street, Moorgate, Barbican, Farringdon, King's Cross St. Pancras, Euston Square, Great Portland Street, Baker Street, Edgware Road, Paddington
- EDGE: label=Central: color = 255,0,0: adjlist=West Ruislip, Ruislip Gardens, South Ruislip, Northolt, Greenford, Perivale, Hanger Lane, North Acton, East Acton, White City, Shepherd's Bush (Central), Holland Park, Notting Hill Gate, Queensway, Lancaster Gate, Marble Arch, Bond Street, Oxford Circus, Tottenham Court Road, Holborn, Chancery Lane, St. Paul's, Bank, Liverpool Street, Bethnal Green, Mile End, Stratford, Leyton, Leytonstone, Snaresbrook, South Woodford, Woodford, Buckhurst Hill, Loughton, Debden, Theydon Bois, Epping

- EDGE: label=Central: color = 255,0,0: adjlist=Ealing Broadway, West Acton, North Acton
- EDGE: label=Piccadilly:color=0,0,200: adjlist=Uxbridge, Hillingdon, Ickenham, Ruislip, Ruislip Manor, Eastcote, Rayners Lane, South Harrow, Sudbury Hill, Sudbury Town, Alperton, Park Royal, North Ealing, Ealing Common, Acton Town, Turnham Green, Hammersmith, Barons Court, Earl's Court, Gloucester Road, South Kensington, Knightsbridge, Hyde Park Corner, Green Park, Piccadilly Circus, Leicester Square, Covent Garden, Holborn, Russell Square, King's Cross St. Pancras, Caledonian Road, Holloway Road, Arsenal, Finsbury Park, Manor House, Turnpike Lane, Wood Green, Bounds Green, Arnos Grove, Southgate, Oakwood, Cockfosters
- $\begin{array}{llll} EDGE: label=Piccadilly: color=0\,,0\,,200: adjlist=Hatton & Cross\,,\\ & Heathrow & Terminals & 1\,, & 2\,\& & 3\,, & Heathrow & Terminal & 4\,, & Hatton & Cross\,, & Hounslow & West\,, & Hounslow & Central\,, & Hounslow & East\,,\\ & Osterley\,\,, & Boston & Manor\,, & Northfields\,\,, & South & Ealing\,\,, & Acton & Town \end{array}$
- EDGE: label=Jubilee: color = 128,128,128: adjlist=Stanmore, Canons Park, Queensbury, Kingsbury, Wembley Park, Neasden, Dollis Hill, Willesden Green, Kilburn, West Hampstead, Finchley Road, Swiss Cottage, St. John's Wood, Baker Street, Bond Street, Green Park, Westminster, Waterloo, Southwark, London Bridge, Bermondsey, Canada Water, Canary Wharf, North Greenwich, Canning Town, West Ham, Stratford
- EDGE: label=Northern: color = 0,0,0: adjlist=Edgware, Burnt Oak, Colindale, Hendon Central, Brent Cross, Golders Green, Hampstead, Belsize Park, Chalk Farm, Camden Town, Mornington Crescent, Euston, Warren Street, Goodge Street, Tottenham Court Road, Leicester Square, Charing Cross, Embankment, Waterloo, Kennington, Oval, Stockwell, Clapham North, Clapham Common, Clapham South, Balham, Tooting Bec, Tooting Broadway, Colliers Wood, South Wimbledon, Morden
- EDGE: label=Northern: color = 0,0,0: adjlist=High Barnet, Totteridge
   \& Whetstone, Woodside Park, West Finchley, Finchley
   Central, East Finchley, Highgate, Archway, Tufnell Park,
   Kentish Town, Camden Town, Euston, King's Cross St. Pancras,
   Angel, Old Street, Moorgate, Bank, Monument, London Bridge,
   Borough, Elephant \& Castle, Kennington
- $\begin{array}{l} {\rm EDGE:\,label = Northern:\,color = 0\,,0\,,0:\,a\,djlis\,t = Mill\ Hill\ East\,,} \\ {\rm Finchley\ Central} \end{array}$
- EDGE: label=Waterloo \& City: color =0,255,160: adjlist=Waterloo , Bank

- EDGE: label=East London: color = 255,190,40: adjlist=Shoreditch ,
   Whitechapel , Shadwell , Wapping , Rotherhithe , Canada Water ,
   Surrey Quays , New Cross Gate
- EDGE: label=East London: color = 255,190,40: adjlist=Surrey Quays, New Cross
- EDGE: label=Docklands Light Rail: color = 0,255,255: adjlist=
   Monument, Shadwell, Limehouse, Westferry, Poplar, Blackwall,
   East India, Canning Town, Royal Victoria, Custom House,
   Prince Regent, Royal Albert, Beckton Park, Cyprus, Gallions
   Reach, Beckton
- EDGE: label=Docklands Light Rail: color = 0,255,255: adjlist=
  Stratford, Pudding Mill Lane, Bow Church, Devons Road, All
  Saints, Poplar, West India Quay, Canary Wharf, Heron Quays,
  South Quay, Crossharbour & London Arena, Mudchute, Island
  Gardens, Cutty Sark, Greenwich, Deptford Bridge, Elverson
  Road, Lewisham
- EDGE: label=Docklands Light Rail: color = 0,255,255: adjlist= Westferry, West India Quay
- EDGE: label=Docklands Light Rail: color = 0,255,255: adjlist=Tower Gateway, Shadwell
- EDGE: label=Docklands Light Rail: color = 0,255,255: adjlist=Canning Town
- EDGE: label=National Rail: color = 0,255,0: adjlist=Richmond, Kew Gardens, Gunnersbury, South Acton, Acton Central, Willesden Junction, Kensal Rise, Brondesbury Park, Brondesbury, West Hampstead, Finchley Road & Frognal, Hampstead Heath, Gospel Oak, Kentish Town West, Camden Road, Caledonian Road & Barnsbury, Highbury & Islington, Canonbury, Dalston Kingsland, Hackney Central, Homerton, Hackney Wick, Stratford, West Ham, Canning Town, Custom House, Silvertown, North Woolwich

#### B.5 Madrid

NODE: label=Anton Martin: x = -3.6987: y = 40.4121

```
NODE: label=Tirso de Molina: x = -3.704: y = 40.4123
NODE: label=Sol: x = -3.7035: y = 40.4167
NODE: label=Gran Via: x = -3.7016: y = 40.4200
NODE: label=Tribunal: x = -3.7009: y = 40.4254
NODE: label=Bilbao: x=-3.7023: y=40.4291
NODE: label=Iglesia: x = -3.6987: y = 40.4349
NODE: label=Rios Rosas: x = -3.7018: y = 40.4420
NODE: label=Cuatro Caminos: x = -3.7038: y = 40.4468
NODE: label=Alvarado: x = -3.7034: y = 40.4502
NODE: label=Estrecho: x = -3.7027: y = 40.4543
NODE: label=Tetuan: x = -3.6981: y = 40.4613
NODE: label=Valdeacederas: x = -3.6957: y = 40.4643
NODE: label=Plaza de Castilla: x=-3.6893: y=40.4660
NODE: label=Canal: x = -3.7043: v = 40.4385
NODE: label=Quevedo: x = -3.7048: y = 40.4334
NODE: label=San Bernardo: x = -3.7062: y = 40.4297
NODE: label=Noviciado: x = -3.7073: y = 40.4255
NODE: label=Santo Domingo: x = -3.7084: y = 40.4202
NODE: label=Opera: x = -3.7101: y = 40.4178
NODE: label=Sevilla: x = -3.6977: y = 40.4184
NODE: label=Banco de Espana: x = -3.6935: y = 40.4194
NODE: label=Retiro: x = -3.6857: y = 40.4206
NODE: label=Principe de Vergara: x = -3.6802: y = 40.4227
NODE: label=Goya: x = -3.6756: y = 40.4248
NODE: label=Manuel Becerra: x = -3.6690: y = 40.4279
NODE: label=Ventas: x = -3.6632: y = 40.4320
NODE: label=Legazpi: x = -3.6950: y = 40.3912
NODE: label=Delicias: x = -3.6942: y = 40.3984
NODE: label=Palos de la Frontera: x = -3.6960: y = 40.4031
NODE: label=Embajadores: x = -3.7022: y = 40.4049
NODE: label=Lavapies: x = -3.7011: y = 40.4087
NODE: label=Callao: x = -3.7058: y = 40.4202
NODE: label=Plaza de Espana: x = -3.7109: y = 40.4234
NODE: label=Ventura Rodriguez: x = -3.7139: y = 40.4272
NODE: label=Arguelles: x = -3.7163: y = 40.4307
NODE: label=Moncloa: x = -3.7193: y = 40.4347
NODE: label=Alonso Martinez: x = -3.6956: y = 40.4275
NODE: label=Colon: x = -3.6907: y = 40.4252
NODE: label=Serrano: x = -3.6878: y = 40.4253
NODE: label=Velazquez : x = -3.6836 : y = 40.4251
NODE: label=Lista: x = -3.6754: y = 40.4299
NODE: label=Diego de Leon: x = -3.6747: y = 40.4347
NODE: label=Avda de America: x = -3.6772: y = 40.4374
NODE: label=Prosperidad : x = -3.6745 : y = 40.4439
NODE: label=Alfonso XIII: x = -3.6678: y = 40.4484
NODE: label=Avda de la Paz: x = -3.6625: y = 40.4534
NODE: label=Arturo Soria: x = -3.6563: y = 40.4560
NODE: label=Esperanza: x = -3.6457: y = 40.4593
```

```
NODE: label = Canillas : x = -3.6354 : y = 40.4646
NODE: label=Mar de Cristal: x = -3.6389: y = 40.4695
NODE: label=San Lorenzo: x = -3.6384: y = 40.4765
NODE: label=Parque de Santa Maria: x = -3.6460: y = 40.4796
NODE: label=Casa de Campo: x = -3.7668: y = 40.3992
NODE: label=Campamento: x = -3.7680: y = 40.3943
NODE: label=Empalme: x = -3.7665: y = 40.3908
NODE: label=Aluche: x = -3.7609: y = 40.3864
NODE: label=Eugenia de Montijo: x = -3.7494: y = 40.3852
NODE: label=Carabanchel: x = -3.7447: y = 40.3878
NODE: label=Vista Alegre: x = -3.7400: y = 40.3889
NODE: label=Oporto: x = -3.7321: y = 40.3883
NODE: label=Urgel: x = -3.7242: y = 40.3930
NODE: label=Marques de Vadillo: x = -3.7163: y = 40.3974
NODE: label=Piramides: x = -3.7146: y = 40.4008
NODE: label=Acacias: x = -3.7064: y = 40.4037
NODE: label=Puerto de Toledo: x=-3.7113: y=40.4065
NODE: label=La Latina: x = -3.7083: y = 40.4111
NODE: label=Chueca: x = -3.6973: y = 40.4228
NODE: label=Ruben Dario: x = -3.6890: y = 40.4334
NODE: label=Nunez de Balboa: x = -3.6830: y = 40.4323
NODE: label=El Carmen: x = -3.6555: y = 40.4321
NODE: label=Quintana: x = -3.6490: y = 40.4329
NODE: label=Pueblo Nuevo: x = -3.6443: y = 40.4352
NODE: label=Ciudad Lineal: x = -3.6384: y = 40.4377
NODE: label=Suanzes: x = -3.6269: y = 40.4409
NODE: label=Torre Arias: x = -3.6172: y = 40.4436
NODE: label = Canillejas : x = -3.6080 : y = 40.4493
NODE: label=Mendez Alvaro: x = -3.6804: y = 40.3957
NODE: label=Conde de Casal: x = -3.6700: y = 40.4084
NODE: label=Sainz de Baranda: x=-3.6692: y=40.4163
NODE: label=O' Donnell: x = -3.6689: y = 40.4230
NODE: label=Republica Argentina: x = -3.6853: y = 40.4450
NODE: label=Nuevos Ministerios: x = -3.6921: y = 40.4465
NODE: label=Guzman el Bueno: x = -3.7125: y = 40.4465
NODE: label=Metropolitano: x = -3.7183: y = 40.4466
NODE: label=Ciudad Universitaria: x = -3.72750: y = 40.4438
NODE: label=Principe Rio: x = -3.72010: y = 40.4203
NODE: label=Puerta del Angel: x = -3.72690: y = 40.4138
NODE: label=Alto de Extremadura: x = -3.73780: y = 40.4101
NODE: label=Lucero: x = -3.74530: y = 40.4046
NODE: label=Laguna: x = -3.74450: y = 40.3990
NODE: label=Carpetana: x = -3.74160: y = 40.3923
NODE: label=Opanel: x = -3.72350: y = 40.3853
NODE: label=Plaza Eliptica: x = -3.71650: y = 40.3851
NODE: label=Usera: x = -3.70750: y = 40.3868
NODE: label=Pitis: x = -3.72760: y = 40.4940
NODE: label=Lacoma: x = -3.72360: y = 40.4851
```

```
NODE: label=Avda Ilustracion: x = -3.71610: y = 40.4769
NODE: label=Penagrande: x = -3.72080: y = 40.4744
NODE: label=Antonio Machado: x = -3.71960: y = 40.4679
NODE: label=Valdezarza: x = -3.7158: y = 40.4665
NODE: label=Francos Rodriguez: x = -3.7115: y = 40.4571
NODE: label=Islas Filipinas: x=-3.7131: y=40.4390
NODE: label=Alonso Cano: x = -3.6982: y = 40.4383
NODE: label=Gregorio Maranon: x = -3.6912: y = 40.4379
NODE: label=Cartagena: x = -3.6726: y = 40.4396
NODE: label=Parque de las Avenidas: x = -3.6630: y = 40.4390
NODE: label=Barrio de la Concepcion: x=-3.6520: y=40.4385
NODE: label=Ascao: x = -3.6416: y = 40.4301
NODE: label=Garcia Noblejas: x = -3.6320: y = 40.4291
NODE: label=Simancas: x = -3.6254: y = 40.4279
NODE: label=San Blas: x = -3.6178: y = 40.4270
NODE: label=Las Musas: x = -3.6077: y = 40.4329
NODE: label=Barajas: x = -3.5826: y = 40.4764
NODE: label=Aeropuerto: x = -3.5710: y = 40.4682
NODE: label=Campo de las Naciones: x = -3.6164: y = 40.4639
NODE: label=Colombia: x = -3.6768: y = 40.4575
NODE: label=Herrera Oria: x = -3.7089: y = 40.4852
NODE: label=Barrio del Pilar: x=-3.7038: y=40.4773
NODE: label=Ventilla: x = -3.6953: y = 40.4701
NODE: label=Duque de Pastrana: x = -3.6791: y = 40.4675
NODE: label=Pio XII: x = -3.6762: y = 40.4627
NODE: label=Concha Espina: x = -3.6779: y = 40.4523
NODE: label=Cruz del Rayo: x = -3.6786: y = 40.4427
NODE: label=Ibiza: x=-3.6778: y=40.4183
NODE: label = Estrella : x = -3.6598 : y = 40.4106
NODE: label=Vinateros: x = -3.6524: y = 40.4103
NODE: label = Artilleros : x = -3.6443 : y = 40.4069
NODE: label=Pavones: x = -3.6353: y = 40.4007
NODE: label=Valdebernardo: x = -3.6215: y = 40.3999
NODE: label=Vicalvaro: x = -3.6090: y = 40.4041
NODE: label=San Cipriano: x = -3.6038: y = 40.4039
NODE: label=Puerta de Arganda: x = -3.5968: y = 40.4017
NODE: label=Rivas Urbanizaciones: x = -3.5443: y = 40.3644
NODE: label=Rivas Vaciamadrid: x = -3.5149: y = 40.3222
NODE: label=La Poveda: x = -3.4786: y = 40.3163
NODE: label=Arganda del Rey: x = -3.4540: y = 40.3017
NODE: label=Fuencarral: x = -3.6931: y = 40.4951
NODE: label=Begona: x = -3.6859: y = 40.4803
NODE: label=Chamartin: x = -3.6825: y = 40.4720
NODE: label=Cuzco: x = -3.6902: y = 40.4585
NODE: label=Santiago Bernabeu: x = -3.6903: y = 40.4521
NODE: label=Lago: x = -3.7347: y = 40.4174
NODE: label=Batan: x = -3.7524: y = 40.4082
NODE: label=Colonia Jardin: x=-3.7744: y=40.3970
```

```
NODE: label=Cuatro Vientos: x = -3.7912: y = 40.3779
NODE: label=Joaquin Vilumbrales: x = -3.8077: y = 40.3498
NODE: label=Puerta del Sur: x = -3.8122: y = 40.3452
NODE: label=Abrantes: x = -3.7273: y = 40.3816
NODE: label=Pan Bendito: x = -3.7346: y = 40.3757
NODE: label=Leganes Central: x = -3.7712: y = 40.3279
NODE: label=San Nicasio: x = -3.7729: y = 40.3361
NODE: label=Hospital Severo Ochoa: x = -3.7672: y = 40.3223
NODE: label=Casa del Reloj: x = -3.7596: y = 40.3268
NODE: label=Julian Besteiro: x = -3.7528: y = 40.3346
NODE: label=El Carrascal: x = -3.7421: y = 40.3362
NODE: label=El Bercial: x = -3.7361: y = 40.3271
NODE: label=Los Espartales: x = -3.7187: y = 40.3247
NODE: label=El Casar : x = -3.7110 : y = 40.3188
NODE: label=Juan de la Cierva: x=-3.7227: y=40.3115
NODE: label=Getafe Central: x = -3.7335: y = 40.3099
NODE: label=Alonso de Mendoza: x = -3.7358: y = 40.3010
NODE: label=Conservatorio: x = -3.7453: y = 40.2934
NODE: label=Arroyo Culebro: x = -3.7568: y = 40.2891
NODE: label=Parque de los Estados: x=-3.7875: y=40.2869
NODE: label=Fuenlabrada Central: x = -3.7992: y = 40.2833
NODE: label=Parque Europa: x = -3.8059: y = 40.2852
NODE: label=Hospital de Fuenlabrada: x = -3.8132: y = 40.2853
NODE: label=Loranca: x = -3.8351: y = 40.2966
NODE: label=Manuela Malasana: x = -3.8648: y = 40.3091
NODE: label=Hospital de Mostoles: x = -3.8744: y = 40.3161
NODE: label=Pradillo: x = -3.8653: y = 40.3217
NODE: label=Mostoles Central: x = -3.8635: y = 40.3288
NODE: label=Universidad Rey Juan Carlos: x = -3.8732: y = 40.3346
NODE: label=Parque Oeste: x = -3.8496: y = 40.3459
NODE: label=Alcorcon Central: x = -3.8319: y = 40.3502
NODE: label=Parque Lisboa: x = -3.8208: y = 40.3492
EDGE: label=1:color=51,153,204: adjlist=Plaza de Castilla,
   Valdeacederas, Tetuan, Estrecho, Alvarado, Cuatro Caminos,
   Rios Rosas, Iglesia, Bilbao, Tribunal, Gran Via, Sol, Tirso
   de Molina, Anton Martin, Atocha, Atocha Renfe, Menendez
   Pelayo, Pacifico, Puente de Vallecas, Nueva Numancia,
   Portazgo, Buenos Aires, Alto del Arenal, Miguel Hernandez,
   Sierra de Guadalupe, Villa de Vallecas, Congosto
EDGE: label = 2: color = 255,34,0: adjlist=Cuatro Caminos, Canal,
   Quevedo, San Bernardo, Noviciado, Santo Domingo, Opera, Sol,
    Sevilla, Banco de Espana, Retiro, Principe de Vergara, Goya
    Manuel Becerra, Ventas
EDGE: label=3: color=255,204,0: adjlist=Moncloa, Arguelles,
   Ventura Rodriguez, Plaza de Espana, Callao, Sol, Lavapies,
```

Embajadores, Palos de la Frontera, Delicias, Legazpi

- EDGE: label=4: color=102,0,0: adjlist=Arguelles, San Bernardo, Bilbao, Alonso Martinez, Colon, Serrano, Velazquez, Goya, Lista, Diego de Leon, Avda de America, Prosperidad, Alfonso XIII, Avda de la Paz, Arturo Soria, Esperanza, Canillas, Mar de Cristal, San Lorenzo, Parque de Santa Maria
- EDGE: label=5: color=153,204,51: adjlist=Casa de Campo, Campamento, Empalme, Aluche, Eugenia de Montijo, Carabanchel, Vista Alegre, Oporto, Urgel, Marques de Vadillo, Piramides, Acacias, Puerto de Toledo, La Latina, Opera, Callao, Gran Via, Chueca, Alonso Martinez, Ruben Dario, Nunez de Balboa, Diego de Leon, Ventas, El Carmen, Quintana, Pueblo Nuevo, Ciudad Lineal, Suanzes, Torre Arias, Canillejas
- EDGE: label=6:color=204,204,204: adjlist=Legazpi, Mendez Alvaro, Pacifico, Conde de Casal, Sainz de Baranda, O'Donnell, Manuel Becerra, Diego de Leon, Avda de America, Republica Argentina, Nuevos Ministerios, Cuatro Caminos, Guzman el Bueno, Metropolitano, Ciudad Universitaria, Moncloa, Arguelles, Principe Rio, Puerta del Angel, Alto de Extremadura, Lucero, Laguna, Carpetana, Oporto, Opanel, Plaza Eliptica, Usera, Legazpi
- EDGE: label=7: color=255,153,0: adjlist=Pitis , Lacoma, Avda Ilustracion , Penagrande , Antonio Machado , Valdezarza , Francos Rodriguez , Guzman el Bueno , Islas Filipinas , Canal , Alonso Cano , Gregorio Maranon , Avda de America , Cartagena , Parque de las Avenidas , Barrio de la Concepcion , Pueblo Nuevo , Ascao , Garcia Noblejas , Simancas , San Blas , Las Musas
- EDGE: label=8: color=255,102,153: adjlist=Barajas , Aeropuerto , Campo de las Naciones , Mar de Cristal , Colombia , Nuevos Ministerios
- EDGE: label = 9: color = 153,51,102: adjlist = Herrera Oria, Barrio del Pilar, Ventilla, Plaza de Castilla, Duque de Pastrana, Pio XII, Colombia, Concha Espina, Cruz del Rayo, Avda de America, Nunez de Balboa, Principe de Vergara, Ibiza, Sainz de Baranda, Estrella, Vinateros, Artilleros, Pavones, Valdebernardo, Vicalvaro, San Cipriano, Puerta de Arganda, Rivas Urbanizaciones, Rivas Vaciamadrid, La Poveda, Arganda del Rey
- EDGE: label=10: color=0,0,102: adjlist=Fuencarral, Begona, Chamartin, Plaza de Castilla, Cuzco, Santiago Bernabeu, Nuevos Ministerios, Gregorio Maranon, Alonso Martinez, Tribunal, Plaza de Espana, Principe Rio, Lago, Batan, Casa de Campo, Colonia Jardin, Cuatro Vientos, Joaquin Vilumbrales, Puerta del Sur

EDGE: label=12: color=221,204,51: adjlist=Puerta del Sur, San Nicasio, Leganes Central, Hospital Severo Ochoa, Casa del Reloj, Julian Besteiro, El Carrascal, El Bercial, Los Espartales, El Casar, Juan de la Cierva, Getafe Central, Alonso de Mendoza, Conservatorio, Arroyo Culebro, Parque de los Estados, Fuenlabrada Central, Parque Europa, Hospital de Fuenlabrada, Loranca, Manuela Malasana, Hospital de Mostoles, Pradillo, Mostoles Central, Universidad Rey Juan Carlos, Parque Oeste, Alcorcon Central, Parque Lisboa, Puerta del Sur

EDGE: label=R: color = 0,0,0: adjlist=Principe Rio, Opera

### B.6 Mexico City

```
NODE: label=General Anaya: x=690.1962319167424: y
   =56.058467001033705650.0
NODE: label = Oceania : x = 940.3502369657688 : y = 432.3965591864863650.0
NODE: label=Terminal Arena: x=950.1078310769819: y
   =394.4713352638818650.0
NODE: label=La Paz: x=1134.7749604373348: y=77.1908618651961650.0
NODE: label=Hangares: x = 965.4931385827108: y
   =359.4471071291963650.0
NODE: label=Pantitlan: x = 985.9116976680533: y
   =333.53450508591675650.0
NODE: label=Agricola Oriental: x=1030.1440231447646: y
   =299.0024221352593650.0
NODE: label=De San Juan: x=1042.1091576036492: y
   =271.6547649235314650.0
NODE: label=Tepalcates: x = 1053.5613631325073: y
   =238.01204706252685650.0
NODE: label=Guelatao: x = 1058.617398590932: y
   =208.69890542341938650.0
NODE: label=Penon Viejo: x=1072.8480727447636: y
   =173.0661885673411650.0
NODE: label=Los Reyes: x = 1101.2718600521146: y
   =95.86844759108476650.0
NODE: label=Santa Marta: x=1072.7428294364531: y
   =105.91539352650011650.0
NODE: label = Acatitla : x = 1075.4690504525017 : y
   =139.75240347255203650.0
NODE: |abe| = Puebla: x = 935.9100000952067: y = 296.41524117850395650.0
NODE: label=Aragon: x=856.1857883404023: y=446.99974340006133650.0
NODE: label=Romero Rubio: x=896.00276758247: y
   =409.4949952355877650.0
NODE: label=Flores Magon: x=867.576285092002: y
   =397.18850897064794650.0
NODE: label=Eduardo Molina: x=803.3824889861783: y
   =448.3661175995914650.0
```

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NODE: label=Consulado: x = 766.1498229872799: y
   =443.6433435239503650.0
NODE: label=Bondojito: x = 766.286701799067: y = 476.317239866421650.0
NODE: label=Deportivo Oceania: x=964.3307797278712: y
   =455.31243973907965650.0
NODE: label=Bosque de Aragon: x=977.9464793780767: y
   =472.36642747282275650.0
NODE: label=Villa de Aragon: x=990.8547338633714: y
   =499.59755706749706650.0
NODE: label=Nezahualcototl:x=1000.3390370617108:y
   =516.5431117667663650.0
NODE: label=Impulsora: x = 997.5489547543752: y
   =532.5055236137931650.0
NODE: label=Rio de los Remedios: x=1000.5679026314724: y
   =547.6603793744362650.0
NODE: label=Muzquaz: x=1012.4373866530839: y
   =568.1026386905862650.0
NODE: label=Technologico:x=1018.1634064339185:y
   =584.8630748762263650.0
NODE: label=Olimpica: x=1026.7100981035865: y
   =600.8988873683857650.0
NODE: label=Plaza Aragon: x=1037.9690008190223: y
   =616.0281610671931650.0
NODE: label=Ciudad Azteca: x=1048.6013714190424: y
   =632.5728622370842650.0
NODE: label=Talisman: x = 786.1707146025242: y
   =529.7202357000715650.0
NODE: label=Martin Carrera: x=799.3689900204648: y
   =557.5666209631888650.0
NODE: label=La Villa-Basilica: x=726.794946699478: y
   =565.18047114417650.0
NODE: label=Deportivo 18 de Marzo: x=676.3000140909905: y
   =562.7214192418963650.0
NODE: label=Indios Verdes: x=685.0369006188668: y
   =602.005187664207650.0
NODE: label=Potrero: x=645.0759244923179: y=510.7858646186287650.0
NODE: label=Lindavista: x = 597.2037301692355: y
   =563.3466532922939650.0
NODE: label=Instituto del Petroleo: x=538.870567962673: y
   =549.3838747123286650.0
NODE: label=Politecnico: x=522.5316901468319: y
   =588.7265267272081650.0
NODE: label=Autobuses del Norte: x=557.1875881750041: y
   =505.815719457291650.0
NODE: label=La Raza: x = 572.6710104117305: y
   =469.11656371892883650.0
NODE: label=Valle Gomez: x=701.8599805101569: y
   =444.81092939823577650.0
```

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NODE: label=Misterios:x=620.7875947196756:y
   =452.7747399281598650.0
NODE: label=Canal del Norte: x=768.6474959324212: y
   =415.6103664793877650.0
NODE: label=Morelos: x = 769.081043677851: y = 396.76246552434424650.0
NODE: label=San Lazaro: x=814.2613867314793: y
   =377.2666308870394650.0
NODE: label=Tepito: x = 693.5537232703792: y = 401.56728382917987650.0
NODE: label=Lagunilla: x=636.8105998909869: y
   =405.4665739121127650.0
NODE: label=Garibaldi:x=591.822372666386:y
   =405.74683230284495650.0
NODE: label=Buenavista: x=497.8501431313742: y
   =435.03393195717206650.0
NODE: label=Cd. Deportiva: x = 883.0400211413211: y
   =282.83269659345365650.0
NODE: label=Velodromo: x=829.2218413264995: y
   =281.4982099721949650.0
NODE: label=Moctezuma: x=845.9103909220333: y
   =357.82039780462975650.0
NODE: label=Balbuena: x=867.6198362226827: y
   =341.53453955404643650.0
NODE: label=Zaragoza: x=945.0509970872313: y
   =327.97343967446295650.0
NODE: label=Gomez Farias: x = 913.5477040393869: y
   =325.91190884312874650.0
NODE: label=Bva. Pto. Aereo:x=889.9864815655966:y
   =332.8650479882195650.0
NODE: label=Candelaria: x = 767.1931856313887: y
   =371.7051559696891650.0
NODE: label=Fray Servando: x=761.4143222420631: y
   =334.0315697821942650.0
NODE: label=Constitucion de 1917: x=995.317124089652: y
   =103.50759487724679650.0
NODE: label=Cerro de la Estrella:x=934.5345267537435:y
   =84.26737865933819650.0
NODE: label=Ztlapalapa:x=896.3013587483638:y
   =92.9733942640031650.0
NODE: label=Uami: x = 962.4489157019518: y = 89.95242994758814650.0
NODE: label = Atlalilco : x = 844.9821996588755 : y
   =71.09925291026866650.0
NODE: label=Escuadron 201:x=817.1791392759446:y
   =93.30209490861102650.0
NODE: label = Aculco: x = 802.2789713858667: y = 128.58126096837123650.0
NODE: label=Apatlaco: x = 795.8640950191474: y
   =164.54774867553732650.0
NODE: label=Iztacalco: x=787.7733206872942: y
   =192.30247870791055650.0
```

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NODE: label=Mixhuca: x = 789.8854590135923: y = 271.6835155959798650.0
NODE: label=Jamaica: x = 755.099826984903: y = 278.25417483640365650.0
NODE: label = Coyuya : x = 778.3846713062158 : y = 224.17244477402062650.0
NODE: label=Santa Anita: x=743.2942348050627: y
   =244.7171345768217650.0
NODE: 1abel = Merced : x = 709.628699793079 : y = 356.76308505728025650.0
NODE: label=Zocalo: x = 663.8286900662175: y = 371.1162596233209650.0
NODE: label=Pino Suarez: x=667.1754622662816: y
   =349.4407369718181650.0
NODE: label=Allende: x=620.7857311906694: y
   =369.13331360426963650.0
NODE: label=Bellas Artes: x = 593.2251170916165: y
   =369.64404832005664650.0
NODE: label=Guerrero:x=546.3165250417469:v
   =404.51332581103617650.0
NODE: label=Tlatelolco: x = 554.371985778633: y
   =441.011505196711650.0
NODE: label=Hidalgo: x = 539.7957163078484: y
   =368.73487977086387650.0
NODE: label=Revolucion: x=518.0148416837726: y
   =370.1878647871303650.0
NODE: label=San Cosme: x = 500.27510457199486: y
   =376.3177768544568650.0
NODE: label=Normal: x=475.876086744038: y=388.19856087655234650.0
NODE: label=Colegio Militar: x=452.0647217099176: y
   =395.59284445512833650.0
NODE: label=Popotla: x=421.80428350307466: y
   =400.25656925213826650.0
NODE: label=San Juaquin: x = 351.17623172926346: y
   =369.15191452798604650.0
NODE: label=Polanco: x = 350.1598712868657: y = 330.788738465861650.0
NODE: label=Auditorio: x = 348.4935191761575: y
   =296.6589047433717650.0
NODE: label=Constituyentes: x=345.00989675046924: y
   =264.5319016691648650.0
NODE: label=Iacubata: x=362.27165066310636: y
   =234.2630601185437650.0
NODE: label=Observatorio:x=312.49682960498774:y
   =221.9559976086067650.0
NODE: label=San Pedro de los Pinos:x=386.3000633547856:y
   =202.9824460941909650.0
NODE: label=San Antonio:x=378.5614148764704:y
   =164.00463448704795650.0
NODE: label=Mixcoac: x = 374.54330902320294: v
   =117.61667399358976650.0
NODE: label=Barranca del Muerto: x=371.24575025609306: y
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```

```
NODE: label=Panteones: x = 294.1375166095565: y
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   =393.7659306549358650.0
NODE: label=Cuitlahuac:x=393.7459990347896:y
   =403.9780261244692650.0
NODE: label=Tacuba: x=356.1203908410537: y=431.4519811269679650.0
NODE: label=Vallejo: x=499.7944848023392: y=533.9620327558863650.0
NODE: label=Norte 49: x = 463.1077184560619: y = 549.443026727866650.0
NODE: label=Refineria:x=342.1713510927749:y
   =471.3770203365727650.0
NODE: label=Camarones: x = 290.55188126805604: y
   =520.3324091986826650.0
NODE: label=Aquiles Serdan: x=233.12755368763635: y
   =547.161494580002650.0
NODE: label=El Rosario: x = 226.67979018388434: y
   =595.6426804223457650.0
NODE: label=Yezozomoc: x=259.2724909545603: y
   =573.1813199311617650.0
NODE: label=Azcapotzalco:x=328.08898769817085:y
   =536.5832172860836650.0
NODE: |abe| = Ferreria : x = 397.082899157643 : y = 524.6010084257157650.0
NODE: label=Juarez: x=537.1363304844184: y=341.13523998073407650.0
NODE: label=Balderas: x=536.1895262493688: y
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NODE: label=Cuauhtemoc: x=505.17880903446894: y
   =306.4869065943534650.0
NODE: label=Juanacatlan: x=387.6205083556217: y
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NODE: label=Chapultepec: x=418.37585716013166: y
   =273.50414814684046650.0
NODE: label=Sevilla: x = 440.10370787897125: y
   =285.24590390586474650.0
NODE: label=Insurgentes: x = 477.0903725732559: y
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NODE: label=San Juan de Letran: x=593.6395399051573: y
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NODE: label=Salto del Agua: x=594.6873459929035: y
   =329.5881663159852650.0
NODE: label=Isabel la Catolica: x=630.4104238219322: y
   =335.45305521842295650.0
NODE: label=La Viga: x = 705.5634498860911: y = 257.1328964931833650.0
NODE: label = Chabacano : x = 669.25797058584 : y = 284.8940835264632650.0
NODE: label=Lazaro Cardenas: x=643.3103155898646: y
   =264.1057034307296650.0
NODE: label=Viaducto: x = 671.6794646798804: y
   =245.7358702841007650.0
```

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NODE: label=Villa de Cortes: x = 672.8004071347966: y
   =176.42760037240555650.0
NODE: label = Iavitas: x = 674.4701256654256: y = 144.9329215850629650.0
NODE: label=Ermita: x = 685.9422490688188: y = 82.87451060439832650.0
NODE: label=Portales: x = 678.6380122545044: y
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NODE: label=Universidad: x=555.6227268668898: y
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NODE: label=Copilco: x=517.5446840628206: y=47.44939640780285650.0
NODE: label=Miguel A de Quevedo: x=504.2832216209025: y
   =66.27853747378867650.0
NODE: label=Viveros: x=504.36684378218297: y
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NODE: label=Coyoacan: x = 505.7737474340197: y
   =108.09908735696342650.0
NODE: label=Patriotismo: x=420.49738751095197: y
   =238.28029987372554650.0
NODE: label=Chilpancingo: x=463.84803694469474: y
   =238.117746223256650.0
NODE: label=Centro Medico: x=538.0754246420652: y
   =244.13860755396337650.0
NODE: label = Obrera: x = 635.660624991185: y = 296.26724806361955650.0
NODE: label=Doctores: x = 612.4215364035742: y
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NODE: label=Ninos Heroes: x=536.2166732386614: y
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NODE: label=Hospital General: x=538.7943380318693: y
   =266.4177784518864650.0
NODE: label=Etiopia: x=535.917635924966: y=215.13388735905932650.0
NODE: label=Eugenia: x=533.9280961771702: y
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NODE: label=Division del Norte: x=516.8483092535726: y
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EDGE: label=1:color=255,59,255: adjlist=Balderas, Salto del Agua
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   Suarez
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   Farias
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EDGE: label=2:color=27,1,229: adjlist=Colegio Militar, Normal
EDGE: label=2:color=27,1,229: adjlist=Normal, San Cosme
EDGE: label=2: color=27,1,229: adjlist=San Cosme, Revolucion
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   Abad
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EDGE: label=3: color=131,158,6: adjlist=Copilco, Miguel A de
   Quevedo
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   Eugenia
```

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   General
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   Heroes
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   Candelaria
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   Autobuses del Norte
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   Raza
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 EDGE: label = 6: color = 255, 0, 0: adjlist = Yezozomoc, \quad Azcapotzalco \\
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EDGE: label=6: color = 255,0,0: adjlist=Norte 49, Vallejo
```

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EDGE: label=6: color=255,0,0: adjlist=Vallejo, Instituto del
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   Lindavista
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   Marzo
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   Villa-Basilica
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   Mixcoac
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EDGE: label=8:color=6,158,17: adjlist=Bellas Artes, San Juan de
   Letran
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   del Agua
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EDGE: label=8: color=6,158,17: adjlist=Obrera, Chabacano
EDGE: label=8:color=6,158,17:adjlist=Chabacano, La Viga
EDGE: label=8:color=6,158,17: adjlist=La Viga, Santa Anita
EDGE: label=8: color=6,158,17: adjlist=Santa Anita, Coyuya
EDGE: label=8: color=6,158,17: adjlist=Coyuya, Iztacalco
EDGE: label=8:color=6,158,17: adjlist=Iztacalco, Apatlaco
EDGE: label=8: color=6,158,17: adjlist=Apatlaco, Aculco
EDGE: label=8: color=6,158,17: adjlist=Aculco, Escuadron 201
EDGE: label=8: color=6,158,17: adjlist=Escuadron 201, Atlalilco
EDGE: label=8: color=6,158,17: adjlist=Atlalilco, Ztlapalapa
EDGE: label=8:color=6,158,17: adjlist=Ztlapalapa, Cerro de la
   Estrella
EDGE: label=8:color=6,158,17:adjlist=Cerro de la Estrella, Uami
EDGE: label=8: color=6,158,17: adjlist=Uami, Constitution de 1917
EDGE: label=9: color=106,66,10: adjlist=Iacubata, Patriotismo
```

```
EDGE: label=9: color=106,66,10: adjlist=Patriotismo, Chilpancingo
EDGE: label=9: color=106,66,10: adjlist=Chilpancingo, Centro
EDGE: label=9: color=106,66,10: adjlist=Centro Medico, Lazaro
   Cardenas
EDGE: label=9: color=106,66,10: adjlist=Lazaro Cardenas, Chabacano
EDGE: label=9: color=106,66,10: adjlist=Chabacano, Jamaica
EDGE: label=9:color=106,66,10:adjlist=Jamaica, Mixhuca
EDGE: label=9: color=106,66,10: adjlist=Mixhuca, Velodromo
EDGE: label=9: color=106,66,10: adjlist=Velodromo, Cd. Deportiva
EDGE: label=9: color=106,66,10: adjlist=Cd. Deportiva, Puebla
EDGE: label=9: color=106,66,10: adjlist=Puebla, Pantitlan
EDGE: label=A: color = 158,6,81: adjlist=Pantitlan, Agricola
   Oriental
EDGE: label=A: color = 158,6,81: adjlist=Agricola Oriental, De San
EDGE: label=A: color = 158,6,81: adjlist=De San Juan, Tepalcates
EDGE: label=A: color = 158,6,81: adjlist=Tepalcates, Guelatao
EDGE: label=A: color = 158,6,81: adjlist=Guelatao, Penon Viejo
EDGE: label=A: color = 158,6,81: adjlist=Penon Viejo, Acatitla
EDGE: label=A: color = 158,6,81: adjlist=Acatitla, Santa Marta
EDGE: label=A: color = 158,6,81: adjlist=Santa Marta, Los Reves
EDGE: label=A: color = 158,6,81: adjlist=Los Reyes, La Paz
EDGE: label=B: color = 200,200,200: adjlist=Buenavista, Guerrero
EDGE: label=B: color = 200,200,200: adjlist=Guerrero, Garibaldi
EDGE: label=B: color = 200,200,200; adjlist=Garibaldi, Lagunilla
EDGE: label=B: color = 200,200,200: adjlist=Lagunilla, Tepito
EDGE: label=B: color = 200,200,200: adjlist=Tepito, Morelos
EDGE: label=B: color = 200,200,200: adjlist=Morelos, San Lazaro
EDGE: label=B: color = 200,200,200: adjlist=San Lazaro, Flores Magon
EDGE: label=B: color = 200,200,200: adjlist=Flores Magon, Romero
EDGE: label=B: color = 200,200,200: adjlist=Romero Rubio, Oceania
EDGE: label=B: color = 200,200,200: adjlist=Oceania, Deportivo
EDGE: label=B: color = 200,200,200: adjlist=Deportivo Oceania,
   Bosque de Aragon
EDGE: label=B: color = 200,200,200: adjlist=Bosque de Aragon, Villa
   de Aragon
EDGE: label=B: color = 200,200,200: adjlist=Villa de Aragon,
   Nezahualcototl
EDGE: label=B: color = 200,200,200: adjlist=Nezahualcototl,
   Impulsora
EDGE: label=B: color = 200,200,200: adjlist=Impulsora, Rio de los
   Remedios
EDGE: label=B: color = 200,200,200: adjlist=Rio de los Remedios,
   Muzquaz
EDGE: label=B: color = 200,200,200: adjlist=Muzquaz, Technologico
```

 $\label=B: color=200\,,200\,,200: adjlist=Technologico\,,\quad Olimpica\\ EDGE: label=B: color=200\,,200\,,200: adjlist=Olimpica\,,\quad Plaza\quad Aragon\\ EDGE: label=B: color=200\,,200\,,200: adjlist=Plaza\quad Aragon\,,\quad Ciudad\quad Azteca$ 

#### B.7 Recife

```
NODE: label=Recife: x=1201.0644337180665: y=431.6944884725592715.0
NODE: label=Joana Bezerra: x=1135.9077461228494: y
   =410.45874565853364715.0
NODE: label=Largo Da Paz:x=1099.7128850411473:y
   =387.49878969858446715.0
NODE: label=Imbiribeira:x=1084.715070045847:y
   =344.84736795623326715.0
NODE: label=Boa Viagem: x=1091.838271925862: y
   =290.8563291108914715.0
NODE: label=Shopping:x=1086.8402682538417:y
   =250.46964425608172715.0
NODE: label=Tancredo Neves: x=1082.2739976291705: y
   =196.98018228614274715.0
NODE: label=Aeroporto: x = 1073.0332819117302: y
   =152.08367187454576715.0
NODE: label=Porta Larga: x=1060.8420362877125: y
   =117.52062179791744715.0
NODE: label=Monte Dos Guararpes: x=1028.352433423444: y
   =75.90633408124972715.0
NODE: label=Prazeres:x=983.0696591851895:v
   =42.891302243654536715.0
NODE: label=Cajueiro Seco: x = 945.9069868898079: y
   =9.33002148337971715.0
NODE: label = A fogadas : x = 1091.499674549303 : y
   =407.7895818241699715.0
NODE: label=Werneck: x=923.8783845210328: y
   =389.28471126622486715.0
NODE: label=Ipiranga: x=1059.3719861694362: y
   =405.6054874357932715.0
NODE: label=Santa Luzia: x = 972.2562314094442: y
   =394.62277233728537715.0
NODE: label=Mangeuira:x=1022.0775620267698:y
   =398.3939739549012715.0
NODE: label=Barro: x=869.5541634144201: y=385.5416453993967715.0
NODE: label=Tejipio: x = 785.998555970139: y = 364.23382066360034715.0
NODE: label=Coqueiral:x=721.8848031982609:y
   =362.64772465616414715.0
NODE: label=Alto Do C u:x=683.8844470722377: y
   =392.6426644274835715.0
NODE: label = Curado: x = 667.8353214211054: y = 450.8290238135853715.0
```

```
NODE: label=Rodovi ria : x=656.4145203444778 : y
   =505.135489117506715.0
NODE: label=Cavaleiro: x=683.699967753158: y
   =338.25259478495923715.0
NODE: label=Floriano:x=591.8788373675591:y
   =273.3332173663129715.0
NODE: label=Engenho Velho: x=541.2395964624005: y
   =264.6159032341998715.0
NODE: label=Jaboatao: x = 487.1167013355696: y
   =256.50345863537973715.0
NODE: label=Timbi: x=563.1661821388152: y=699.40619713756715.0
EDGE: label=Red Line: color = 255,0,0: adjlist=Recife, Joana Bezerra
EDGE: label=Red Line: color = 255,0,0: adjlist=Joana Bezerra,
   Afogadas
EDGE: label=Red Line: color = 255,0,0: adjlist=Afogadas, Ipiranga
EDGE: label=Red Line: color = 255,0,0: adjlist=Ipiranga, Mangeuira
EDGE: label=Red Line: color = 255,0,0: adjlist=Mangeuira, Santa
   Luzia
EDGE: label=Red Line: color = 255,0,0: adjlist=Santa Luzia, Werneck
EDGE: label=Red Line: color = 255,0,0: adjlist=Werneck, Barro
EDGE: label=Red Line: color = 255,0,0: adjlist=Barro, Tejipio
EDGE: label=Red Line: color = 255,0,0: adjlist=Tejipio, Coqueiral
EDGE: label=Red Line: color = 255,0,0: adjlist=Coqueiral, Cavaleiro
EDGE: label=Red Line: color = 255,0,0: adjlist=Cavaleiro, Floriano
EDGE: label=Red Line: color = 255,0,0: adjlist=Floriano, Engenho
   Velho
EDGE: label=Red Line: color = 255,0,0: adjlist=Engenho Velho,
   Jaboatao
EDGE: label=Red Line: color = 255,0,0: adjlist=Coqueiral, Alto Do
EDGE: label=Red Line: color = 255,0,0: adjlist=Alto Do C u , Curado
EDGE: label=Red Line: color = 255,0,0: adjlist=Curado, Rodovi ria
EDGE: label=Red Line: color = 255,0,0: adjlist=Rodovi ria, Timbi
EDGE: label=Blue Line: color = 0,0,255: adjlist=Recife, Joana
EDGE: label=Blue Line: color = 0,0,255: adjlist=Joana Bezerra, Largo
EDGE: label=Blue Line: color = 0,0,255: adjlist=Largo Da Paz,
   Imbiribeira
Viagem
EDGE: label=Blue Line: color = 0,0,255: adjlist=Boa Viagem, Shopping
EDGE: label=Blue Line: color = 0,0,255: adjlist=Shopping, Tancredo
   Neves
EDGE: label=Blue Line: color = 0,0,255: adjlist=Tancredo Neves,
   Aeroporto
EDGE: label=Blue Line: color = 0,0,255: adjlist=Aeroporto, Porta
   Larga
```

EDGE: label=Blue Line: color = 0,0,255: adjlist=Porta Larga, Monte Dos Guararpes

 $\begin{array}{ll} \texttt{EDGE: label=Blue Line: color=0\,,0\,,255: adjlist=Monte Dos Guararpes\,,} \\ \texttt{Prazeres} \end{array}$ 

EDGE: label=Blue Line: color = 0,0,255: adjlist=Prazeres , Cajueiro Seco

#### B.8 San Francisco

```
NODE: label=Oakland City Center/12th St: x=-122.2715: y=37.8031
NODE: label=19th St/Oakland: x = -122.2687: y = 37.8075
NODE: label=MacArthur: x = -122.2673: y = 37.8284
NODE: label=Ashby: x = -122.2698: y = 37.8530
NODE: label=Downtown Berkeley: x = -122.2680: y = 37.8699
NODE: label=North Berkeley: x = -122.2839: y = 37.8739
NODE: label=El Cerrito Plaza: x = -122.2992: y = 37.9028
NODE: label=El Cerrito del Norte: x = -122.3176: y = 37.9258
NODE: label=Richmond: x = -122.3538: y = 37.9373
NODE: label=Rockridge: x = -122.2527: y = 37.8442
NODE: label=Orinda: x = -122.1833: y = 37.8785
NODE: label=Lafayette: x = -122.1236: y = 37.8934
NODE: label=Walnut Creek: x = -122.0681: y = 37.9046
NODE: label=Pleansant Hill: x = -122.0567: y = 37.9276
NODE: label=Concord: x = -122.0297: y = 37.9721
NODE: label=Morth Concord/Martinez: x = -122.0251: y = 38.0027
NODE: label=Pittsburgh/Bay Point:x = -121.9418:y = 38.0190
NODE: label=Lake Merritt: x = -122.2655: y = 37.7975
NODE: label=West Oakland: x = -122.2944: y = 37.8047
NODE: label=Embarcadero: x = -122.3970: y = 37.7929
NODE: label=Fruitvale: x = -122.2241: y = 37.7746
NODE: label=Fremont: x = -121.9763: y = 37.5572
NODE: label=Dublin/Pleasanton: x = -121.9004: y = 37.7015
NODE: label=Union City: x = -122.0178: y = 37.5913
NODE: label=South Hayward: x = -122.0578: y = 37.6350
NODE: label=Hayward: x = -122.0878: y = 37.6705
NODE: label=Bay Fair: x = -122.1278: y = 37.6978
NODE: label=San Leandro: x = -122.1613: y = 37.7228
NODE: label=Coliseum/Oakland Airport: x = -122.1980: y = 37.7543
NODE: label=Castro Valley: x = -122.0776: y = 37.6909
NODE: label=Montgomery St: x = -122.4020: y = 37.7892
NODE: label=Powell St: x = -122.4068: y = 37.7850
NODE: label=Civic Center: x = -122.4136: y = 37.7795
NODE: label=16th St Mission: x = -122.4197: y = 37.7650
NODE: label = 24th St Mission: x = -122.4183: y = 37.7523
NODE: label=Millbrae: x = -122.3868: y = 37.6002
NODE: label=San Francisco International Airport (SFO):x
    =-122.3925: y=37.6157
NODE: label=San Bruno: x = -122.4157: y = 37.6371
```

City, South Hayward

Hayward, Hayward

```
NODE: label=Glen Park: x = -122.4339: y = 37.7329
NODE: label=Balboa Park: x = -122.4475: y = 37.7217
NODE: label=Daly City: x = -122.4690: y = 37.7062
NODE: label=Colma: x = -122.4676: y = 37.6847
NODE: label=South San Francisco: x = -122.4439: y = 37.6642
EDGE: label=BART Richmond-Daly City: color = 255,0,0: adjlist=
   Richmond, El Cerrito del Norte
EDGE: label=BART Richmond-Daly City: color = 255,0,0: adjlist=El
   Cerrito del Norte, El Cerrito Plaza
EDGE: label=BART Richmond-Daly City: color = 255,0,0: adjlist=El
   Cerrito Plaza, North Berkeley
EDGE: label=BART Richmond-Daly City: color = 255,0,0: adjlist=North
   Berkeley, Downtown Berkeley
EDGE: label=BART Richmond-Daly City: color = 255,0,0: adjlist=
   Downtown Berkeley, Ashby
EDGE: label=BART Richmond-Daly City: color = 255,0,0: adjlist=Ashby,
    MacArthur
EDGE: label=BART Richmond-Daly City: color = 255,0,0: adjlist=
   MacArthur, 19th St/Oakland
EDGE: label=BART Richmond-Daly City: color = 255,0,0: adjlist=19th
   St/Oakland, Oakland City Center/12th St
EDGE: label=BART Richmond-Daly City: color = 255,0,0: adjlist=
   Oakland City Center/12th St, West Oakland
EDGE: label=BART Richmond-Daly City: color = 255,0,0: adjlist=West
   Oakland, Embarcadero
EDGE: label=BART Richmond-Daly City: color = 255,0,0: adjlist=
   Embarcadero, Montgomery St
EDGE: label=BART Richmond-Daly City: color = 255,0,0: adjlist=
   Montgomery St, Powell St
EDGE: label=BART Richmond-Daly City: color = 255,0,0: adjlist=Powell
    St, Civic Center
EDGE: label=BART Richmond-Daly City: color = 255,0,0: adjlist=Civic
   Center, 16th St Mission
EDGE: label=BART Richmond-Daly City: color = 255,0,0: adjlist=16th
   St Mission, 24th St Mission
EDGE: label=BART Richmond-Daly City: color = 255,0,0: adjlist = 24th
   St Mission, Glen Park
EDGE: label=BART Richmond-Daly City: color = 255,0,0: adjlist=Glen
   Park, Balboa Park
EDGE: label=BART Richmond-Daly City: color = 255,0,0: adjlist=Balboa
    Park, Daly City
EDGE: label=BART Fremont-Daly City: color = 0,123,49: adjlist=
   Fremont, Union City
EDGE: label=BART Fremont-Daly City: color = 0,123,49: adjlist=Union
```

EDGE: label=BART Fremont-Daly City: color = 0,123,49: adjlist=South

```
EDGE: label=BART Fremont-Daly City: color = 0,123,49: adjlist= Hayward, Bay Fair
```

EDGE: label=BART Fremont-Daly City: color =0,123,49: adjlist=Bay Fair, San Leandro

EDGE: label=BART Fremont-Daly City: color = 0,123,49: adjlist=San Leandro, Coliseum/Oakland Airport

EDGE: label=BART Fremont-Daly City: color = 0,123,49: adjlist = Coliseum/Oakland Airport, Fruitvale

EDGE: label=BART Fremont-Daly City: color =0,123,49: adjlist=Fruitvale, Lake Merritt

EDGE: label=BART Fremont-Daly City: color =0,123,49: adjlist=Lake Merritt, West Oakland

EDGE: label=BART Fremont-Daly City: color =0,123,49: adjlist=West Oakland, Embarcadero

 $\begin{array}{ll} \text{EDGE: label=} \text{BART Fremont-Daly City: color} = 0\,, 123\,, 49 \colon \text{adjlist} = \\ \text{Embarcadero}\,\,, & \text{Montgomery St} \end{array}$ 

EDGE: label=BART Fremont-Daly City: color = 0,123,49: adjlist=Powell St, Civic Center

EDGE: label=BART Fremont-Daly City: color =0,123,49: adjlist=Civic Center, 16th St Mission

EDGE: label=BART Fremont-Daly City: color =0,123,49: adjlist=16th St Mission, 24th St Mission

EDGE: label=BART Fremont-Daly City: color =0,123,49: adjlist=24th St Mission, Glen Park

EDGE: label=BART Fremont-Daly City: color =0,123,49: adjlist=Glen Park, Balboa Park

EDGE: label=BART Fremont-Daly City: color =0,123,49: adjlist=Balboa Park, Daly City

EDGE: label=BART Richmond-Fremont: color = 255,200,0: adjlist = Richmond, El Cerrito del Norte

EDGE: label=BART Richmond-Fremont: color = 255,200,0: adjlist=El Cerrito del Norte, El Cerrito Plaza

EDGE: label=BART Richmond-Fremont: color = 255,200,0: adjlist=El Cerrito Plaza, North Berkeley

EDGE: label=BART Richmond-Fremont: color = 255,200,0: adjlist=North Berkeley, Downtown Berkeley

EDGE: label=BART Richmond-Fremont: color = 255,200,0: adjlist = Downtown Berkeley, Ashby

 $\begin{tabular}{ll} EDGE: label = BART & Richmond - Fremont: color = 255, 200, 0: adjlist = Ashby, \\ & MacArthur \end{tabular}$ 

EDGE: label=BART Richmond-Fremont: color = 255,200,0: adjlist=MacArthur, 19th St/Oakland

 $\begin{array}{l} \texttt{EDGE: label=BART \ Richmond-Fremont: color=255,200,0: adjlist=19th \\ \texttt{St/Oakland, Oakland \ City \ Center/12th \ St} \end{array}$ 

EDGE: label=BART Richmond-Fremont: color = 255,200,0: adjlist = Oakland City Center/12th St, Lake Merritt

- EDGE: label=BART Richmond-Fremont: color = 255,200,0: adjlist=Lake Merritt, Fruitvale
- EDGE: label=BART Richmond-Fremont: color = 255,200,0: adjlist = Fruitvale, Coliseum/Oakland Airport
- EDGE: label=BART Richmond-Fremont: color = 255,200,0: adjlist = Coliseum/Oakland Airport, San Leandro
- EDGE: label=BART Richmond-Fremont: color = 255,200,0: adjlist=San Leandro, Bay Fair
- $\begin{array}{l} \texttt{EDGE: label=\!BART\ Richmond-Fremont: color=255,200,0: adjlist=\!Bay \\ Fair\ ,\ Hayward } \end{array}$
- $\label = BART \ Richmond Fremont: color = 255, 200, 0: adjlist = Hayward, \ South \ Hayward$
- $\label = BART \ Richmond Fremont: color = 255, 200, 0: adjlist = South \ Hayward, \ Union \ City$
- EDGE: label=BART Richmond-Fremont: color = 255,200,0: adjlist=Union City, Fremont
- EDGE: label=BART Pittsburgh/Bay Point-Millbrae: color = 255,255,0: adjlist=Pittsburgh/Bay Point, Morth Concord/Martinez
- EDGE: label=BART Pittsburgh/Bay Point-Millbrae: color = 255,255,0: adjlist=Morth Concord/Martinez, Concord
- EDGE: label=BART Pittsburgh/Bay Point-Millbrae: color = 255,255,0: adjlist=Concord, Pleansant Hill
- EDGE: label=BART Pittsburgh/Bay Point-Millbrae: color = 255,255,0: adjlist=Pleansant Hill, Walnut Creek
- EDGE: label=BART Pittsburgh/Bay Point-Millbrae: color = 255,255,0: adjlist=Walnut Creek, Lafayette
- EDGE: label=BART Pittsburgh/Bay Point-Millbrae: color = 255,255,0: adjlist=Lafayette, Orinda
- EDGE: label=BART Pittsburgh/Bay Point-Millbrae: color = 255,255,0: adjlist=Orinda, Rockridge
- EDGE: label=BART Pittsburgh/Bay Point-Millbrae: color = 255,255,0: adjlist=Rockridge, MacArthur
- EDGE: label=BART Pittsburgh/Bay Point-Millbrae: color = 255,255,0: adjlist=MacArthur, 19th St/Oakland
- $\begin{array}{lll} \text{EDGE: label=BART & Pittsburgh/Bay & Point-Millbrae: color = $255,255,0$:} \\ \text{adjlist=19th & St/Oakland, & Oakland & City & Center/12th & St} \end{array}$
- EDGE: label=BART Pittsburgh/Bay Point-Millbrae: color = 255,255,0: adjlist=Oakland City Center/12th St, West Oakland
- EDGE: label=BART Pittsburgh/Bay Point-Millbrae: color = 255,255,0: adjlist=West Oakland, Embarcadero
- EDGE: label=BART Pittsburgh/Bay Point-Millbrae: color = 255,255,0: adjlist=Embarcadero, Montgomery St
- $\begin{array}{ll} \text{EDGE: label=} \text{BART Pittsburgh/Bay Point-Millbrae: color} = 255, 255, 0: \\ \text{adjlist=} \text{Montgomery St, Powell St} \\ \end{array}$
- $\begin{array}{ll} \text{EDGE: label=BART Pittsburgh/Bay Point-Millbrae: color = 255,255,0:} \\ \text{adjlist=Powell St, Civic Center} \end{array}$
- EDGE: label=BART Pittsburgh/Bay Point-Millbrae: color = 255,255,0: adjlist=Civic Center, 16th St Mission

```
EDGE: label=BART Pittsburgh/Bay Point-Millbrae: color = 255,255,0:
   adjlist=16th St Mission, 24th St Mission
EDGE: label=BART Pittsburgh/Bay Point-Millbrae: color = 255,255,0:
   adjlist=24th St Mission, Glen Park
EDGE: label=BART Pittsburgh/Bay Point-Millbrae: color = 255,255,0:
   adjlist=Glen Park, Balboa Park
EDGE: label=BART Pittsburgh/Bay Point-Millbrae: color = 255,255,0:
   adjlist=Balboa Park, Daly City
EDGE: label=BART Pittsburgh/Bay Point-Millbrae: color = 255,255,0:
   adilist=Daly City, Colma
EDGE: label=BART Pittsburgh/Bay Point-Millbrae: color = 255,255,0:
   adjlist=Colma, South San Francisco
EDGE: label=BART Pittsburgh/Bay Point-Millbrae: color = 255,255,0:
   adjlist=South San Francisco, San Bruno
EDGE: label=BART Pittsburgh/Bay Point-Millbrae: color = 255,255,0:
   adjlist=San Bruno, Millbrae
EDGE: label=BART Dublin/Pleasanton-SF Airport (SFO): color
   =0,56,255: adjlist=Dublin/Pleasanton, Castro Valley
EDGE: label=BART Dublin/Pleasanton-SF Airport (SFO): color
   =0,56,255: adjlist=Castro Valley, Bay Fair
EDGE: label=BART Dublin/Pleasanton-SF Airport (SFO): color
   =0,56,255: adjlist=Bay Fair, San Leandro
EDGE: label=BART Dublin/Pleasanton-SF Airport (SFO): color
   =0,56,255: adjlist=San Leandro, Coliseum/Oakland Airport
EDGE: label=BART Dublin/Pleasanton-SF Airport (SFO): color
   =0.56.255: adjlist=Coliseum/Oakland Airport, Fruitvale
EDGE: label=BART Dublin/Pleasanton-SF Airport (SFO): color
   =0,56,255: adjlist=Fruitvale, Lake Merritt
EDGE: label=BART Dublin/Pleasanton-SF Airport (SFO): color
   =0,56,255: adjlist=Lake Merritt, West Oakland
EDGE: label=BART Dublin/Pleasanton-SF Airport (SFO): color
   =0,56,255: adjlist=West Oakland, Embarcadero
\verb|EDGE: label= BART Dublin/Pleasanton-SF Airport (SFO): color \\
   =0,56,255: adjlist=Embarcadero, Montgomery St
EDGE: label=BART Dublin/Pleasanton-SF Airport (SFO): color
   =0,56,255: adjlist=Montgomery St, Powell St
EDGE: label=BART Dublin/Pleasanton-SF Airport (SFO): color
   =0,56,255: adjlist=Powell St, Civic Center
EDGE: label=BART Dublin/Pleasanton-SF Airport (SFO): color
   =0,56,255: adjlist=Civic Center, 16th St Mission
EDGE: label=BART Dublin/Pleasanton-SF Airport (SFO): color
   =0,56,255: adjlist=16th St Mission, 24th St Mission
EDGE: label=BART Dublin/Pleasanton-SF Airport (SFO): color
   =0,56,255: adjlist=24th St Mission, Glen Park
EDGE: label=BART Dublin/Pleasanton-SF Airport (SFO): color
   =0,56,255: adjlist=Glen Park, Balboa Park
EDGE: label=BART Dublin/Pleasanton-SF Airport (SFO): color
   =0,56,255: adjlist=Balboa Park, Daly City
```

#### B.9 Stockholm

```
NODE: label=Norsborg: x = 17.8145: y = 59.2438
NODE: label=Slussen: x=18.0724: y=59.3195
NODE: label=M\" orby Centrum: x=18.0361: y=59.3987
NODE: label=Mariatorget: x=18.0633: y=59.3169
NODE: label=Zinkensdamm: x = 18.0502: y = 59.3178
NODE: label=Hornstull: x=18.0341: y=59.3158
NODE: label=Liljeholmen: x=18.0229: y=59.3107
NODE: label=Medborgarplatsen: x=18.0735: y=59.3143
NODE: label=Skanstull: x=18.0762: y=59.3079
NODE: label=Gullmarsplan: x=18.0805: y=59.2990
NODE: label=Gamla Stan: x=18.0671: y=59.3233
NODE: label=T-Centralen: x=18.0593: y=59.3310
NODE: label=Kungst\" adgarden: x=18.0735: y=59.3307
NODE: label=H\"ortorget: x=18.0637: y=59.3354
NODE: label=\"Ostermalmstorg: x=18.0741: y=59.3350
NODE: label=Odenplan: x=18.0497: y=59.3430
NODE: label=Sank Eriksplan: x=18.0365: y=59.3398
NODE: label=Fridhemsplan : x = 18.0324 : y = 59.3343
NODE: label=Radhuset: x=18.0421: y=59.3303
NODE: label=Thorildsplan: x=18.0155: y=59.3319
NODE: label=Kristineberg: x=18.0035: y=59.3329
NODE: label=Alvik: x=17.9801: y=59.3337
NODE: label=Stora Mossen: x=17.9662: y=59.3345
NODE: label=Stadshagen: x=18.0175: y=59.3370
NODE: label=V\" astra Skogen: x=18.0041: y=59.3475
NODE: label=Huvudsta: x = 17.9857: y = 59.3495
NODE: label=Vreten: x=17.9740: y=59.3542
NODE: label=Sundbybergs Centrum: x=17.9722: y=59.3609
NODE: label=Duvbo: x = 17.9647: y = 59.3678
NODE: label=Rissne: x=17.9400: y=59.3758
NODE: label=Brommaplan: x = 17.9391: y = 59.3383
NODE: label=Abrahamsberg: x=17.9528: y=59.3366
NODE: label = \ \ Akeshov : x = 17.9249 : y = 59.3421
NODE: label = \ ''Angbyplan : x = 17.9070 : y = 59.3418
```

```
NODE: label=Islandstorget: x=17.8939: y=59.3459
NODE: label=Blackeberg: x=17.8827: y=59.3482
NODE: label=Racksta: x=17.8818: y=59.3547
NODE: label=V\" alling by: x=17.8721: y=59.3633
NODE: label=Johannelund: x = 17.8575: y = 59.3679
NODE: label=H\" asselby Gard: x = 17.8438: y = 59.3669
NODE: label=H\" asselby Strand: x=17.8326: y=59.3612
NODE: label=Solna Centrum: x = 17.9989: y = 59.3588
NODE: label=N\" ackrosen: x=17.9834: y=59.3667
NODE: label=Hallonbergen: x=17.9693: y=59.3754
NODE: label=Kista: x=17.9424: y=59.4030
NODE: label=Husby: x = 17.9255: y = 59.4102
NODE: label=Akalla: x=17.9128: y=59.4148
NODE: label=Stadion: x = 18.0817: y = 59.3430
NODE: label=Tekniska H\" ogskolan: x=18.0718: y=59.3458
NODE: label=Universitet : x = 18.0550 : y = 59.3655
NODE: label=Bergshamra: x = 18.0367: y = 59.3815
NODE: label=Danderyds Sjukhus: x=18.0414: y=59.3918
NODE: label=Karlaplan: x=18.0908: y=59.3388
NODE: label=G\" ardet: x=18.0996: y=59.3466
NODE: label=Ropsten: x = 18.1023: y = 59.3573
NODE: label=Blasut: x=18.0915: y=59.2904
NODE: label=Midsommarkransen: x=18.0120: y=59.3018
NODE: label=Enskede Gard: x = 18.0703: y = 59.2894
NODE: label=Skarmarbrink: x=18.0905: y=59.2954
NODE: label=Fru angen: x=17.9650: y=59.2859
NODE: label=V astertorp: x = 17.9667: v = 59.2914
NODE: label=Telefonplan: x=17.9972: y=59.2983
NODE: label=H\" agerstens\" asen: x=17.9792: y=59.2956
NODE: label=Aspudden: x=18.0014: y=59.3064
NODE: label=Satra: x=17.9214: y=59.2850
NODE: label=Bred\" ang: x = 17.9338: y = 59.2948
NODE: label=M\" alarh\" olden: x=17.9573: y=59.3010
NODE: label = \ "Ornsberg: x = 17.9892: y = 59.3055
NODE: label=Axelsberg: x=17.9755: y=59.3044
NODE: label=Sk arholmen: x=17.9071: y=59.2771
NODE: label=Varberg: x=17.8902: y=59.2759
NODE: label=Varby Gard: x=17.8845: y=59.2646
NODE: label=Masmo: x=17.8802: y=59.2497
NODE: label=Fittja: x=17.8610: y=59.2475
NODE: label=Hallunda: x=17.8255: y=59.2433
NODE: label=Alby: x=17.8457: y=59.2395
NODE: label=Radmansgatan: x = 18.0588: y = 59.3406
NODE: label=Rinkeby: x=17.9288: y=59.3881
NODE: label=Tensta: x=17.9012: y=59.3944
NODE: label=Hjulsta: x=17.8886: y=59.3968
NODE: label=Ragsved: x = 18.0283: y = 59.2566
NODE: label=H\" ogdalen: x=18.0429: y=59.2638
```

```
NODE: label=Bandhagen: x = 18.0495: y = 59.2704
NODE: label=Globen: x=18.0779: y=59.2942
NODE: label=Sockenplan: x=18.0701: y=59.2835
NODE: label=Stureby: x = 18.0556: y = 59.2746
NODE: label=Svedmyra: x = 18.0673: y = 59.2777
NODE: label=Hags\" atra: x = 18.0124: y = 59.2628
NODE: label=Sandsborg: x=18.0923: y=59.2848
NODE: label=Hammarbyh\" ojden: x = 18.1045: y = 59.2947
NODE: label=Bj\" orkhagen: x=18.1156: y=59.2912
NODE: label=K\" arrtorp: x = 18.1144: y = 59.2845
NODE: label=Bargarmossen: x = 18.1315: y = 59.2762
NODE: label=Skarpn \ "ack: x=18.1334: y=59.2668
NODE: label=Skogskyrkogarden: x=18.0955: y=59.2792
NODE: label=Farsta: x=18.0932: y=59.2435
NODE: label=Farsta Strand: x=18.1019: y=59.2350
NODE: label=H\" okar\" angen: x=18.0826: y=59.2579
NODE: label=Gubb\" angen: x = 18.0820: y = 59.2629
NODE: label=Tallkrogen:x=18.0853:y=59.2711
EDGE: label=Red: color = 234,80,0: adjlist=M\" orby Centrum,
   Danderyds Sjukhus
EDGE: label=Red: color = 234,80,0: adjlist=Danderyds Sjukhus,
   Bergshamra
EDGE: label=Red: color = 234,80,0: adjlist=Bergshamra, Universitet
EDGE: label=Red: color = 234,80,0: adjlist=Universitet, Tekniska H\"
   ogskolan
EDGE: label=Red: color = 234,80,0: adjlist=Tekniska H\" ogskolan,
EDGE: label=Red: color = 234,80,0: adjlist=Stadion, \"Ostermalmstorg
EDGE: label=Red: color = 234,80,0: adjlist =\"Ostermalmstorg, T-
    Centralen
EDGE: label=Red: color = 234,80,0: adjlist=T-Centralen, Gamla Stan
EDGE: label=Red: color = 234,80,0: adjlist=Gamla Stan, Slussen
EDGE: label=Red: color = 234,80,0: adjlist=Slussen, Mariatorget
EDGE: label=Red: color = 234,80,0: adjlist=Mariatorget, Zinkensdamm
EDGE: label=Red: color = 234,80,0: adjlist=Zinkensdamm, Hornstull
EDGE: label=Red: color = 234,80,0: adjlist=Hornstull, Liljeholmen
EDGE: label=Red: color = 234,80,0: adjlist=Liljeholmen, Aspudden
EDGE: label=Red: color = 234,80,0: adjlist=Aspudden, \"Ornsberg
EDGE: label=Red: color = 234,80,0: adjlist = "Ornsberg, Axelsberg"
EDGE: label=Red: color = 234,80,0: adjlist=Axelsberg, M\" alarh\"
   olden
EDGE: label=Red: color = 234,80,0: adjlist=M\" alarh\" olden, Bred\"
EDGE: label=Red: color = 234,80,0: adjlist=Bred\" ang, Satra
EDGE: label=Red: color = 234,80,0: adjlist=Satra, Sk\" arholmen
EDGE: label=Red: color = 234,80,0: adjlist=Sk\" arholmen, Varberg
EDGE: label=Red: color = 234,80,0: adjlist=Varberg, Varby Gard
EDGE: label=Red: color = 234,80,0: adjlist=Varby Gard, Masmo
```

```
EDGE: label=Red: color = 234,80,0: adjlist=Masmo, Fittja
EDGE: label=Red: color = 234,80,0: adjlist=Fittja, Alby
EDGE: label=Red: color = 234,80,0: adjlist=Alby, Hallunda
EDGE: label=Red: color = 234,80,0: adjlist=Hallunda, Norsborg
EDGE: label=Red: color = 234,80,0: adjlist=Liljeholmen,
   Midsommarkransen
EDGE: label=Red: color = 234,80,0: adjlist=Midsommarkransen,
   Telefonplan
EDGE: label=Red: color = 234,80,0: adjlist=Telefonplan, H\" agerstens
EDGE: label=Red: color = 234,80,0: adjlist=H\" agerstens\" asen, V\"
   astertorp
EDGE: label=Red: color = 234,80,0: adjlist=V\" astertorp, Fru\" angen
EDGE: label=Red: color = 234,80,0: adjlist=Ropsten, G\" ardet
EDGE: label=Red: color = 234,80,0: adjlist=G\" ardet, Karlaplan
EDGE: label=Red: color = 234,80,0: adjlist=Karlaplan, \"
   Ostermalmstorg
EDGE: label=Blue: color = 32,42,132: adjlist=Akalla, Husby
EDGE: label=Blue: color = 32,42,132: adjlist=Husby, Kista
EDGE: label=Blue: color = 32,42,132: adjlist=Kista, Hallonbergen
EDGE: label=Blue: color = 32,42,132: adjlist=Hallonbergen, N\"
EDGE: label=Blue: color = 32,42,132: adjlist=N\" ackrosen, Solna
   Centrum
EDGE: label=Blue: color = 32,42,132: adjlist=Solna Centrum, V\" astra
EDGE: label=Blue: color = 32,42,132: adjlist=V\" astra Skogen,
   Stadshagen
EDGE: label=Blue: color = 32,42,132: adjlist=Stadshagen,
   Fridhemsplan
EDGE: label=Blue: color = 32,42,132: adjlist=Fridhemsplan, Radhuset
EDGE: label=Blue: color = 32,42,132: adjlist=Radhuset, T-Centralen
EDGE: label=Blue: color = 32,42,132: adjlist=T-Centralen, Kungst\"
   adgarden
EDGE: label=Blue: color = 32,42,132: adjlist=Hjulsta, Tensta
EDGE: label=Blue: color = 32,42,132: adjlist=Tensta, Rinkeby
EDGE: label=Blue: color = 32,42,132: adjlist=Rinkeby, Rissne
EDGE: label=Blue: color = 32,42,132: adjlist=Rissne, Duvbo
EDGE: label=Blue: color = 32,42,132: adjlist=Duvbo, Sundbybergs
   Centrum
EDGE: label=Blue: color = 32,42,132: adjlist=Sundbybergs Centrum,
EDGE: label=Blue: color = 32,42,132: adjlist=Vreten, Huvudsta
EDGE: label=Blue: color = 32,42,132: adjlist=Huvudsta, V\" astra
EDGE: label=Green: color = 133,186,13: adjlist=H\" asselby Strand, H
   \"asselby Gard
```

```
EDGE: label=Green: color = 133,186,13: adjlist=H\" asselby Gard,
   Johannelund
EDGE: label=Green: color = 133,186,13: adjlist=Johannelund, V\"
    allingby
EDGE: label=Green: color = 133,186,13: adjlist=V\" allingby, Racksta
EDGE: label=Green: color = 133,186,13: adjlist=Racksta, Blackeberg
EDGE: label=Green: color = 133,186,13: adjlist=Blackeberg,
    Islandstorget
EDGE: label=Green: color = 133,186,13: adjlist=Islandstorget, \"
   Angbyplan
EDGE: label=Green: color = 133,186,13: adjlist = \"Angbyplan, \"
   Akeshov
EDGE: label=Green: color = 133,186,13: adjlist = \"Akeshov, Brommaplan
EDGE: label=Green: color = 133,186,13: adjlist=Brommaplan,
   Abrahamsberg
EDGE: label=Green: color = 133,186,13: adjlist=Abrahamsberg, Stora
EDGE: label=Green: color = 133,186,13: adjlist=Stora Mossen, Alvik
EDGE: label=Green: color = 133,186,13: adjlist=Alvik, Kristineberg
EDGE: label=Green: color = 133,186,13: adjlist=Kristineberg,
    Thorildsplan
EDGE: label=Green: color = 133,186,13: adjlist=Thorildsplan,
   Fridhemsplan
EDGE: label=Green: color = 133,186,13: adjlist=Fridhemsplan, Sank
   Eriksplan
EDGE: label=Green: color = 133,186,13: adjlist=Sank Eriksplan,
   Odenplan
EDGE: label=Green: color = 133,186,13: adjlist=Odenplan,
   Radmansgatan
EDGE: label=Green: color = 133,186,13: adjlist=Radmansgatan, H\"
   ortorget
EDGE: label=Green: color = 133,186,13: adjlist=H\" ortorget, T-
   Centralen
EDGE: label=Green: color = 133,186,13: adjlist=T-Centralen, Gamla
EDGE: label=Green: color = 133,186,13: adjlist=Gamla Stan, Slussen
EDGE: label=Green: color = 133,186,13: adjlist=Slussen,
   Medborgarplatsen
EDGE: label=Green: color = 133,186,13: adjlist=Medborgarplatsen,
    Skanstull
EDGE: label=Green: color = 133,186,13: adjlist=Skanstull,
   Gullmarsplan
EDGE: label=Green: color = 133,186,13: adjlist=Gullmarsplan, Globen
EDGE: label=Green: color = 133,186,13: adjlist=Globen, Enskede Gard
EDGE: label=Green: color = 133,186,13: adjlist=Enskede Gard,
   Sockenplan
EDGE: label=Green: color = 133,186,13: adjlist=Sockenplan, Svedmyra
EDGE: label=Green: color = 133,186,13: adjlist=Svedmyra, Stureby
```

```
EDGE: label=Green: color = 133,186,13: adjlist=Stureby, Bandhagen
EDGE: label=Green: color = 133,186,13: adjlist=Bandhagen, H\" ogdalen
EDGE: label=Green: color = 133,186,13: adjlist=H\" ogdalen, Ragsved
EDGE: label=Green: color = 133,186,13: adjlist=Ragsved, Hags\" atra
EDGE: label=Green: color = 133,186,13: adjlist=Gullmarsplan,
    Skarmarbrink
EDGE: label=Green: color = 133,186,13: adjlist=Skarmarbrink, Blasut
EDGE: label=Green: color = 133,186,13: adjlist=Blasut, Sandsborg
EDGE: label=Green: color = 133,186,13: adjlist=Sandsborg,
    Skogskyrkogarden
EDGE: label=Green: color = 133,186,13: adjlist=Skogskyrkogarden,
    Tallkrogen
EDGE: label=Green: color = 133,186,13: adjlist=Tallkrogen, Gubb\"
    angen
\label{eq:color} \begin{tabular}{l} EDGE: label=Green: color = 133,186,13: adjlist=Gubb\" angen, H\" okar \end{tabular}
    \"angen
EDGE: label=Green: color = 133,186,13: adjlist=H\" okar\" angen,
    Farsta
EDGE: label=Green: color = 133,186,13: adjlist=Farsta, Farsta Strand
EDGE: label=Green: color = 133,186,13: adjlist=Skarmarbrink,
   Hammarbyh\" ojden
EDGE: label=Green: color = 133,186,13: adjlist=Hammarbyh\"ojden, Bj
    \"orkhagen
EDGE: label=Green: color = 133,186,13: adjlist=Bj\"orkhagen, K\"
    arrtorp
EDGE: label=Green: color = 133,186,13: adjlist=K\" arrtorp,
    Bargarmossen
\label{eq:edge} \footnotesize \textbf{EDGE: label=Green: color=133,186,13: adjlist=Bargarmossen}, \;\; Skarpn
   \"ack
```

## B.10 Sydney

```
NODE: label=Berowra: x = 151.1526: y = -33.6238
NODE: label=Mount Kuring-gai: x = 151.1367: y = -33.6533
NODE: label=Mount Colah: x = 151.1150: y = -33.6717
NODE: label=Asquith: x = 151.1081: y = -33.6886
NODE: label=Hornsby: x = 151.0984: y = -33.7027
NODE: label=Waitara: x = 151.1045: y = -33.7100
NODE: label=Wahroonga: x = 151.1170: y = -33.7177
NODE: label=Warrawee: x = 151.1217: y = -33.7243
NODE: label=Turramurra: x = 151.1288: y = -33.7324
NODE: label=Pymble: x = 151.1422: y = -33.7449
NODE: label=Gordon: x = 151.1543: y = -33.7560
NODE: label=Killara: x=151.1619: y=-33.7656
NODE: label=Lindfield: x=151.1692: y=-33.7752
NODE: label=Roseville: x=151.1765: y=-33.7837
NODE: label=Chatswood: x = 151.1810: y = -33.7974
NODE: label=Artarmon: x = 151.1857: y = -33.8091
```

```
NODE: label=St Leonards: x=151.1944: y=-33.8233
NODE: label=Wollstonecraft: x = 151.1918: y = -33.8319
NODE: label=Waverton: x = 151.1977: y = -33.8379
NODE: label=North Sydney: x = 151.2073: y = -33.8411
NODE: label=Milsons Point: x = 151.2119: y = -33.8461
NODE: label=Wynyard: x = 151.2056: y = -33.8660
NODE: label=Circular Quay: x = 151.2110: y = -33.8613
NODE: label=Martin Place: x = 151.2118: y = -33.8679
NODE: label=St James: x=151.2121: y=-33.8702
NODE: label=Kings Cross: x=151.2225: y=-33.8749
NODE: label = Edgecliff : x = 151.2371 : y = -33.8799
NODE: label=Bondi Junction: x = 151.2474: y = -33.8910
NODE: label=Museum: x = 151.2098: y = -33.8763
NODE: label=Town Hall: x=151.2069: y=-33.8739
NODE: label=Central: x = 151.2064: y = -33.8832
NODE: label=Redfern: x = 151.1988: y = -33.8919
NODE: label=Macdonaldtown: x = 151.1863: y = -33.8967
NODE: label=Newtown: x = 151.1796: y = -33.8977
NODE: label=Stanmore: x = 151.1639: y = -33.8928
NODE: label=Petersham : x = 151.1556 : y = -33.8938
NODE: label=Lewisham: x = 151.1473: y = -33.8932
NODE: label=Summer Hill: x=151.1390: y=-33.8904
NODE: label = A shfield : x = 151.1254 : y = -33.8874
NODE: label=Croydon: x = 151.1159: y = -33.8836
NODE: label=Burwood: x = 151.1039: y = -33.8771
NODE: label=Strathfield: x=151.0941: y=-33.8715
NODE: label=North Strathfield: x=151.0883: y=-33.8590
NODE: label=Concord West: x = 151.0856: y = -33.8489
NODE: label=Rhodes: x = 151.0872: y = -33.8300
NODE: 1abel=Meadowbank: x = 151.0900: y = -33.8168
NODE: label=West Ryde: x = 151.0903: y = -33.8071
NODE: label=Denistone: x = 151.0872: y = -33.7998
NODE: label=Eastwood: x = 151.0821: y = -33.7901
NODE: label=Epping: x = 151.0819: y = -33.7727
NODE: label=Cheltenham : x = 151.0787 : y = -33.7557
NODE: label=Beecroft: x=151.0662: y=-33.7503
NODE: label=Pennant Hills: x = 151.0718: y = -33.7387
NODE: label=Thornleigh: x=151.0781: y=-33.7320
NODE: label=Normanhurst: x = 151.0965: y = -33.7212
NODE: label = Carlingford : x = 151.0472 : y = -33.7820
NODE: label=Telopea: x = 151.0414: y = -33.7940
NODE: label=Dundas: x = 151.0344: y = -33.8033
NODE: label=Rydalmere: x = 151.0290: y = -33.8103
NODE: label=Camellia: x = 151.0243: y = -33.8188
NODE: label=Rosehill: x=151.0219: y=-33.8236
NODE: label=Clyde: x=151.0169: y=-33.8359
NODE: label=Lidcombe: x = 151.0453: y = -33.8638
NODE: label=Auburn: x = 151.0328: y = -33.8494
```

```
NODE: label=Olympic Park: x = 151.0694: y = -33.8466
NODE: label=Flemington: x = 151.0698: y = -33.8649
NODE: label=Homebush: x = 151.0869: y = -33.8669
NODE: label=Berala: x = 151.0318: y = -33.8721
NODE: label=Regents Park: x = 151.0239: y = -33.8830
NODE: label=Granville: x=151.0126: y=-33.8331
NODE: label=Harris Park: x=151.0077: y=-33.8235
NODE: label=Parramatta: x = 151.0056: y = -33.8177
NODE: label=Westmead: x = 150.9876: y = -33.8085
NODE: label=Wentworthville: x = 150.9727: y = -33.8072
NODE: label=Pendle Hill: x=150.9564: y=-33.8014
NODE: label=Toongabbie: x = 150.9516: y = -33.7875
NODE: label=Seven Hills: x = 150.9357: y = -33.7743
NODE: label=Blacktown: x=150.9096: y=-33.7690
NODE: label=Marayong: x = 150.9002: y = -33.7467
NODE: label=Quakers Hill: x = 150.8870: y = -33.7282
NODE: label=Schofields: x=150.8698: y=-33.6968
NODE: label=Riverstone: x = 150.8605: y = -33.6796
NODE: label=Vineyard: x = 150.8513: y = -33.6508
NODE: label=Mulgrave: x = 150.8304: y = -33.6266
NODE: label=Windsor: x = 150.8112: y = -33.6138
NODE: label=Clarendon: x = 150.7880: y = -33.6088
NODE: label=East Richmond: x = 150.7590: y = -33.6016
NODE: 1abel=Richmond: x=150.7525: y=-33.5989
NODE: label=Emu Plains: x = 150.6720: y = -33.7455
NODE: label=Penrith: x = 150.6960: y = -33.7501
NODE: label=Kingswood: x = 150.7195: y = -33.7582
NODE: label=Werrington: x = 150.7574: y = -33.7591
NODE: label=St Marys: x = 150.7753: y = -33.7621
NODE: label=Mount Druitt: x = 150.8202: y = -33.7697
NODE: label=Rooty Hill: x = 150.8443: y = -33.7716
NODE: label=Doonside: x = 150.8692: y = -33.7638
NODE: label=Merrylands: x=150.9925: y=-33.8366
NODE: label=Guildford: x = 150.9845: y = -33.8543
NODE: label=Yennora: x = 150.9706: y = -33.8649
NODE: label=Fairfield: x=150.9575: y=-33.8721
NODE: label=Canley Vale: x=150.9440: y=-33.8868
NODE: label=Cabramatta: x = 150.9387: y = -33.8949
NODE: label=Warwick Farm: x = 150.9351: y = -33.9135
NODE: label=Liverpool: x=150.9268: y=-33.9253
NODE: label=Casula: x = 150.9118: y = -33.9501
NODE: label=Glenfield: x = 150.8932: y = -33.9722
NODE: label=Macquarie Fields: x=150.8787: y=-33.9852
NODE: label=Ingleburn: x=150.8648: y=-33.9974
NODE: label=Minto: x = 150.8424: y = -34.0275
NODE: label=Leumeah: x = 150.8302: y = -34.0509
NODE: label=Campbelltown: x = 150.8143: y = -34.0637
NODE: label=Macarthur: x = 150.7969: y = -34.0720
```

```
NODE: label=Carramar: x = 150.9610: y = -33.8845
NODE: label=Villawood: x=150.9760: y=-33.8809
NODE: label=Leightonfield: x = 150.9849: y = -33.8815
NODE: label=Chester Hill: x = 150.9996: y = -33.8835
NODE: label=Sefton: x = 151.0116: y = -33.8851
NODE: label=Birrong: x = 151.0239: y = -33.8932
NODE: label=Yagoona: x = 151.0242: y = -33.9068
NODE: label=Bankstown: x = 151.0342: y = -33.9179
NODE: label=Punchbowl: x = 151.0553: y = -33.9255
NODE: label=Wiley Park: x=151.0672: y=-33.9230
NODE: label=Lakemba: x = 151.0757: y = -33.9201
NODE: label=Belmore: x = 151.0886: y = -33.9172
NODE: label=Campsie : x = 151.1026 : y = -33.9105
NODE: label=Canterbury: x=151.1187: y=-33.9121
NODE: label=Hurlstone Park: x = 151.1318: y = -33.9104
NODE: label=Dulwich Hill: x = 151.1410: y = -33.9111
NODE: label=Marrickville: x = 151.1547: y = -33.9144
NODE: label=Erskineville: x=151.1855: y=-33.9001
NODE: label=St Peters: x=151.1810: y=-33.9071
NODE: label=Sydenham: x = 151.1663: y = -33.9146
NODE: label=Tempe: x = 151.1563: y = -33.9244
NODE: label=Wolli Creek: x=151.1535: y=-33.9282
NODE: label=Green Square: x = 151.2026: y = -33.9061
NODE: label=Mascot: x = 151.1875: y = -33.9228
NODE: label=Domestic Airport: x=151.1834: y=-33.9341
NODE: label=International Airport: x=151.1647: y=-33.9339
NODE: label=Holsworthy: x = 150.9574: y = -33.9632
NODE: label=East Hills: x=150.9850: y=-33.9616
NODE: label=Panania: x = 150.9982: y = -33.9544
NODE: label=Revesby: x = 151.0148: y = -33.9524
NODE: label=Padstow: x = 151.0320: y = -33.9519
NODE: label=Riverwood: x=151.0521: y=-33.9515
NODE: label=Narwee: x = 151.0701: y = -33.9475
NODE: label=Beverly Hills: x = 151.0809: y = -33.9491
NODE: label=Kingsgrove: x = 151.1007: y = -33.9406
NODE: label=Bexley North: x = 151.1138: y = -33.9375
NODE: label=Bardwell Park: x = 151.1252: y = -33.9315
NODE: label=Turrella: x = 151.1403: y = -33.9299
NODE: label = Arncliffe : x = 151.1473 : y = -33.9365
NODE: label=Banksia: x = 151.1403: y = -33.9454
NODE: label=Rockdale: x=151.1368: y=-33.9521
NODE: label=Kogarah : x = 151.1328 : y = -33.9617
NODE: label=Carlton: x = 151.1235: y = -33.9683
NODE: label=Allawah: x = 151.1145: y = -33.9697
NODE: label=Hurstville: x=151.1024: y=-33.9675
NODE: label=Penshurst: x=151.0887: y=-33.9661
NODE: label=Mortdale: x = 151.0812: y = -33.9708
NODE: label=Oatley: x = 151.0790: y = -33.9808
```

```
NODE: label=Como: x = 151.0680: y = -34.0046
NODE: label=Jannali: x=151.0647: y=-34.0171
NODE: label=Sutherland: x = 151.0572: y = -34.0319
NODE: label=Loftus: x = 151.0513: y = -34.0455
NODE: label=Engadine: x = 151.0150: y = -34.0679
NODE: label=Heathcote: x = 151.0081: y = -34.0881
NODE: label=Waterfall: x=150.9947: y=-34.1348
NODE: label=Kirrawee: x = 151.0712: y = -34.0349
NODE: label=Gymea: x = 151.0853: y = -34.0349
NODE: label=Miranda: x = 151.1022: y = -34.0363
NODE: label=Caringbah: x=151.1221: y=-34.0415
NODE: label=Woolooware: x = 151.1443: y = -34.0477
NODE: label=Cronulla: x = 151.1514: y = -34.056
EDGE: label=Eastern Suburbs and Illawarra Line: color = 7,148,255:
   adjlist=Bondi Junction, Edgecliff, Kings Cross, Martin Place
     Town Hall, Central, Redfern, Erskineville, St Peters,
   Sydenham, Tempe, Wolli Creek, Arncliffe, Banksia, Rockdale,
   Kogarah, Carlton, Allawah, Hurstville, Penshurst, Mortdale,
   Oatley, Como, Januali, Sutherland, Loftus, Engadine,
   Heathcote, Waterfall
EDGE: label=Eastern Suburbs and Illawarra Line: color = 7,148,255:
   adjlist=Sutherland, Kirrawee, Gymea, Miranda, Caringbah,
   Woolooware, Cronulla
EDGE: label=Bankstown Line: color = 255,82,0: adjlist=Town Hall,
   Wynyard, Circular Quay, St James, Museum, Central, Redfern,
   Erskineville, St Peters, Marrickville, Dulwich Hill,
   Hurlstone Park, Canterbury, Campsie, Belmore, Lakemba, Wiley
    Park, Punchbowl, Bankstown, Yagoona, Birrong, Sefton,
   Chester Hill, Leightonfield, Villawood, Carramar, Cabramatta
    , Warwick Farm, Liverpool
```

- EDGE: label=Bankstown Line: color = 255,82,0: adjlist=Birrong, Regents Park, Berala, Lidcombe
- EDGE: label=Inner West Line: color = 122,115,255: adjlist=Liverpool, Warwick Farm, Cabramatta, Carramar, Villawood, Leightonfield, Chester Hill, Sefton, Regents Park, Berala, Lidcombe, Flemington, Homebush, Strathfield, Burwood, Croydon, Ashfield, Summer Hill, Lewisham, Petersham, Stanmore, Newtown, Macdonaldtown, Redfern, Central, Town Hall, Wynyard, Circular Quay, St James, Museum
- EDGE: label=Cumberland Line: color = 168,27,255: adjlist= Campbelltown, Leumeah, Minto, Ingleburn, Macquarie Fields, Glenfield, Casula, Liverpool, Warwick Farm, Cabramatta, Canley Vale, Fairfield, Yennora, Guildford, Merrylands, Harris Park, Parramatta, Westmead, Wentworthville, Pendle Hill, Toongabbie, Seven Hills, Blacktown

EDGE: label=Airport and East Hills Line: color = 35,227,0: adjlist= Macarthur, Campbelltown, Leumeah, Minto, Ingleburn, Macquarie Fields, Glenfield, Holsworthy, East Hills, Panania, Revesby, Padstow, Riverwood, Narwee, Beverly Hills, Kingsgrove, Bexley North, Bardwell Park, Turrella, Wolli Creek, International Airport, Domestic Airport, Mascot, Green Square, Central, Museum, St James, Circular Quay, Wynyard, Town Hall

EDGE: label=South Line: color = 121,255,255: adjlist=Campbelltown, Leumeah, Minto, Ingleburn, Macquarie Fields, Glenfield, Casula, Liverpool, Warwick Farm, Cabramatta, Canley Vale, Fairfield, Yennora, Guildford, Merrylands, Granville, Clyde, Auburn, Lidcombe, Flemington, Homebush, Strathfield, Burwood, Croydon, Ashfield, Summer Hill, Lewisham, Petersham, Stanmore, Newtown, Macdonaldtown, Redfern, Central, Town Hall, Wynyard, Circular Quay, St James, Museum

EDGE: label=North Shore and Western Lines: color = 254,179,0:
 adjlist=Emu Plains, Penrith, Kingswood, Werrington, St Marys, Mount Druitt, Rooty Hill, Doonside, Blacktown, Seven Hills, Toongabbie, Pendle Hill, Wentworthville, Westmead, Parramatta, Harris Park, Granville, Clyde, Auburn, Lidcombe, Flemington, Homebush, Strathfield, Burwood, Croydon, Ashfield, Summer Hill, Lewisham, Petersham, Stanmore, Newtown, Macdonaldtown, Redfern, Central, Town Hall, Wynyard, Milsons Point, North Sydney, Waverton, Wollstonecraft, St Leonards, Artarmon, Chatswood, Roseville, Lindfield, Killara, Gordon, Pymble, Turramurra, Warrawee, Wahroonga, Waitara, Hornsby, Asquith, Mount Colah, Mount Kuring-gai, Berowra

EDGE: label=Northern Line: color = 247,0,55: adjlist=Hornsby, Normanhurst, Thornleigh, Pennant Hills, Beecroft, Cheltenham, Epping, Eastwood, Denistone, West Ryde, Meadowbank, Rhodes, Concord West, North Strathfield, Strathfield, Burwood, Croydon, Ashfield, Summer Hill, Lewisham, Petersham, Stanmore, Newtown, Macdonaldtown, Redfern, Central, Town Hall, Wynyard, Milsons Point, North Sydney

 $\begin{array}{l} {\rm EDGE: label = Carling ford \ \ \, Line: color = 0\,,11\,,198: adjlist = Clyde\,, \\ {\rm Rosehill} \;\;, \;\; {\rm Rosehill} \;\;, \;\; {\rm Rydalmere} \;\;, \;\; {\rm Dundas} \;\;, \;\; {\rm Telopea} \;\;, \;\; {\rm Carling ford} \\ {\rm EDGE: label = Olympic \ \, Park \ \, Sprint: color = 0\,,0\,,0: adjlist = Lidcombe\,, } \\ {\rm Olympic \ \, Park} \end{array}$ 

#### B.11 Toronto

NODE: label=North York Centre: x=548.3656148413377: y = 637.6924963871645692.0

```
NODE: label=Union: x=534.9854062654489: y=149.41603630634563692.0
NODE: label=St Andrew: x = 512.9701663477811: y
   =162.119309569601692.0
NODE: label=Osgoode: x = 513.6985360369094: y
   =190.22721650690892692.0
NODE: label=St Patrick: x=513.6101189821484: y
   =215.288297837154692.0
NODE: label=Wellesley: x=564.2825154560209: y
   =268.4576291498545692.0
NODE: label=Rosedale: x=549.4386125984867: y=323.007699818422692.0
NODE: label=Summerhill:x=551.3083169666285:y
   =359.92153347510896692.0
NODE: label=St Clair:x = 552.0416506817132: y
   =395.0265476351312692.0
NODE: label=Davisville: x=548.2876809114198: y
   =442.26962390056184692.0
NODE: label=Spadina: x=480.8517476767438: y
   =291.37480302812986692.0
NODE: label=Eglington: x=547.8382397801869: y
   =486.5443880694138692.0
NODE: label=Lawrence: x = 549.3916631335366: y
   =525.6352244419058692.0
NODE: label=York Mills: x=549.367640778923: y
   =566.4804252255301692.0
NODE: label=Sheppard-Yonge:x=551.284816355123:y
   =605.1201681694863692.0
NODE: label=Finch: x=550.8371768151151: y=671.0981218621517692.0
NODE: label=Bessarion: x = 673.6638726496225: y
   =637.2966514837284692.0
NODE: label=Bayview: x=619.2563527217508: y=627.9832570106405692.0
NODE: label=Leslie: x = 729.1709737664762: y = 654.8582831767869692.0
NODE: label=Don Mills: x = 779.6605791855211: y
   =658.7592630811425692.0
NODE: label=Dupont: x = 475.10963401099434: y = 326.82204897579692.0
NODE: label=St Clair West: x=420.7467101798818: y
   =393.31365872394656692.0
NODE: label=Bathurst: x=459.433874576835: y=288.4817577354929692.0
NODE: label=Christie:x=425.7317277352348:y
   =288.24399723160866692.0
NODE: label=Ossington: x=390.4904490149398: y
   =290.6641745650783692.0
NODE: label=Dufferin: x=363.5208404208381: y
   =289.70859128315396692.0
NODE: label=Lansdowne: x=337.05411555862634: y
   =295.5685832707462692.0
NODE: label=Dundas West: x = 308.63675433094295: y
   =298.63264130070627692.0
NODE: label=Keele: x=278.1608021637177: y=297.4022257844125692.0
```

```
NODE: label=High Park: x=250.89186053236813: y
   =299.3530140074223692.0
NODE: label=Runnymede: x = 218.42970985990746: y
   =299.77668153357126692.0
NODE: label=Jane: x=182.5962614468292: y=300.66328230433544692.0
NODE: label=Old\ Mill: x=147.30234879174463: y
   =299.6652910497543692.0
NODE: label=Royal York: x=110.4733633922744: y
   =298.13705994154395692.0
NODE: label=Islington:x=77.15119181226339:y
   =298.21038567972073692.0
NODE: label=Kipling: x=42.33248533112442: y=300.8690008314476692.0
NODE: label=Bloor-Yonge: x=554.2434471922602: y
   =296.0001374120451692.0
NODE: label=Bay: x=535.7131858086171: y=295.57957021216055692.0
NODE: label=St George: x=509.18001734150425: y
   =294.37532458834033692.0
NODE: label=Museum: x = 513.5099140132093: y = 266.78771450949955692.0
NODE: label=Queen's Park: x = 513.6889089330482: y
   =238.70549958688895692.0
NODE: label=College: x=559.2061257777948: y
   =238.90679448436816692.0
NODE: label=Dundas: x = 561.1316717043912: y = 213.42064344042603692.0
NODE: label=Queen: x=562.1244442947623: y=188.80487496273702692.0
NODE: label=King: x=559.1061008130134: y=164.22731135568415692.0
NODE: label=Sherbourne: x=583.1827965800443: y
   =297.069553286791692.0
NODE: label=Castle Frank: x = 615.0495542265041: y
   =293.6085436965329692.0
NODE: label=Broadview: x=650.101774858699: y
   =293.66215574191847692.0
NODE: label=Chester: x = 680.0663347230288: y
   =292.81886037740753692.0
NODE: label=Pape: x = 703.5411023844736: y = 295.4480271603864692.0
NODE: label=Donlands: x=729.7289981176939: y
   =297.93355121967716692.0
NODE: label=Greenwood: x = 753.2786480302722: y
   =297.8691433725385692.0
NODE: label=Coxwell: x = 780.1063759895965: y
   =297.20369839133514692.0
NODE: label=Woodbine: x = 812.1914533588848: y
   =305.2910061438438692.0
NODE: label=Main Street: x=838.3992982377013: v
   =320.83416566832886692.0
NODE: label=Victoria Park: x=853.3786830714891: y
   = 339.77212999281164692.0
NODE: label=Warden: x = 869.5270225475806: y = 364.1712868241165692.0
NODE: label=Kennedy: x = 875.9828865340006: y = 393.1506888781412692.0
```

```
NODE: label=Lawrence East: x=876.6193897026717: y
   =424.8275297596737692.0
NODE: label=Ellesmere: x=874.053448555486: y
   =464.2684808438154692.0
NODE: label=Midland: x = 908.6953883467754: y
   =488.48023901669956692.0
NODE: label=Scarborough Centre: x=943.8338711405282: y
   =488.5509340626954692.0
NODE: label=McCowan: x = 989.7821300689836: y = 491.5011334766272692.0
NODE: label=Eglington West: x=375.2912193114021: y
   =477.0711599507095692.0
NODE: label=Glencairn: x=341.0749312294223: y=515.79228944121692.0
NODE: label=Eginton West: x = 312.1464905532554: y
   =563.3174345442742692.0
NODE: label=Yorkdale: x = 308.43619446331184: y
   =589.8486207689948692.0
NODE: label=Wilson: x=306.82276942709404: y=624.1184864518065692.0
NODE: label=Downsview: x=305.6772253459253: y
   =655.9688768792022692.0
EDGE: label=Bloor-Darnforth: color = 0,150,35: adjlist=Kipling,
   Islington
EDGE: label=Bloor-Darnforth: color = 0,150,35: adjlist=Islington,
   Royal York
EDGE: label=Bloor-Darnforth: color = 0,150,35: adjlist=Royal York,
   Old Mill
EDGE: label=Bloor-Darnforth: color=0,150,35: adjlist=Old Mill,
EDGE: label=Bloor-Darnforth: color = 0,150,35: adjlist=Jane,
   Runnymede
EDGE: label=Bloor-Darnforth: color = 0,150,35: adjlist=Runnymede,
   High Park
EDGE: label=Bloor-Darnforth: color = 0,150,35: adjlist=High Park,
   Keele
EDGE: label=Bloor-Darnforth: color = 0,150,35: adjlist=Keele, Dundas
EDGE: label=Bloor-Darnforth: color=0,150,35: adjlist=Dundas West,
   Lansdowne
EDGE: label=Bloor-Darnforth: color = 0,150,35: adjlist=Lansdowne,
   Dufferin
EDGE: label=Bloor-Darnforth: color = 0,150,35: adjlist=Dufferin,
   Ossington
EDGE: label=Bloor-Darnforth: color = 0,150,35: adjlist=Ossington,
   Christie
EDGE: label=Bloor-Darnforth: color = 0,150,35: adjlist=Christie,
   Bathurst
EDGE: label=Bloor-Darnforth: color = 0,150,35: adjlist=Bathurst,
   Spadina
```

- $\begin{array}{l} {\rm EDGE: \, lab \, el = Bloor Darmforth: \, color = 0\,, 150\,, 35: \, adj \, lis \, t = Spadina\,, \quad St \\ {\rm George} \end{array}$
- EDGE: label=Bloor-Darnforth: color = 0,150,35: adjlist=St George, Bay
- $\begin{array}{l} \texttt{EDGE: label=Bloor-Darnforth: color=0\,,150\,,35: adjlist=Bay\,, \quad Bloor-Yonge} \end{array}$
- $$\begin{split} \texttt{EDGE: label=Bloor-Darnforth: color=0\,,150\,,35: adjlist=Bloor-Yonge\,,} \\ \texttt{Sherbourne} \end{split}$$
- $\begin{array}{l} {\rm EDGE:\ label = Bloor Darnforth:\ color = 0\,,150\,,35:\ adjlist = Sherbourne\,,} \\ {\rm Castle\ Frank} \end{array}$
- $$\label{eq:edge:edge:bound} \begin{split} \text{EDGE: label=Bloor-Darmforth: color} = 0\,, 150\,, 35\colon \text{adjlist=Broadview}\,\,, \\ \text{Chester} \end{split}$$
- EDGE: label=Bloor-Darnforth: color=0,150,35: adjlist=Chester, Pape
- $\begin{array}{l} \texttt{EDGE: label=Bloor-Darnforth: color=0\,,150\,,35: adjlist=Pape\,,} \\ \texttt{Donlands} \end{array}$
- $$\label{eq:edge:edge:edge} \begin{split} \text{EDGE: label=Bloor-Darnforth: color} = 0\,, 150\,, 35\colon \text{adjlist=Donlands}\,\,, \\ \text{Greenwood} \end{split}$$
- $\begin{array}{l} \texttt{EDGE: label=Bloor-Darnforth: color=0\,,150\,,35: adjlist=Greenwood\,,} \\ \texttt{Coxwell} \end{array}$
- $\begin{array}{l} {\rm EDGE:\ label = Bloor Darnforth:\ color = 0\,,150\,,35:\ adjlist = Coxwell\,\,,} \\ {\rm Woodbine} \end{array}$
- EDGE: label=Bloor-Darnforth: color = 0,150,35: adjlist=Woodbine, Main Street
- $\begin{array}{l} {\rm EDGE: label = Bloor Darmforth: color = 0\,, 150\,, 35: adjlist = Main \ Street \,\,,} \\ {\rm Victoria\ Park} \end{array}$
- $$\label{eq:edge:edge:edge:edge} \begin{split} \text{EDGE: label=Bloor-Darnforth: color} = 0\,, 150\,, 35\colon \text{adjlist=Warden}\,, \\ \text{Kennedy} \end{split}$$
- EDGE: label=Yonge-University-Spadina: color = 255,255,0: adjlist = Downsview, Wilson
- EDGE: label=Yonge-University-Spadina: color = 255,255,0: adjlist=Wilson, Yorkdale
- EDGE: label=Yonge-University-Spadina: color = 255,255,0: adjlist=Yorkdale, Eginton West
- EDGE: label=Yonge-University-Spadina: color = 255,255,0: adjlist = Eginton West, Glencairn
- EDGE: label=Yonge-University-Spadina: color = 255,255,0: adjlist=Glencairn, Eglington West
- EDGE: label=Yonge-University-Spadina: color = 255,255,0: adjlist = Eglington West, St Clair West
- $\begin{array}{l} {\tt EDGE: label = Yonge-University-Spadina: color = 255, 255, 0: adjlist = St} \\ {\tt Clair\ West,\ Dupont} \end{array}$
- EDGE: label=Yonge-University-Spadina: color = 255,255,0: adjlist=Dupont, Spadina

- $\begin{array}{l} {\rm EDGE: \, label = Yonge-University Spadina: \, color = 255\,, 255\,, 0: \, adjlist = } \\ {\rm Spadina\,, \ \, St \ \, George} \end{array}$
- $$\label{eq:edge} \begin{split} \text{EDGE: label=Yonge-University-Spadina: color=255,255,0: adjlist=St} \\ \text{George} \;,\;\; \text{Museum} \end{split}$$
- EDGE: label=Yonge-University-Spadina: color = 255,255,0: adjlist=Museum, Queen's Park
- EDGE: label=Yonge-University-Spadina: color = 255,255,0: adjlist=Queen's Park, St Patrick
- EDGE: label=Yonge-University-Spadina: color = 255,255,0: adjlist=St Patrick, Osgoode
- EDGE: label=Yonge-University-Spadina: color = 255,255,0: adjlist = Osgoode, St Andrew
- EDGE: label=Yonge-University-Spadina: color = 255,255,0: adjlist=St Andrew, Union
- EDGE: label=Yonge-University-Spadina: color = 255,255,0: adjlist=Union, King
- $$\label{eq:edge} \begin{split} \text{EDGE: label=Yonge-University-Spadina: color=255,255,0: adjlist=} \\ \text{King, Queen} \end{split}$$
- EDGE: label=Yonge-University-Spadina: color = 255,255,0: adjlist = Queen, Dundas
- EDGE: label=Yonge-University-Spadina: color = 255,255,0: adjlist = Dundas, College
- EDGE: label=Yonge-University-Spadina: color = 255,255,0: adjlist = College, Wellesley
- EDGE: label=Yonge-University-Spadina: color = 255,255,0: adjlist= Wellesley, Bloor-Yonge
- $$\label{eq:edge} \begin{split} \text{EDGE: label=Yonge-University-Spadina: color=255,255,0: adjlist=Bloor-Yonge}\,, & \text{Rosedale} \end{split}$$
- $$\label{eq:edge} \begin{split} \text{EDGE: label=Yonge-University-Spadina: color=255,255,0: adjlist=Rosedale}\,, & \text{Summerhill} \end{split}$$
- EDGE: label=Yonge-University-Spadina: color = 255,255,0: adjlist = Summerhill, St Clair
- EDGE: label=Yonge-University-Spadina: color = 255,255,0: adjlist=St Clair, Davisville
- EDGE: label=Yonge-University-Spadina: color = 255,255,0: adjlist=Davisville, Eglington
- EDGE: label=Yonge-University-Spadina: color = 255,255,0: adjlist = Eglington, Lawrence
- EDGE: label=Yonge-University-Spadina: color = 255,255,0: adjlist = Lawrence, York Mills
- EDGE: label=Yonge-University-Spadina: color = 255,255,0: adjlist=York Mills, Sheppard-Yonge
- EDGE: label=Yonge-University-Spadina: color = 255,255,0: adjlist=Sheppard-Yonge, North York Centre
- EDGE: label=Yonge-University-Spadina: color = 255,255,0: adjlist= North York Centre, Finch
- $\begin{array}{l} {\rm EDGE: \, label = S \, carborough \, \, RT: \, color = 0 \, , 131 \, , 212: \, adj \, list = \! \! Kennedy \, , } \\ {\rm Lawrence \, \, \, East} \end{array}$

- $\begin{tabular}{ll} EDGE: label = Scarborough & RT: color = 0\,, 131\,, 212: adjlist = Lawrence & East \\ , & Ellesmere \\ \end{tabular}$
- $\begin{array}{l} {\rm EDGE: \, lab \, el = S \, carborough \, \, RT: \, color = 0 \, , 131 \, , 212: \, adj \, lis \, t = Elles mere \, , } \\ {\rm Midland} \end{array}$
- $\begin{array}{l} {\rm EDGE: \, label = S \, carborough \, \, RT: \, color = 0 \, , 131 \, , 212: \, a \, dj \, list = Midland \, ,} \\ {\rm S \, carborough \, \, \, Centre} \end{array}$
- EDGE: label=Scarborough RT: color = 0,131,212: adjlist=Scarborough Centre, McCowan
- EDGE: label=Sheppard: color = 123,0,95: adjlist=Sheppard-Yonge, Bayview
- EDGE: label=Sheppard: color = 123,0,95: adjlist=Bayview, Bessarion
- EDGE: label=Sheppard: color = 123,0,95: adjlist=Bessarion, Leslie
- EDGE: label=Sheppard: color = 123,0,95: adjlist=Leslie, Don Mills

### B.12 Washington

```
NODE: label=Gallery Pl-Chinatown: x = -77.0214: y = 38.8983
NODE: label=White Flint:x = -77.1130:y = 39.0482
NODE: label=Arlington Cemetery: x = -77.0627: y = 38.8845
NODE: label=Pentagon: x = -77.0536: y = 38.8694
NODE: label=Virginia Sq-GMU: x = -77.1037: y = 38.8830
NODE: label=Shady Grove: x = -77.1647: y = 39.1199
NODE: label=Court House: x = -77.0835: y = 38.8915
NODE: label=Clarendon: x = -77.0964: y = 38.8861
NODE: label=Vienna/Fairfax-GMU: x = -77.2713: y = 38.8778
NODE: label=Dunn Loring-Merrifield: x = -77.2287: y = 38.8832
NODE: label=West Falls Church-VT/UVA: x = -77.1888: y = 38.9008
NODE: label=East Falls Church: x = -77.1565: y = 38.8861
NODE: label=Baliston-MU: x = -77.1111: y = 38.8825
NODE: label=Cleveland Park: x = -77.0578: y = 38.9348
NODE: label=Van Ness-UDC: x = -77.0631: y = 38.9436
NODE: label=Tenleytown-AU: x = -77.0793: y = 38.9478
NODE: label=Medical Center: x = -77.0969: y = 38.9999
NODE: label=Grosvenor-Strathmore: x = -77.1039: y = 39.0293
NODE: label=Twinbrook: x = -77.1208: y = 39.0624
NODE: label=Rockville: x = -77.1460: y = 39.0846
NODE: label=Friendship Heights: x = -77.0853: y = 38.9598
NODE: label=Bethesda: x = -77.0942: y = 38.9845
NODE: label=Metro Center: x = -77.0272: y = 38.8987
NODE: label=Judiciary Sq:x = -77.0163:y = 38.8962
NODE: label=Union Station: x = -77.0063: y = 38.8981
NODE: label=Dupont Circle: x = -77.0429: y = 38.9098
NODE: label=Woodley Park-Zoo/Adams Morgan: x = -77.0524: y = 38.9249
NODE: label=McPherson Sq: x = -77.0332: y = 38.9014
NODE: label=Farragut North: x = -77.0395: y = 38.9033
NODE: label=Farragut West: x = -77.0394: y = 38.9013
NODE: label=Foggy Bottom-GWU: x = -77.0500: y = 38.9007
```

NODE: label=Rosslyn: x = -77.0718: y = 38.8953

```
NODE: label=Rhode Island Ave: x = -76.9957: y = 38.9208
NODE: label=Brookland-CUA: x = -76.9945: y = 38.9331
NODE: label=Fort Totten: x = -77.0018: y = 38.9518
NODE: label=Mt Vernon Sq/7th St-Convention Center: x = -77.0218:y
    =38.9056
NODE: label=Shaw-Howard U: x = -77.0219: y = 38.9129
NODE: label=U St/African-American Civil War Memorial/Cardozo:x
    =-77.0284: y=38.9165
NODE: label=Columbia Heights: x = -77.0325: y = 38.9287
NODE: label=Georgia Ave-Petworth: x = -77.0245: y = 38.9361
NODE: label=Takoma: x = -77.0175: y = 38.9753
NODE: label=Silver Spring: x = -77.0267: y = 38.9895
NODE: label=Forest Glen: x = -77.0427: y = 39.0155
NODE: label=Wheaton: x = -77.0508: y = 39.0386
NODE: label=Glenmont: x = -77.0530: y = 39.0616
NODE: label=West Hyattsville: x = -76.9694: y = 38.9550
NODE: label=Prince George's Plaza: x = -76.9561: y = 38.9652
NODE: label=College Park-U of Md: x = -76.9282: y = 38.9785
NODE: label=Greenbelt: x = -76.9111: y = 39.0111
NODE: label=Federal Triangle: x = -77.0272: y = 38.8987
NODE: label=Smithsonian: x = -77.0279: y = 38.8880
NODE: label=L'Enfant Plaza: x = -77.0216: y = 38.8849
NODE: label=Archives-Navy Memorial: x = -77.0215: y = 38.8939
NODE: label=Federal Center SW: x = -77.0156: y = 38.8849
NODE: label=Capitol South: x = -77.0048: y = 38.8850
NODE: label=Eastern Market: x = -76.9952: y = 38.8841
NODE: label=Potomac Ave: x = -76.9853: y = 38.8808
NODE: label=Stadium-Armory: x = -76.9771: y = 38.8860
NODE: label=Benning Road: x = -76.9379: y = 38.8903
NODE: label=Capitol Heights: x = -76.9132: y = 38.8895
NODE: label=Minnesota Ave: x = -76.9476: y = 38.8980
NODE: label=Deanwood: x = -76.9355: y = 38.9080
NODE: label=Cheverly: x = -76.9151: y = 38.9165
NODE: label=Landover: x = -76.8900: y = 38.9339
NODE: label=New Carroliton: x = -76.8718: y = 38.9483
NODE: label=Addison Road-Seat Pleasant:x = -76.8932:y = 38.8866
NODE: label=Waterfront-SEU: x = -77.0170: y = 38.8764
NODE: label=Navy Yard: x = -77.0047: y = 38.8765
NODE: label=Anacostia : x = -76.9954 : y = 38.8620
NODE: label=Branch Ave: x = -76.9117: y = 38.8270
NODE: label=Congress Heights: x = -76.9879: y = 38.8453
NODE: label=Suitland: x = -76.9315: y = 38.8437
NODE: label=Naylor Road: x = -76.9564: y = 38.8511
NODE: label=Southern Ave: x = -76.9749: y = 38.8411
NODE: label=Pentagon City:x = -77.0590:y = 38.8629
NODE: label=Crystal City:x = -77.0503:y = 38.8577
NODE: label=Ronald Reagan Washington National Airport:x
    =-77.0436: y=38.8529
```

```
NODE: label=Braddock Road: x = -77.0533: y = 38.8142
NODE: label=Van Dorn Street: x = -77.1293: y = 38.7993
NODE: label=Franconia-Springfield: x = -77.1685: y = 38.7660
NODE: label=Eisenhower Ave: x = -77.0708: y = 38.8001
NODE: label=Huntington: x = -77.0750: y = 38.7940
NODE: label=King Street: x = -77.0609: y = 38.8064
EDGE: label=Red: color = 255,0,0: adjlist=Rockville, Shady Grove
EDGE: label=Red: color = 255,0,0: adjlist=Rockville, Twinbrook
EDGE: label=Red: color = 255,0,0: adjlist=Twinbrook, White Flint
EDGE: label=Red: color = 255,0,0: adjlist=White Flint, Grosvenor-
   Strathmore
EDGE: label=Red: color = 255,0,0: adjlist=Grosvenor-Strathmore,
   Medical Center
EDGE: label=Red: color = 255,0,0: adjlist=Medical Center, Bethesda
EDGE: label=Red: color = 255,0,0: adjlist=Bethesda, Friendship
   Heights
EDGE: label=Red: color = 255,0,0: adjlist=Friendship Heights,
   Tenleytown-AU
EDGE: label=Red: color = 255,0,0: adjlist=Tenleytown-AU, Van Ness-
EDGE: label=Red: color = 255,0,0: adjlist=Van Ness-UDC, Cleveland
EDGE: label=Red: color = 255,0,0: adjlist=Cleveland Park, Woodley
   Park-Zoo/Adams Morgan
EDGE: label=Red: color = 255,0,0: adjlist=Woodley Park-Zoo/Adams
   Morgan, Dupont Circle
EDGE: label=Red: color = 255,0,0: adjlist=Farragut North, Dupont
    Circle
EDGE: label=Red: color = 255,0,0: adjlist=Farragut North, Metro
EDGE: label=Red: color = 255,0,0: adjlist=Metro Center, Gallery Pl-
   Chinatown
EDGE: label=Red: color = 255,0,0: adjlist=Gallery Pl-Chinatown,
   Judiciary Sq
EDGE: label=Red: color = 255,0,0: adjlist=Union Station, Judiciary
EDGE: label=Red: color = 255,0,0: adjlist=Rhode Island Ave, Union
   Station
EDGE: label=Red: color = 255,0,0: adjlist=Rhode Island Ave,
   Brookland-CUA
EDGE: label=Red: color = 255,0,0: adjlist=Brookland-CUA, Fort Totten
EDGE: label=Red: color = 255,0,0: adjlist=Takoma, Fort Totten
EDGE: label=Red: color = 255,0,0: adjlist=Silver Spring, Takoma
EDGE: label=Red: color = 255,0,0: adjlist=Forest Glen, Silver Spring
EDGE: label=Red: color = 255,0,0: adjlist=Wheaton, Forest Glen
EDGE: label=Red: color = 255,0,0: adjlist=Wheaton, Glenmont
EDGE: label=Orange: color = 255,190,40: adjlist=Dunn Loring-
   Merrifield, Vienna/Fairfax-GMU
```

```
EDGE: label=Orange: color = 255,190,40: adjlist=Dunn Loring-Merrifield, West Falls Church-VT/UVA
```

EDGE: label=Orange: color = 255,190,40: adjlist=West Falls Church-VT /UVA, East Falls Church

 $\begin{array}{l} {\rm EDGE: \, lab \, el = \, Orange: \, color \, = \, 255 \, , 190 \, , 40: \, a \, dj \, lis \, t = \, East \quad Falls \quad Church \, ,} \\ {\rm Baliston \, = \! MU} \end{array}$ 

EDGE: label=Orange: color = 255,190,40: adjlist=Baliston-MU, Virginia Sq-GMU

EDGE: label=Orange: color = 255,190,40: adjlist=Virginia Sq-GMU, Clarendon

$$\label{eq:color} \begin{split} \text{EDGE: label=Orange: color=255,} 190, &40: \texttt{adjlist=Court House}\,, \\ \text{Clarendon} \end{split}$$

EDGE: label=Orange: color = 255,190,40: adjlist=Court House, Rosslyn

 $\begin{array}{l} \texttt{EDGE: label=Orange: color=255,} 190, 40: \texttt{adjlist=Rosslyn} \ , \ \ \texttt{Foggy} \\ \texttt{Bottom-GWU} \end{array}$ 

 $\begin{tabular}{l} EDGE: label=Orange: color=255, 190, 40: adjlist=Farragut West, Foggy Bottom-GWU \\ \end{tabular}$ 

 $\begin{array}{l} {\rm EDGE:\ lab\,el = Orange:\ color = 255\,, 190\,, 40:\ a\,dj\,li\,s\,t = McPherson\ Sq\,,\ Metro\ Center} \end{array}$ 

$$\label{eq:edge:edge:edge:edge} \begin{split} \text{EDGE: label=Orange: color} = &255,190,40 \text{: adjlist=Federal Triangle} \;, \\ \text{Metro Center} \end{split}$$

EDGE: label=Orange: color = 255,190,40: adjlist=Smithsonian, Federal Triangle

EDGE: label=Orange: color = 255,190,40: adjlist=L'Enfant Plaza, Smithsonian

EDGE: label=Orange: color = 255,190,40: adjlist=Federal Center SW, L'Enfant Plaza

EDGE: label=Orange: color = 255,190,40: adjlist=Federal Center SW, Capitol South

EDGE: label=Orange: color = 255,190,40: adjlist=Capitol South, Eastern Market

EDGE: label=Orange: color = 255,190,40: adjlist=Potomac Ave, Eastern Market

EDGE: label=Orange: color = 255,190,40: adjlist=Stadium-Armory, Potomac Ave

$$\label{eq:color} \begin{split} \text{EDGE: label=Orange: color} = &255,190,40: \texttt{adjlist=Minnesota} \ \ \, \text{Ave}\,, \\ \text{Stadium-Armory} \end{split}$$

EDGE: label=Orange: color = 255,190,40: adjlist=Deanwood, Minnesota Ave

EDGE: label=Orange: color = 255,190,40: adjlist=Cheverly, Deanwood

EDGE: label=Orange: color = 255,190,40: adjlist=Landover, Cheverly

EDGE: label=Orange: color = 255,190,40: adjlist=Landover, New Carroliton

- EDGE: label=Blue: color = 0,0,255: adjlist=Van Dorn Street , King Street
- EDGE: label=Blue: color = 0,0,255: adjlist=King Street , Braddock Road
- $$\label{eq:edge:edge:edge} \begin{split} \text{EDGE: label=Blue: color} = 0\,, 0\,, 255 \colon \text{adjlist=Ronald Reagan Washington} \\ \text{National Airport}\,\,,\,\,\, \text{Braddock Road} \end{split}$$
- EDGE: label=Blue: color = 0,0,255: adjlist=Crystal City , Pentagon City
- EDGE: label=Blue: color = 0,0,255: adjlist=Pentagon City, Pentagon
- EDGE: label=Blue: color = 0,0,255: adjlist=Pentagon, Arlington Cemetery
- $$\begin{split} \texttt{EDGE: label=Blue: color=0\,,0\,,255: adjlist=Arlington\ Cemetery\,\,,} \\ &Rosslyn \end{split}$$
- EDGE: label=Blue: color = 0,0,255: adjlist=Foggy Bottom-GWU, Rosslyn
- EDGE: label=Blue: color = 0,0,255: adjlist=Foggy Bottom-GWU, Farragut West
- $\begin{array}{l} {\rm EDGE: \, lab \, el = } Blue: color = 0\,, 0\,, 255: a \, dj \, lis \, t = \! McPherson \ \, Sq\,, \quad Farragut \\ West \end{array}$
- EDGE: label=Blue: color = 0,0,255: adjlist=Metro Center, McPherson Sq
- EDGE: label=Blue: color = 0,0,255: adjlist=Metro Center, Federal Triangle
- EDGE: label=Blue: color = 0,0,255: adjlist=Federal Triangle , Smithsonian
- EDGE: label=Blue: color =0 ,0 ,255: adjlist=Smithsonian , L'Enfant Plaza
- $\begin{array}{l} {\rm EDGE: \, label = B \, lue: \, color = 0 \, , 0 \, , 255: \, a \, dj \, list = \!\! L' \, Enfant \; \; Plaza \, , \; \; Federal \, \\ {\rm Center \; \; SW} \end{array}$
- $\begin{array}{l} {\rm EDGE: label = Blue: color = 0 \,, 0 \,, 255: adjlist = Capitol \ \, South \,, \ \, Federal \,\, \\ {\rm Center \ \, SW} \end{array}$
- $\begin{array}{l} {\rm EDGE: \, lab \, el = \, Blue: \, color \, = 0 \, , 0 \, , 255: \, a \, dj \, lis \, t = \, Eastern \quad Market \, , \quad Capitol \\ {\rm South} \end{array}$
- EDGE: label=Blue: color = 0,0,255: adjlist=Eastern Market , Potomac Ave
- EDGE: label=Blue: color = 0,0,255: adjlist=Potomac Ave, Stadium-Armory
- EDGE: label=Blue: color = 0,0,255: adjlist=Benning Road, Stadium-Armory
- EDGE: label=Blue: color = 0,0,255: adjlist=Capitol Heights , Benning Road
- EDGE: label=Blue: color = 0,0,255: adjlist=Addison Road-Seat Pleasant, Capitol Heights
- $$\label{eq:edge:edge:edge:edge} \begin{split} \text{EDGE: label=Yellow: color} = &255\,,255\,,0\text{: adjlist} = \\ \text{Eisenhower Ave}\,, \\ \text{Huntington} \end{split}$$

- EDGE: label=Yellow: color = 255,255,0: adjlist=Braddock Road, King Street
- EDGE: label=Yellow: color = 255,255,0: adjlist=Braddock Road, Ronald Reagan Washington National Airport
- EDGE: label=Yellow: color = 255,255,0: adjlist=Crystal City, Ronald Reagan Washington National Airport
- $\begin{array}{l} {\rm EDGE: \, lab \, el = \, Yellow: \, color \, = \, 255 \,, 255 \,, 0: \, adj \, lis \, t = \, Pentagon \,\,, \quad Pentagon \,\,, \\ {\rm City} \end{array}$
- $\begin{array}{l} {\rm EDGE: \, lab \, el = \, Yellow: \, color \, = \, 255 \,, 255 \,, 0: \, adj \, lis \, t = \, Pentagon \,\,, \ \, L' \, Enfant \,\, \\ {\rm Plaza} \end{array}$
- EDGE: label=Yellow: color = 255,255,0: adjlist=Archives-Navy Memorial, L'Enfant Plaza
- $$\begin{split} \text{EDGE: label=Yellow: color=255,255,0: adjlist=Gallery } & \text{Pl-Chinatown,} \\ & \text{Archives-Navy Memorial} \end{split}$$
- $\begin{array}{l} \hbox{EDGE: label=Yellow: color=255,255,0: adjlist=Mt\ Vernon\ Sq/7th\ St-Convention\ Center\,,\ Gallery\ Pl-Chinatown} \end{array}$
- EDGE: label=Green: color = 20,150,0: adjlist=Suitland, Branch Ave
- EDGE: label=Green: color = 20,150,0: adjlist=Suitland, Naylor Road
- EDGE: label=Green: color = 20,150,0: adjlist=Naylor Road, Southern Ave
- EDGE: label=Green: color = 20,150,0: adjlist=Southern Ave, Congress Heights
- EDGE: label=Green: color = 20,150,0: adjlist=Congress Heights, Anacostia
- $EDGE: label = Green: color = 20\,, 150\,, 0: adjlist = Anacostia\,\,,\,\,\, Navy\,\,\, Yard = 100\,, 100$
- EDGE: label=Green: color = 20,150,0: adjlist=Navy Yard, Waterfront SEU
- EDGE: label=Green: color = 20,150,0: adjlist=Waterfront-SEU, L' Enfant Plaza
- $\begin{array}{l} {\rm EDGE: \, lab \, el = Green: \, color = 20\,, 150\,, 0: \, adj \, list = \!\! L' \, Enfant \ \ \, Plaza\,,} \\ {\rm Archives Navy \ \, Memorial} \end{array}$
- $\begin{array}{l} {\rm EDGE: \, lab \, el = Green: \, color \, = \, 20 \, , 150 \, , 0: \, adj \, list = \, Gallery \quad Pl-Chinatown \, ,} \\ {\rm Mt \ \, Vernon \ \, Sq/7th \ \, St-Convention \ \, Center} \end{array}$
- EDGE: label=Green: color = 20,150,0: adjlist=Shaw-Howard U, Mt Vernon Sq/7th St-Convention Center
- $\begin{array}{l} \texttt{EDGE: label=Green: color=20\,,} 150\,, 0: \texttt{adjlist=} U \ \texttt{St/African-American} \\ \texttt{Civil War Memorial/Cardozo} \,, \ \texttt{Shaw-Howard} \,\, U \end{array}$
- EDGE: label=Green: color = 20,150,0: adjlist=Columbia Heights, U St/African-American Civil War Memorial/Cardozo
- $\begin{array}{l} {\rm EDGE: \, lab \, el = Green: \, color \, = \, 20 \, , 150 \, , 0: \, adj \, list \, = \, Georgia \ \ \, Ave-Petworth \, ,} \\ {\rm Columbia \ \, Heights} \end{array}$
- $\begin{array}{l} {\rm EDGE: \, lab \, el = Green: \, color \, = \, 20 \, , 150 \, , 0: \, a \, dj \, lis \, t \, = \, Georgia \quad Ave-Petworth \, ,} \\ {\rm Fort \ \, Totten} \end{array}$

- EDGE: label=Green: color = 20,150,0: adjlist=Fort Totten, West Hyattsville
- $$\label{local_energy} \begin{split} \text{EDGE: label=Green: color} = &20\,,\!150\,,\!0\text{: adjlist=Prince George's Plaza}\,,\\ \text{West Hyattsville} \end{split}$$
- EDGE: label=Green: color = 20,150,0: adjlist=Prince George's Plaza, College Park-U of Md
- $$\label{eq:color} \begin{split} \text{EDGE: label=Green: color} = &20\,,\!150\,,\!0\text{: adjlist} \!=\! \text{College Park-U of Md}, \\ \text{Greenbelt} \end{split}$$

# Appendix C

# R Code for Empirical Experiment Analysis

This Appendix contains the R code that was used to carry out the statistical analysis as part of our empirical experiment. The analysis is described in Section 7.4.

```
library (foreign)
library (survival)
library (session)
library (frailtypack)
# Reading the data
MapTypeP <- read.csv("map-type-p.csv", header = TRUE, sep =
   ",", quote="\"", dec="\"", fill=TRUE)
MapTypeG <- read.csv("map-type-g.csv", header = TRUE, sep =
   ",", quote="\"", dec=".", fill = TRUE)
MapTypeF <- read.csv("map-type-f.csv", header = TRUE, sep =
   ",", quote="\",", dec=".", fill = TRUE
# Merging the data in a single object and defining factors
attach (MapTypeP)
attach (MapTypeG)
attach (MapTypeF)
MetroMap <- rbind (MapTypeP, MapTypeG)
MetroMap <- rbind (MetroMap, MapTypeF)
# Location of map
MapTypeP$Map <- as.factor(MapTypeP$Map)
# Group of individuals
MapTypeP$Qgrp <- as.factor(MapTypeP$Qgrp)
# Repeated observations of individuals
MapTypeP$Login <- as.factor(MapTypeP$Login)
# Location of map
MapTypeG$Map <- as.factor(MapTypeG$Map)
# Group of individuals
```

```
MapTypeG$Qgrp <- as.factor(MapTypeG$Qgrp)
# Repeated observations of individuals
MapTypeG$Login <- as.factor(MapTypeG$Login)
# Location of map
MapTypeF$Map <- as.factor(MapTypeF$Map)
# Group of individuals
MapTypeF$Qgrp <- as.factor(MapTypeF$Qgrp)
# Repeated observations of individuals
MapTypeF$Login <- as.factor(MapTypeF$Login)
# Location of map
MetroMap$Map <- as.factor(MetroMap$Map)
# Group of individuals
MetroMap$Qgrp <- as . factor (MetroMap$Qgrp)
# Repeated observations of individuals
MetroMap$Login <- as.factor(MetroMap$Login)
# Type of map
MetroMap$MapType <- as.factor(MetroMap$MapType)
# Analysis for map type P (official published map)
MetroMap.P. Surv <- frailtyPenal(Surv(Time.s., Correct) ~ Map +
   cluster (Login), n.knots=13, kappa1=1000, data=MapTypeP,
   cross.validation=TRUE, Frail=TRUE)
print.frailtyPenal(MetroMap.P.Surv)
summary. frailty Penal (MetroMap.P. Surv, level = 0.95, len = 6)
plot (MetroMap.P. Surv, type="surv", conf=FALSE)
# Analysis for map type G (geographic map)
MetroMap.G. Surv <- frailtyPenal(Surv(Time.s., Correct) ~ Map +
   cluster (Login), n.knots=13, kappa1=1000, data=MapTypeG,
   cross.validation=TRUE, Frail=TRUE)
print.frailtyPenal(MetroMap.G.Surv)
summary. frailtyPenal (MetroMap.G. Surv, level = 0.95, len = 6)
lines (MetroMap.G. Surv, type="surv", conf=FALSE)
# Analysis for map type F (automatically-drawn map)
MetroMap.F. Surv <- frailtyPenal(Surv(Time.s., Correct) ~ Map +
   cluster (Login), n.knots=13, kappa1=1000, data=MapTypeF,
   cross.validation=TRUE, Frail=TRUE)
print.frailtyPenal(MetroMap.F.Surv)
summary. frailtyPenal (MetroMap.F. Surv, level = 0.95, len = 6)
lines (MetroMap.F. Surv, type="surv", conf=FALSE)
# Analysis with aggregated and grouped data
MetroMap.Surv <- frailtyPenal(Surv(Time.s., Correct) ~ Map +
   cluster (Login) + MapType, n.knots=10, kappa1=1000, data=
   MetroMap, cross.validation=TRUE, Frail=TRUE)
```

```
print.frailtyPenal (MetroMap.Surv)
summary.frailtyPenal (MetroMap.Surv,level = 0.95, len = 6)
plot (MetroMap.Surv, type="surv", conf=TRUE)

# Analysis with aggregated and grouped data
MetroMap.Surv <- frailtyPenal (Surv(Time.s.,Correct) ~ Map +
    cluster(Login) + MapType, n.knots=10, kappa1=1000, data=
    MetroMap, cross.validation=TRUE, Frail=TRUE)
print.frailtyPenal (MetroMap.Surv)
summary.frailtyPenal (MetroMap.Surv,level = 0.95, len = 6)
plot (MetroMap.Surv, type="surv", conf=TRUE)</pre>
```